CONEM 2016 CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
ABCM 40 anos Associação Brasileira de Engenharia e Ciências Mecânicas
21 - 25 AGOSTO DE 2016 FORTALEZA - CEARÁ

ATTITUDE ESTIMATION ANALYSIS OBTAINED FROM MEMs AND INERTIAL CLASS GYROS USING QUATERNIONS AND EULER ANGLES APPROACH

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Abstract: *The direction which a vehicle is pointing at can be represented in the Cartesian coordinates system and is also known as vehicle attitude. The attitude parameters depend on angular velocities and the inertial sensor that measures the vehicle's angular velocity is the gyro. Attitude parameters, estimated in Euler angles and quaternions provide these Cartesian coordinates and are going to be discussed in this paper. The accuracy of both estimations is campared. The attitude parameters are provided by two inertial sensors (MEMS-based and velocity sensor gyros) calculated in a real time computer installed in a motion simulating table.*

Keywords: *Attitude, Quaternions, Euler Angles, MEMS-based gyro*

1. INTRODUCTION

The vehicle's attitude can be represented by three mutually perpendicular unit vectors, Fig. 1, called a coordinate frame (A'). We also consider another coordinate frame as a reference (A). So, the relationship between the vehicle's frame (A') and the reference frame (A) can be defined by Eq (1):

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} i' \cdot i & i' \cdot j & i' \cdot k \\ j' \cdot i & j' \cdot j & j' \cdot k \\ k' \cdot i & k' \cdot j & k' \cdot k \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \tag{1}$$

The inner matrix represents a scalar product between the frame vectors and is known as Rotation Matrix or Direction Cosine Matrix, C.

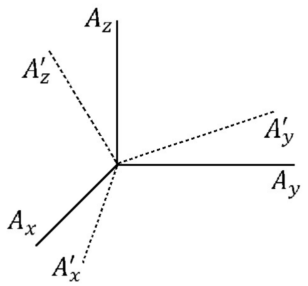

Figure 1. Vehicle's attitude frame representation in a reference attitude frame

We can rewrite the Rotation Matrix utilizing attitude parameters, by quaternions the Rotation Matrix is presented in Eq.(2).

$$C = \begin{pmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{pmatrix} \tag{2}$$

For Euler angles we have chosen θ - ψ - ϕ rotation sequence, then the Rotation matrix is represented in Eq.(3).

$$C = \begin{pmatrix} \cos\psi\cos\theta & \sin\phi & -\cos\psi\sin\theta \\ -\sin\psi\cos\theta\cos\phi + \sin\theta\sin\phi & \cos\psi\cos\phi & \sin\psi\sin\theta\cos\phi + \cos\theta\sin\phi \\ -\sin\psi\cos\theta\sin\phi + \sin\theta\cos\phi & -\cos\psi\sin\phi & \sin\psi\sin\theta\sin\phi + \cos\theta\cos\phi \end{pmatrix} \tag{3}$$

1.1. Attitude Kinematics

When the vehicle's attitude is changing with time, the rotation matrix is time dependent, C(t). In order to represent the evolution of vehicle's attitude, we have to acquire the angular velocities for each axis in every time step. We can use any attitude representation, in this paper we are comparing quaternions and Euler angles. If the computational system is fast enough to produce small time steps, we will obtain the attitude parameters by adding these small parameters. This mathematic procedure is the same as adding very small rotations discussed above.

The quaternions for each time step is calculated as follows in the Eq.(4):

$$\begin{pmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \\ \dot{q_4} \end{pmatrix} = \begin{pmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_z & -\omega_y & 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} \tag{4}$$

The Euler angles for each time step, with our sequence of rotation, are presented in the Eq.(5).

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & -\tan\psi\cos\phi & \tan\psi\sin\phi \\ 0 & \cos\phi\,/\cos\psi & -\sin\phi\,/\cos\psi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \tag{5}$$

The evolution of attitude parameters can be calculated by the integration of the infinitesimal parameters obtained above.

An important procedure that is required is the forced normalization of the quaternions parameters. This is necessary to ensure that the properties of the calculated quaternions are valid.

2. DATA ACQUISITION

The National Instrument compactRIO NI cRIO-9014 is the real time controller utilized. It uses a FPGA module cRIO-9113 for the communication with the MEMs gyro. This controller enables us to acquire the angular velocities at 100Hz sample rate with determinism.

The computational system diagram is depicted in Fig. 2.

The angular velocities provided by the MEMs gyros (IMU) are acquired in FPGA VI. It has a sample rate of 1ms. This VI communicates with RT VI by DMA (Direct Memory Access) and with MEMs gyros by serial port at a bit rate of 921600 bits/s. Every 10ms the RT VI reads the angular velocities in the FPGA VI.

The velocity sensor provides the angular velocities by an analogical output, these values have a sample rate of 10ms.

As the RT VI runs in a timed loop of 100Hz, it also estimates the attitude parameters by the implementation of the previously discussed equations (4) and (5) every 10ms. The angular velocities, the Euler angles, the quaternions and time step are stored in the internal memory of the CRIO controller. A host computer runs a GUI (Graphical user interface) showing the instantaneous values and commands of the system. This communication is through a WI-FI (TCP/IP) communication.

This set-up is installed in a motion simulating table, which is also commanded by the real-time controller by its analogical inputs. Figure 3 presents a picture of the set-up.

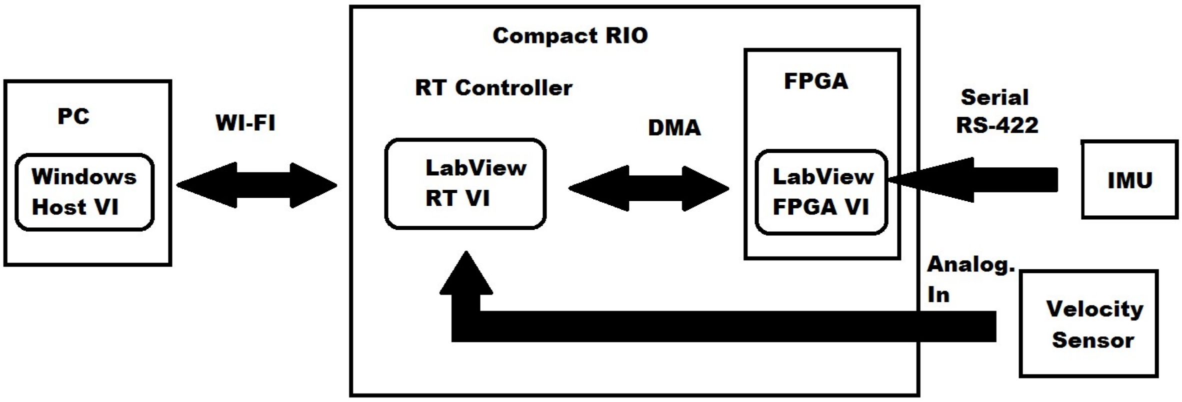

Figure 2. Computational System diagram

Figure 3. Test set-up

2.1. Angular Velocity sensors

2.1.1. IMU MEMs gyro

STIM300 is an IMU consisting of 3 high accuracy MEMS-based gyros, 3 high stability accelerometers and 3 high stability inclinometers in a miniature package. Each axis is factory-calibrated for bias, sensitivity and compensated for temperature effects to provide high-accuracy measurements in the temperature range -40°C to +85°C, (Sensonor, 2013).

Specifications: Full Scale: ±400º/s; Resolution: 0.22º/h; Scale Factor Accuracy: ±500ppm; Sample Rate: 1000 samples/s; Bias Range: ±250º/h; Bias error over static temperature: 5º/h; Bias Instability: 0.5º/h; Angular Random Walk: 0.15º/√h; Misalignment: 1mrad.

2.1.2. Velocity Sensor Unit

The sensor unit consists of two angular velocity sensors (each has two sensitive axes), a servo amplifier, an internal power supply, motor power supply and a temperature sensor.

Specifications: Full Scale: Axes Y, Z = ±20º/s, Axis X = ±40º/s; Bias independent on acceleration: 0.1º/s; Bias proportional to acceleration: 0.1º/s; Misalignment: 3º.

3. RESULTS

The objective of this study was to verify if quaternions could produce an attitude as reliable as Euler angles. We have commanded the motion simulating table to perform a rotation of 90º and return to 0º around X axis during 58.50s. The Euler angle's time histories are represented in Fig 4. The quaternion's time histories are represented in Fig. 5. The final attitude parameters are presented in Tab. 1 produced after 58.50s.

Table 1. Attitude parameters after 58.50s

Euler angles		Quaternions	
MEMs gyro	Velocity sensor	MEMs gyro	Velocity sensor
ϕ = -1.7911º	ϕ = -1.2663º	q1 = -0.0154	q1 = -0.0121
θ = 1.9539º	θ = 3.9982º	q2 = 0.0168	q2 = 0.0352
ψ = 1.4586º	ψ = -3.4817º	q3 = 0.0130	q3 = -0.0300
		q4 = 0.9997	q4 = 0.9989

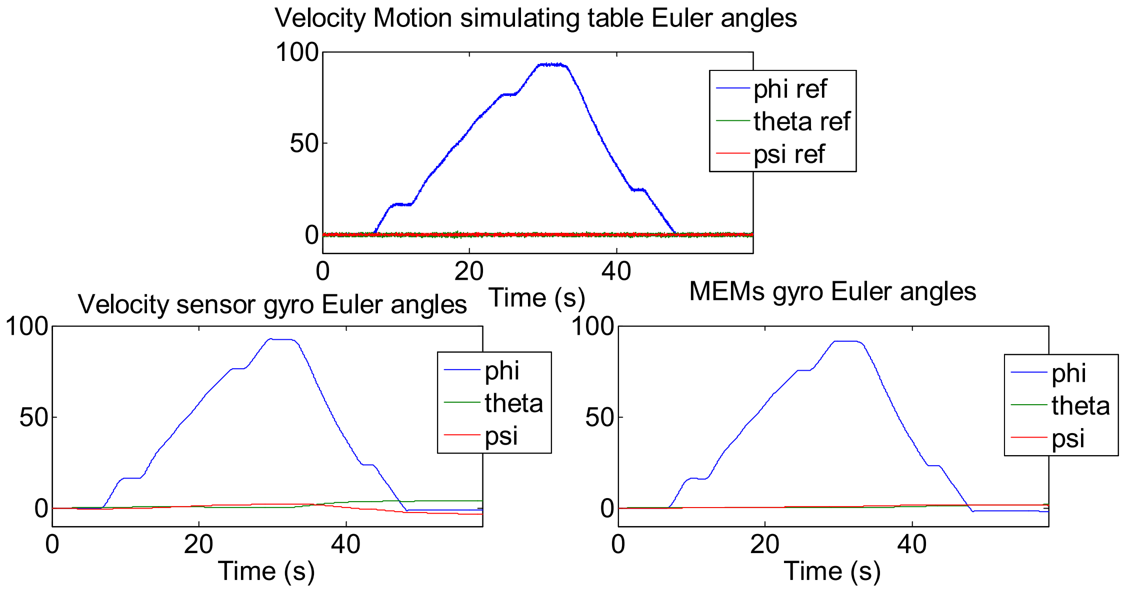

Figure 4. Euler Angles time histories

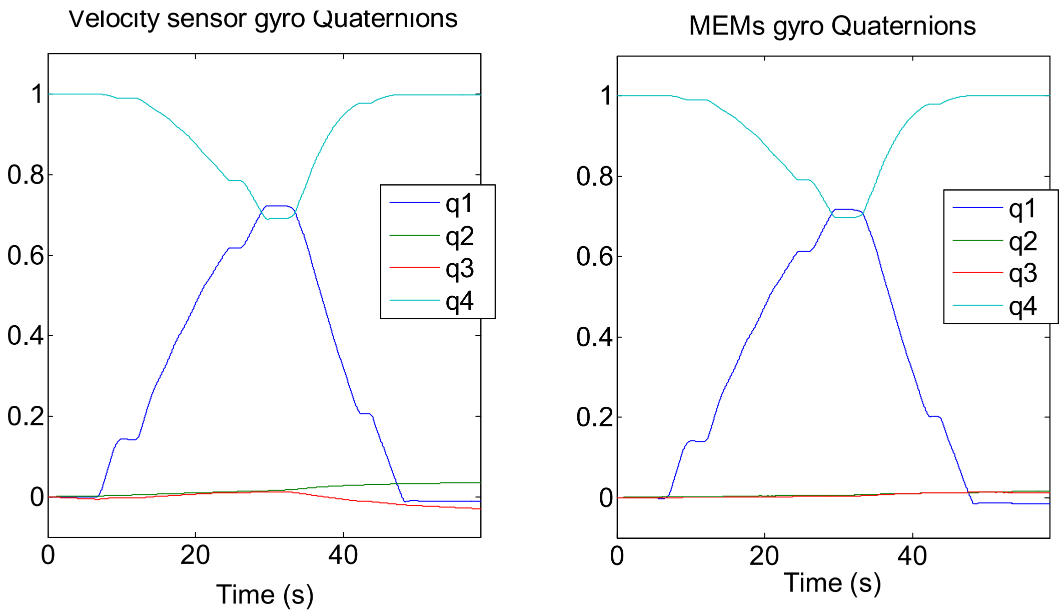

Figure 5. Quaternion time histories

The rotation matrix “Cea” provided by the final Euler angles and the rotation matrix “Cq” provided by the final quaternions produced through the velocity sensors are, presented in Fig. 6,

$$Cea = \begin{pmatrix} 0.9957 & -0.0607 & -0.0696 \\ 0.0590 & 0.9979 & -0.0263 \\ 0.0710 & 0.0221 & 0.9972 \end{pmatrix} \qquad Cq = \begin{pmatrix} 0.9957 & -0.0608 & -0.0695 \\ 0.0591 & 0.9979 & -0.0263 \\ 0.0710 & 0.0221 & 0.9972 \end{pmatrix}$$

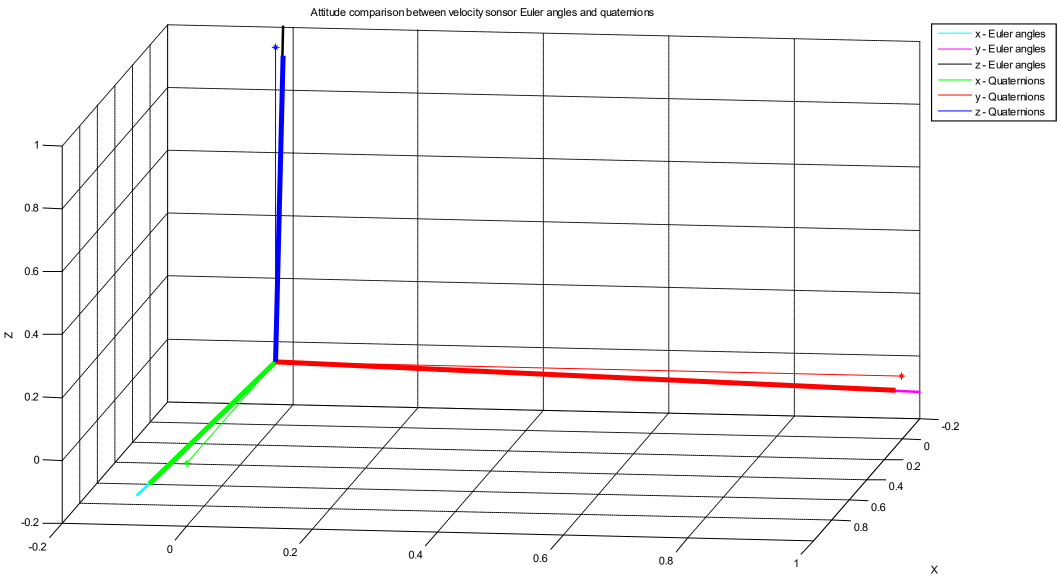

Figure 6. Attitude comparison between velocity sensor Euler angles and Quaternions

The rotation matrix “Cea” provided by the final Euler angles and the rotation matrix “Cq” provided by the final quaternions through MEMs gyro are, presented in Fig. 7,

$$Cea = \begin{pmatrix} 0.9991 & 0.0255 & -0.0341 \\ -0.0265 & 0.9992 & -0.0304 \\ 0.0333 & 0.0312 & 0.9990 \end{pmatrix} \qquad Cq = \begin{pmatrix} 0.9991 & 0.0254 & -0.0341 \\ -0.0265 & 0.9992 & -0.0304 \\ 0.0333 & 0.0312 & 0.9990 \end{pmatrix}$$

It is important to note that, at the final attitude, there is an attitude error in comparison to the reference motion simulating table attitude. But, the generated Rotation matrices are almost the same. Figures 6 and 7 presents the Cartesian coordinates generated by the Rotation matrix created by Euler angles “Cea” and created by Quaternions “Cq”.

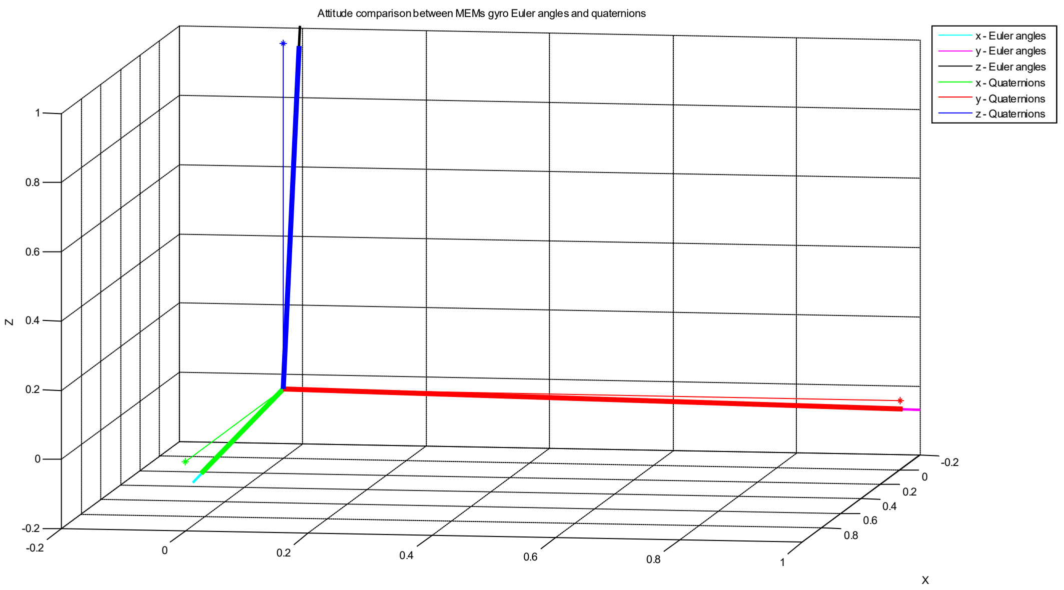

Figure 7. Attitude comparison between MEMs gyro Euler angles and Quaternions

A second contribution of this study is to demonstrate that there is no singularity when we use Quaternions as attitude parameters. We have commanded the motion simulating table to perform a single rotation near 90º around Z axis during 25s. With the rotation sequence selected in the Euler angles representation, θ - ψ - ϕ, singularity occurs in ψ. This can be seen in Eq. (5). The Euler angle's time histories are represented in Fig 8. The quaternion's time histories are represented in Fig. 9. The final attitude parameters are presented in Tab. 2 produced after 25s.

Table 2. Attitude parameters after 25s

Euler angles		Quaternions	
MEMs gyro	Velocity sensor	MEMs gyro	Velocity sensor
ϕ = -69.9688º	ϕ = 894.9837º	q1 = -0.0090	q1 = -0.0005
θ = 70.0391º	θ = -894.7538º	q2 = 0.0099	q2 = 0.0033
ψ = 88.6025º	ψ = 92.1532º	q3 = 0.7042	q3 = 0.6936
		q4 = 0.7098	q4 = 0.7203

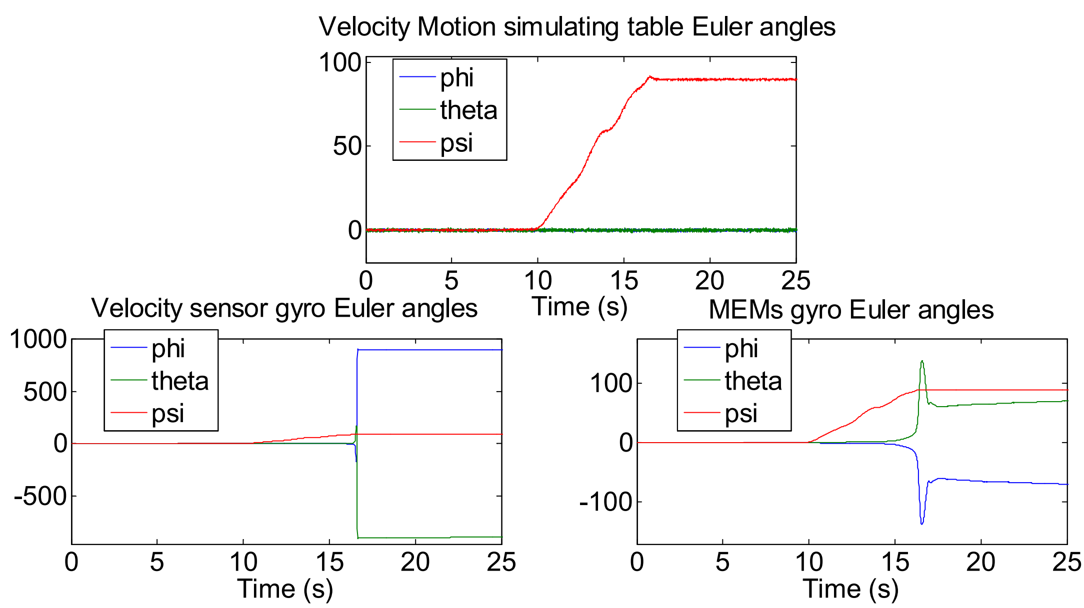

Figure 8. Euler Angles time histories

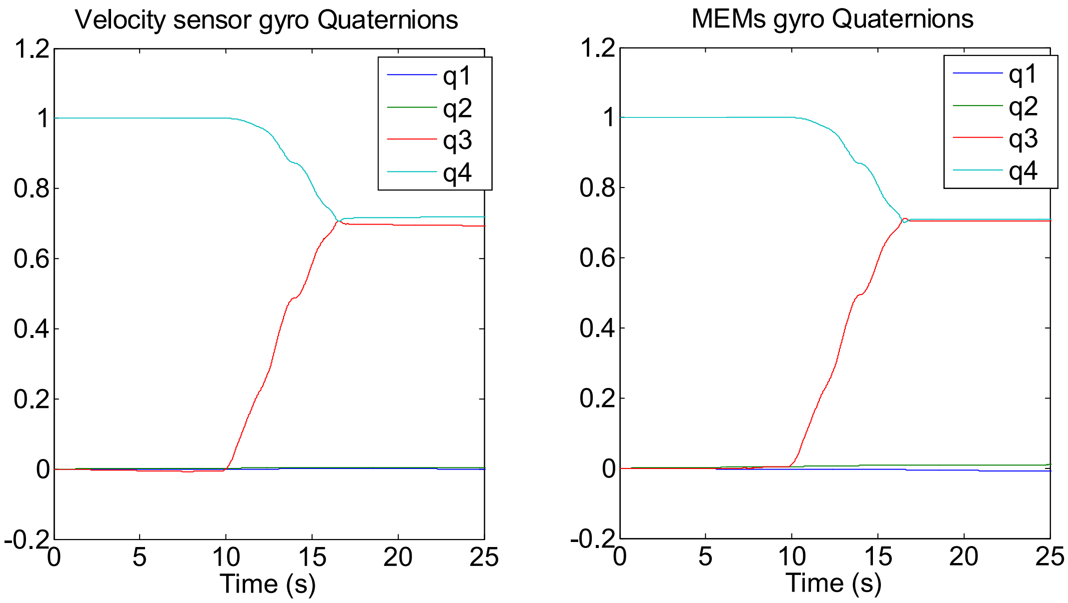

Figure 9. Quaternion time histories

When ψ is near to 90º we can see that Euler angles become instable, while Quaternions have a better behavior. This is an advantage, using quaternions as attitude parameters to control a vehicle.

The rotation matrix “Cea2” provided by the final Euler angles and the rotation matrix “Cq2” provided by the final quaternions produced through the velocity sensors are, presented in Fig. 10,

$$Cea2 = \begin{pmatrix} 0.0374 & 0.9993 & -0.0034 \\ -0.9993 & 0.0374 & 0.0039 \\ 0.0041 & 0.0033 & 1.0000 \end{pmatrix} \quad Cq2 = \begin{pmatrix} 0.0378 & 0.9993 & -0.0055 \\ -0.9993 & 0.0378 & 0.0039 \\ 0.0041 & 0.0054 & 1.0000 \end{pmatrix}$$

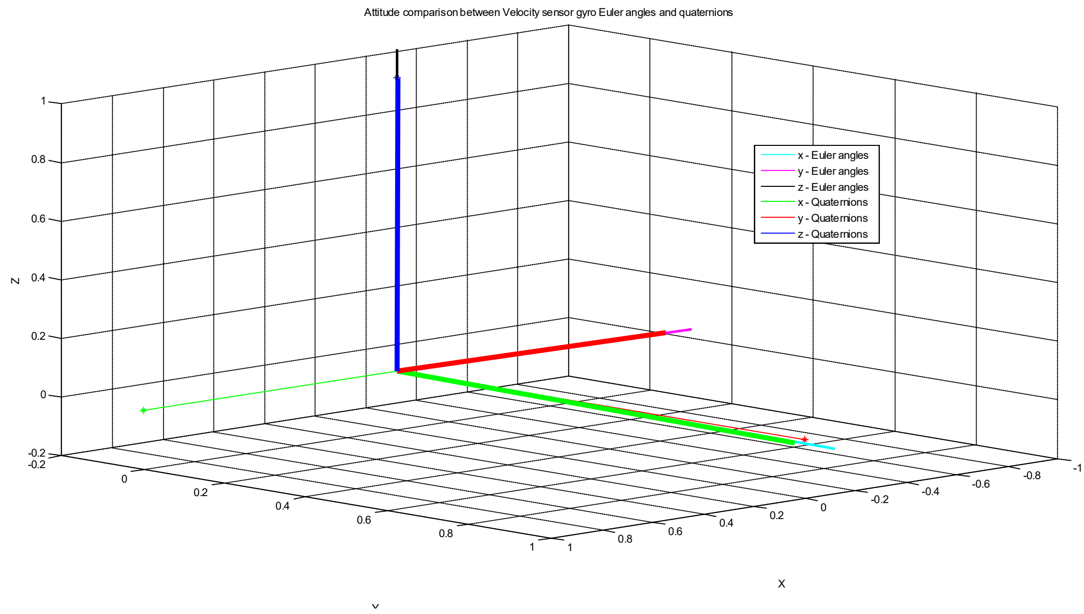

Figure 10. Attitude comparison between velocity sensor Euler angles and Quaternions

The rotation matrix “Cea2” provided by the final Euler angles and the rotation matrix “Cq2” provided by the final quaternions through MEMs gyro are, presented in Fig. 11,

$$Cea2 = \begin{pmatrix} 0.0083 & 0.9997 & -0.0229 \\ -1.0000 & 0.0084 & 0.0011 \\ 0.0013 & 0.0229 & 0.9997 \end{pmatrix} \quad Cq2 = \begin{pmatrix} 0.0079 & 0.9996 & -0.0268 \\ -1.0000 & 0.0079 & 0.0012 \\ 0.0014 & 0.0268 & 0.9996 \end{pmatrix}$$

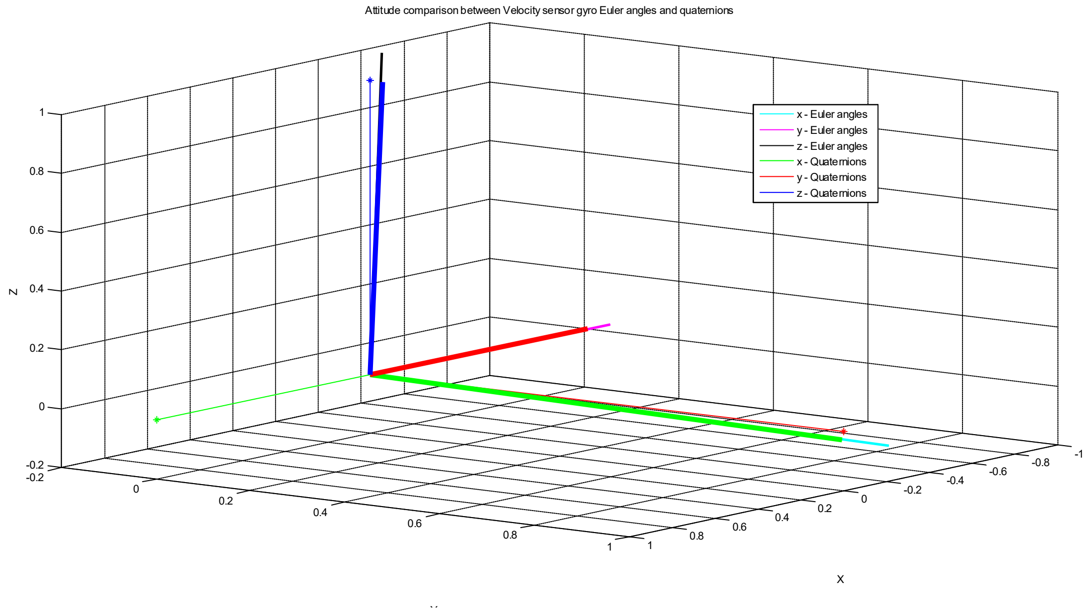

Figure 11. Attitude comparison between MEMs gyro Euler angles and Quaternions

4. CONCLUSIONS

This paper presents a study about the Quaternions as attitude parameters. It was demonstrated that quaternions parameters generate the same Cartesian coordinates as Euler angles. Quaternions have the advantage of do not produce unexpected attitude values, for any vehicle's orientation, which makes it more suitable for control attitude.

Interesting aspects were observed in the tests. In one axis rotations the evolution of corresponding quaternion parameter is very similar to the corresponding Euler angle. When singularity occurs, despite MEMs Euler angles got instable, it returns to lower values comparing with velocity sensor's Euler angles.

We have obtained final attitude errors due our implemented attitude estimation algorithm. It does not consider critical corrections that are required when using gyros. Further studies should be made to compensate misalignment, bias, Earth rotation, and so on.

These attitude estimation algorithms were developed and implemented in a real-time computer for future studies of attitude control.

5. ACKNOWLEDGEMENTS

The authors acknowledge the Hilton Cleber Pietrobom, Joana and Timo Wekerle contributions.

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