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Evolutionary Topology Optimization for Acoustic Mufflers

Abstract: CON-2016-1639

Abstract: *This article introduces an evolutionary topology optimization approach applied to design acoustic mufflers. The main goal is to find the best configuration of barriers inside typical acoustic mufflers used in the automotive industry. The acoustic medium is governed by Helmholtz equation and rigid wall boundary conditions are introduced to represent acoustic barriers. In order to find a configuration of acoustic barriers that minimize the Transmission Loss function of the muffler an Evolutionary Structural Optimization (ESO) method will be used.*
To represent the acoustic domain inside the muffler, several boundary conditions are implemented, such as, normal imposed velocity, imposed pressure and imposed impedance as anechoic termination or infinite outlet. The continuum problem is solved in the frequency domain and it is discretized using the finite element method. Using ESO method a simple topology is reached, which maximizes the Transmission Loss function for different frequencies. The results reached are then compared with results available in the literature, considering different boundary conditions and excitation frequencies. The influence of fluid element discretization and of rejection parameters in the ESO method in the obtained topologies are presented. This work will be escalated to solve acoustic cloaking device problems.
Palavras-chave: *Topology Optimization, ESO Method,Transmission Loss, Acoustic Mufflers*

1. INTRODUCTION

In the past few decades, topology optimization has been used to improve and find the best responses in vibroacoustic systems using coupled and uncoupled formulation (Bendsoe and Sigmund, 2004; Huang and Xie, 2010). One of those systems is represented by the expansion chamber named mufflers, currently used in sound attenuation in vehicle exaust systems, that will be studied in this paper and has been studied in several publications (Munjal, 1987; Lee and Kim, 2009).

A number of authors have been devoted to study the problem of acoustic mufflers modeling and to propose different design approaches to optimize the acoustic muffler. Yedeg *et al.* (2015) muffler model has a perforated pipe connecting inlet and outlet and Lee and Jang (2012) muffler model couples flow and acoustic equations for the optimization problem. The principal design approaches used are based on Solid Isotropic Material with Penalization (SIMP) method (Dühring *et al.*, 2008; Yedeg *et al.*, 2015), or evolutionary structural optimization (ESO) method (Silva, 2007; Silva and Pavanello, 2010).

Actually the optimization of acoustic domains has been an important topic and investigations concerning noise attenuation became a major issue in the present days. In this scenario muffler type devices studies grew in importance, Barbieri and Barbieri (2005) used base shape optimization to improve Transmission Loss (TL) in a muffler, Lee and Kim (2009) used MMA to create barriers or partitions in expansion chambers maximizing transmission loss effectively reducing sound pressure levels in the outlet portion of the chamber.

In this work an evolutionary structural optimization (ESO) method is proposed to maximize TL and minimize acoustic pressure level in the outlet of a single expansion chamber of one automotive muffler.

2. ANALYTICAL AND APPROXIMATE TRANSMISSION LOSS IN A MUFFLER

An automotive exhaust system, two-dimensional muffler Fig. 1, presents a Transmission Loss (TL) function that can be analytically calculated by the following equation (Selamet and Radavich, 1997),

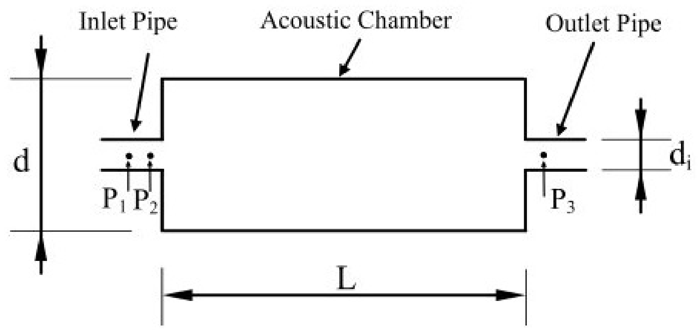


Figure 1: **Single Expansion Chamber Muffler**

$$TL = 10 \times Log_{10}\left(1 + \frac{1}{4}\left(m - \frac{1}{m}\right)^2 \cdot sin^2\big(k \cdot L\big)\right) \tag{1}$$

where,

$$m = \frac{d}{d_i}, k = \frac{2\pi f}{c} \tag{2}$$

and where d is the acoustic chamber diameter, d_i is the inlet and outlet pipes diameter, k is the wave number, f is the frequency and c is the speed of sound in the medium.

Using Eq. (1) to predict the TL function for a muffler with, $L = 0.5$m, $d = 0.15$m and $d_i = 0.03$m, considering that the domain is filled with air ($c = 343$m/s) for a frequency range, results shown in Fig. 2 are obtained.

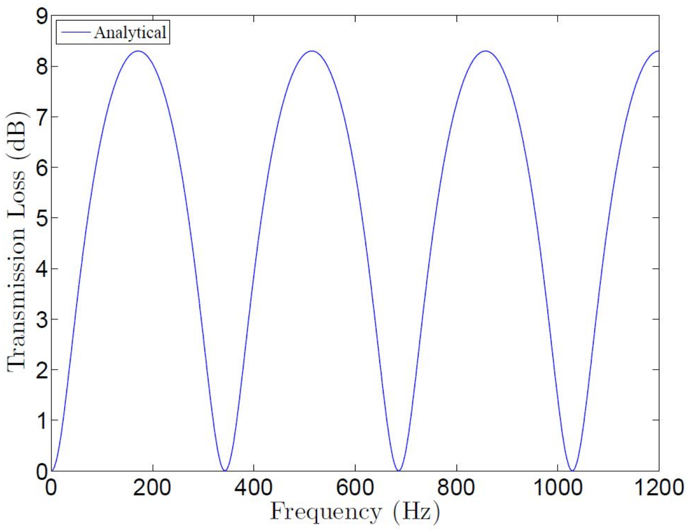


Figure 2: **TL analytical function of a single expansion chamber Eq.** (1)

The analytical model for TL function is no longer valid when rigid wall conditions are included inside the muffler acoustic chamber (Lee and Kim, 2009). In order to obtain an approximated solution the Finite Element Method is used and another equation is stated to find the TL function for an acoustic chamber with rigid barriers, Eq. (3);

$$TL = 20 \times Log_{10}\left(\left|\frac{1}{P_3}\frac{P_1 - P_2 \cdot e^{-jk \cdot l}}{1 - e^{-j2k \cdot l}}\right|\right) \tag{3}$$

where l is the distance between points 1 and 2, P_1 and P_2 are pressures in the inlet pipe and P_3 is the pressure in the outlet pipe as shown in Fig. 1. The $j = \sqrt{-1}$ is the imaginary unit.

Problem formulation and corresponding finite element discretization are discussed in the next section.

3. MUFFLER DEVICE: GOVERNING EQUATION AND FINITE ELEMENT MODEL

The governing equation of steady-state linear acoustic problems in frequency domain, valid inside the muffler acoustic chambers is the Helmholtz equation (Dühring *et al.*, 2008):

$$\nabla^2 P_{(x,y)} + \frac{\omega^2}{c^2} P_{(x,y)} = 0 \tag{4}$$

where ω is the angular frequency in (rad/s), P is the acoustic pressure and ∇^2 is the Laplacian operator. Equation (4) is valid in the acoustic domain shown in Fig. 3 in $(\Omega_d \mathrm{U} \Omega_f)$.

The weighted residual formulation for Eq. (4) can be written as;

$$\frac{1}{\rho}\int_{\Omega} \Phi \cdot \nabla^2 P d\Omega + \frac{\omega^2}{\rho c^2}\int_{\Omega} \Phi \cdot P d\Omega = 0 \tag{5}$$

where ρ is the air density, ϕ is the Weight Function and Ω_f is the fluid domain with Ω_d being a partition of it, represented in Fig. 3.

Rearranging the first term of Eq. (5) and using the Divergence Theorem, Eq. (5) can be written as:

$$\frac{1}{\rho}\int_{\Gamma} \Phi \cdot \nabla P \cdot \vec{n} d\Gamma - \frac{1}{\rho}\int_{\Omega} \nabla\Phi \cdot \nabla P d\Omega + \frac{\omega^2}{\rho c^2}\int_{\Omega} \Phi \cdot P d\Omega = 0 \tag{6}$$

Since,

$$\nabla P \cdot \vec{n} = \frac{\partial P}{\partial n} \tag{7}$$

Eq. (6) can be rearranged as:

$$\frac{1}{\rho}\int_{\Omega} \nabla\Phi \cdot \nabla P d\Omega - \frac{\omega^2}{\rho c^2}\int_{\Omega} \Phi \cdot P d\Omega = \frac{1}{\rho}\int_{\Gamma} \Phi \cdot \frac{\partial P}{\partial n} d\Gamma \tag{8}$$

where Γ is the boundary of the acoustic domain.

For the problem stated in this paper two boundary conditions will be introduced, see Fig. 3, Ω_f is the full domain filled with fluid and Ω_d represents part of the domain where partitions will be introduced. The Ω_d region is the design domain for the evolutionary optimization method. Two boundary conditions are imposed: Normal velocity V_n in boundary Γ_i, expressed in Eq. (9), and normal impedance $\bar{Z}$ in boundary Γ_o, expressed in Eq. (10).

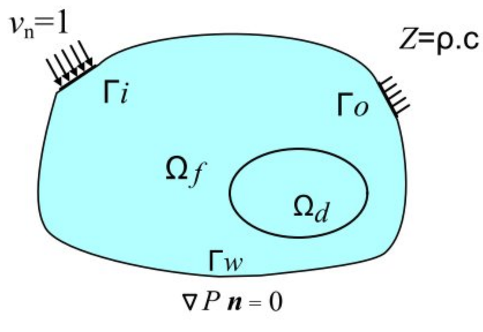


Figure 3: **Acoustic domain and Boundary conditions**

$$V_n = -\frac{1}{j\rho\omega}\frac{\partial P}{\partial n} \tag{9}$$

$$P = \bar{Z} V_n = -\frac{\bar{Z}}{j\rho\omega}\frac{\partial P}{\partial n} = -\frac{1}{j\rho\omega\bar{A}}\frac{\partial P}{\partial n} \tag{10}$$

where V_n is a vector of imposed velocities in Γ_i and $\bar{A}$ is acoustic admittance in Γ_o. To completely represent the whole boundary boundary problem rigid wall natural conditions $(\nabla P n = 0)$ are imposed in Γ_w.

Applying Eq. (9) and Eq. (10) in Eq. (8) we have

$$\frac{1}{\rho}\int_{\Omega}\nabla\Phi\cdot\nabla P d\Omega-\frac{\omega^2}{\rho c^2}\int_{\Omega}\Phi\cdot P d\Omega=-\frac{1}{\rho}\left(\int_{\Gamma_i} j\rho\omega\Phi V_n d\Gamma_i+\int_{\Gamma_o} j\rho\omega\Phi\bar{A}P d\Gamma_o\right) \tag{11}$$

Using the finite element method is possible to approximate Eq. (11). Using Galerkin's method the system response equation for the stated problem is:

$$\left(\left[\mathbf{K}\right]+j\omega\left[\mathbf{C}\right]-\omega^2\left[\mathbf{M}\right]\right)\cdot\left\{\mathbf{P}\right\}=\left\{\mathbf{V_{ni}}\right\} \tag{12}$$

where,

$$\left[\mathbf{K}\right]=\int_{\Omega}\left[\mathbf{B}\right]^t\left[\mathbf{B}\right]d\Omega \tag{13}$$

$$\left[\mathbf{C}\right]=\rho\bar{A}\int_{\Gamma_o}\left[\mathbf{N}\right]^t\left[\mathbf{N}\right]d\Gamma_o \tag{14}$$

$$\left[\mathbf{M}\right]=\frac{1}{c^2}\int_{\Omega}\left[\mathbf{N}\right]^t\left[\mathbf{N}\right]d\Omega \tag{15}$$

$$\left\{\mathbf{V_{ni}}\right\}=-j\rho\omega\int_{\Gamma_i}\left[\mathbf{N}\right]^t V_n d\Gamma_i \tag{16}$$

and $[\mathbf{N}]$ is the vector containing the shape functions for the acoustic element discretization with $[\mathbf{B}]$ being its derivatives (Desmet and Vandepitte, 2005), $[\mathbf{K}]$ is the acoustic stiffness matrix, $[\mathbf{C}]$ is the equivalent damping matrix relative to the output boundary condition, $[\mathbf{M}]$ is the mass matrix and $[\mathbf{V_{ni}}]$ is the vector relative to inlet conditions.

In this work, linear 4 node elements are used. Figure 4 shows the mesh used.

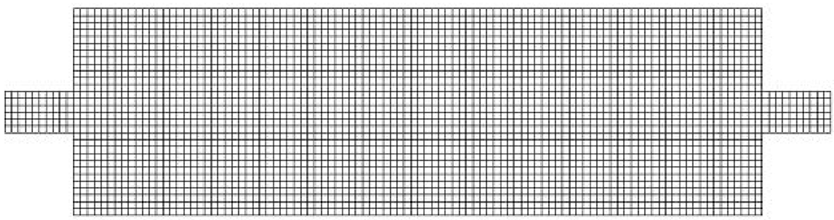


Figure 4: **Muffler Mesh with 3120 elements measuring 0.005m.**

Equation (12) can be used to verify the TL behavior for our domain previous to the addition of partitions, using the boundary conditions presented in Fig. 3, ($V_n = 1$) in the entire inlet and ($Z = \rho c$) in the entire outlet. Figure 5 shows a comparison of results achieved with Eq. (1) and results from the formulation developed in this section.

4. EVOLUTIONARY ACOUSTIC OPTIMIZATION PROBLEM FOR TL MAXIMIZATION:

In this section the evolutionary acoustic optimization problem is stated and the sensitivity analysis is derived. In this work an Evolutionary Topology optimization strategy is adopted, considering discrete values 1 or 0 which correspond to acoustic and void elements respectively, in a discretized form of the equation system, this approach is called "hard kill" in the literature. This method avoids intermediate density elements during the optimization procedure Huang and Xie (2010).

4.1 Problem statement

The objective is to maximize the amplitude of the TL function for a predefined frequency. The desing variable x_i represents the presence ($x_i = 1$) or absence ($x_i = 0$) of our particular fluid element. When the fluid element is removed, a rigid wall boundary condition is naturally imposed on the system, representing the local inclusion of a rigid barrier.

Considering volume constraints for barriers inclusion, the evolutionary optimization problem can be stated as:

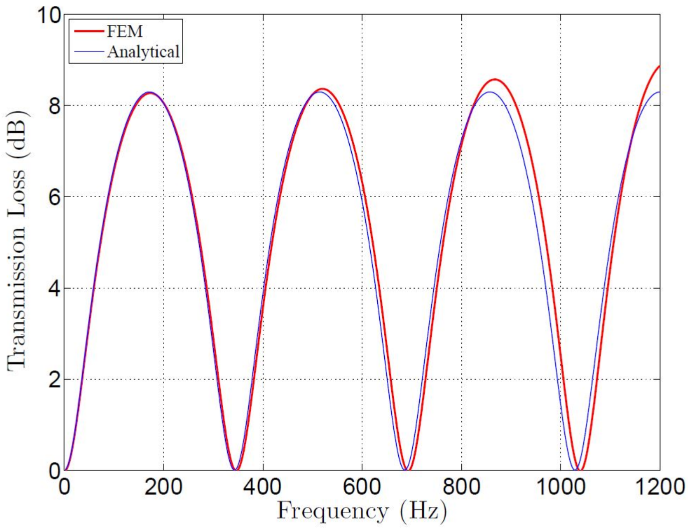


Figure 5: **Comparison between analytical TL and FEM approximated TL.**

Maximize $\mathbf{TL} = 20 \times Log_{10}\left(\left|\frac{1}{P_3}\frac{P_1 - P_2 \cdot e^{-jk \cdot l}}{1 - e^{-j2k \cdot l}}\right|\right)$

Subject to: $\left(\left[\mathbf{K}\right] + j\omega\left[\mathbf{C}\right] - \omega^2\left[\mathbf{M}\right]\right) \cdot \left\{\mathbf{P}\right\} = \left\{\mathbf{V_{ni}}\right\}$

$V_f - \sum_{i=1}^{nel} v_i x_i = 0$

$x_i = 0$ or 1

In the ESO method a gradient-based optimizer is necessary in order to evaluate the relevance of each element after the model analysis. This optimizer is generally called sensitivity and its calculation is described in the next section.

4.2 Transmission Loss Sensitivity Analysis

The sensitivity number is the objective function derivative with respect to the design variable, since transmission loss can be written as,

$$TL(\omega, x_i) = 10 \times log_{10}\left(\frac{\mid Pin \mid^2}{\mid Pout \mid^2}\right) \tag{17}$$

the derivative is,

$$\frac{\partial TL(\omega, x_i)}{\partial x_i} = \frac{10}{ln10} \times \left(\frac{\partial \mid P_{in} \mid^2}{\partial x_i} \cdot \frac{1}{\mid P_{in} \mid^2} - \frac{\partial \mid P_{out} \mid^2}{\partial x_i} \cdot \frac{1}{\mid P_{out} \mid^2}\right) \tag{18}$$

where P_{in} is,

$$P_{in} = \mid \left(\frac{P_1 - P_2 \cdot e^{-jk \cdot l}}{1 - e^{-j2k \cdot l}}\right) \mid \tag{19}$$

and P_{out} is,

$$P_{out} = \mid P_3 \mid \tag{20}$$

Since P_{in} and P_{out} in Eq. (19) and Eq. (20) are complex numbers, they can be expanded as follows:

$$P_{in}^2 = \frac{1}{\gamma}\big(P_{1Re} - P_{2Re} \cdot cos(k \cdot l) - P_{2Im} \cdot sin(k \cdot l)\big)^2 + \frac{1}{\gamma}\big(P_{1Im} - P_{2Im} \cdot cos(k \cdot l) + P_{2Re} \cdot sin(k \cdot l)\big)^2 \tag{21}$$

$$P_{out}^2 = P_{3Re}^2 + P_{3Im}^2 \tag{22}$$

with γ defined as,

$$\gamma = \big(1 - cos\big(2k \cdot l\big)\big)^2 + \big(sin\big(2k \cdot l\big)\big)^2 \tag{23}$$

where subscripts (Re) and (Im) mean real and imaginary parts respectively.

To calculate Eq. (21) and Eq. (22) derivatives for sensitivity analysis, the differentiations of acoustic pressure in the three points, P_1, P_2 and P_3 are necessary. Those can be found by differentiating the governing equation system Eq. (24) as:

$$\left\{\frac{d\mathbf{P}}{dx_i}\right\} = -\left(\left[\mathbf{K}\right] + j\omega\left[\mathbf{C}\right] - \omega^2\left[\mathbf{M}\right]\right)^{-1} \cdot \frac{d}{dx_i}\left(\left[\mathbf{K}\right] + j\omega\left[\mathbf{C}\right] - \omega^2\left[\mathbf{M}\right]\right) \cdot \left\{\mathbf{P}\right\} \tag{24}$$

where $\left\{\mathbf{V_{ni}}\right\}$ is constant with respect to x_i.

According to Chu *et al.* (1996), Vicente *et al.* (2015) and Vicente *et al.* (2016) in order to determine the change of response in especific degrees of freedomP_j of the system due to the i_{th} element removal, a vector F_j can be introduced. F_j is composed of zeros and a single one for the specific degree of freedom.

Thus we can write the following equations:

$$\frac{dP_j}{dx_i} = \mathbf{F}_j^t \cdot \frac{dP}{dx_i} \tag{25}$$

$$Z = \left(\left[\mathbf{K}\right] + j\omega\left[\mathbf{C}\right] - \omega^2\left[\mathbf{M}\right]\right) \tag{26}$$

Substituting Eq. (24) in Eq. (25):

$$\frac{dP_j}{dx_i} = F_j^t \cdot Z^{-1} \cdot \frac{dZ}{dx_i} \cdot \left\{P\right\} \tag{27}$$

Since the systems response where only F_j acts can be written as,

$$P_j = Z^{-1} \cdot F_j \tag{28}$$

Eq. (27) becomes:

$$\frac{dP_j}{dx_i} = -P_j^t \cdot \frac{dZ}{dx_i} \cdot \left\{P\right\} \tag{29}$$

where $\frac{dZ}{dx_i}$ is the response matrix of the i_{th} element, and can be calculated in local form, element by element.

Equation (29) aproximates $\frac{dP_i}{dx_i}$ avoiding huge computational costs and enabling $\frac{dTL}{dx_i}$ evaluation.

5. NUMERICAL RESULTS

The objective of this section is to validate the formulation and implementation of the optimization method, finite element model and TL evaluation, the numerical results presented by Lee and Kim (2009) are used for comparison.

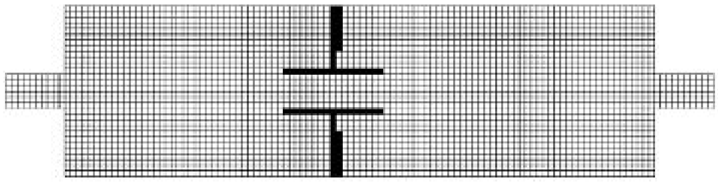
The proposed example is represented in Fig. 1, with the configuration presented in section 2. It is a bi-dimensional representative acoustic model, for a simple automotive muffler. Table 1 shows ESO parameters used to optimize the system.

Table 1: **ESO Parameters.**

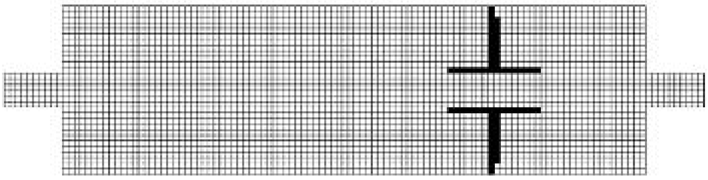
Evolutionary Ratio (ER)	0.166%
Final Volume (V_f)	97 %

The topology optimization strategy proposed in this article is applied for four different frequencies, those are relative to peak and valley values in Fig. 5.

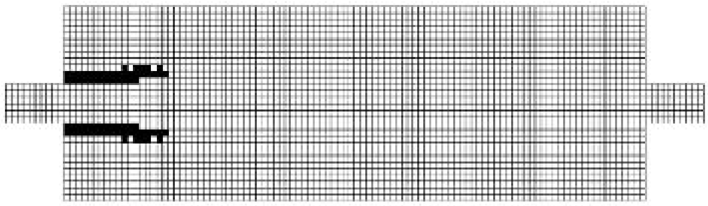
It is possible to see that the final topologies produced in this work, shown in Fig. 6, are similar to the ones presented by Lee and Kim (2009). Figure 7 shows the evolution of the Transmission loss function amplitude through the optimization process for ($\omega = 519.5$ Hz).



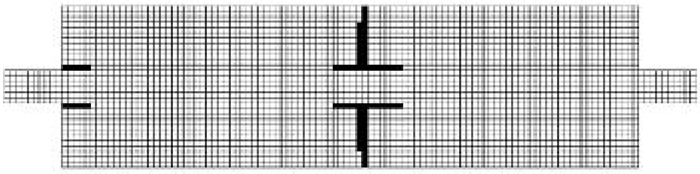
(a)



(b)



(c)



(d)

Figure 6: **Final topologies using ESO method ; (a): topology for a frequency of 346 Hz; (b): topology for a frequency of 693 Hz; (c): topology for a frequency of 173 Hz; (d): topology for a frequency of 519.5 Hz.**

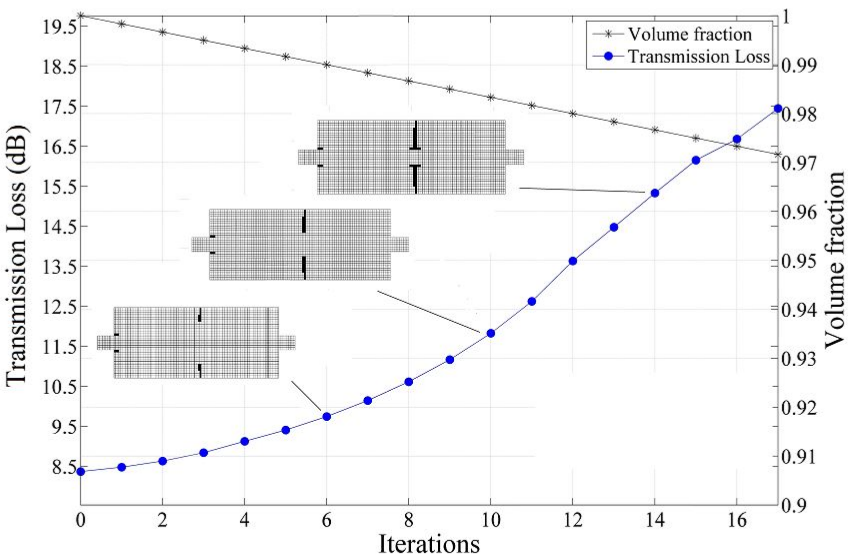
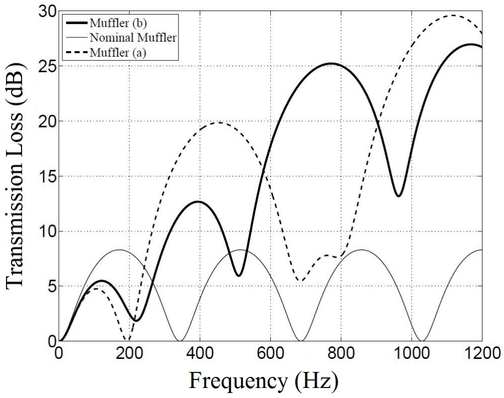
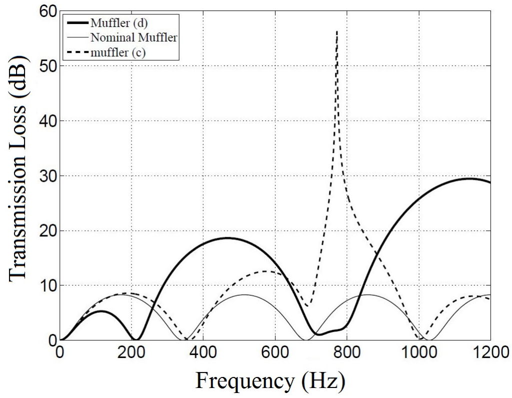


Figure 7: **TL evolution for 520 Hz example**

At the end of the optimization the maximum TL obtained for 519.5 Hz was 18 dB. Figure 8 shows TL response for a range of frequencies for mufflers presented in Fig. 6.



(a)



(b)

Figure 8: **(a) TL responses for Mufflers (a) and (b); (b)TL responses for Mufflers (c) and (d)**

According to the results presented here the ESO approach was validated. One of the reasons results displayed in this paper do not agree completely with other publications is different methods being used, Lee and Kim (2009) used SIMP method to find the barrier layout, and different numerical parameters, in this work an ESO approach was used, an approach prior to Bi-directional Evolutionary Structural Optimization (BESO) method.

6. CONCLUSION

The main objective of this work is to propose an evolutionary methodology for acoustic otimization of automotive mufflers. A sensitivity analysis of the optimization problem has been presented considering an evolutionary approach. In order to maximize the Tranmission Loss, the optimiztion process has added attenuation barriers in the expansion chamber interior. For different frequencies, the results presented show the capability of the proposed methodology to maximize noise attenuation in the studied system. As a further work, the Bi-directional evolutionary structure optimization should be implemented for a more detailed analysis of the muffler optimization and different acoustic problems, for example, cloaking devices design.

7. ACKNOWLEDGMENTS

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9. AUTHORAL RESPONSIBILITIES

The authors are solely responsible for the content of this work.