CONEM 2016
CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
21 - 25 AGOSTO DE 2016
FORTALEZA - CEARÁ
ABCM 40 anos
Associação Brasileira de Engenharia e Ciências Mecânicas

SPACECRAFT ATTITUDE ESTIMATION USING THE REGULARIZED PARTICLE FILTER WITH ROUGHENING

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Abstract. *In this work, will be described the attitude determination and the gyros bias estimation using the Regularized Particle Filter with Roughening for nonlinear systems. The Particle Filter is a statistical, brute-force approach to estimation that often works well for problems that are difficult for the conventional Kalman Filter. In real time applications their estimation accuracy and efficiency are significantly affected by numbers of particle. The Regularized Particle Filter was used for preventing the sample impoverishment, this problem occurs when the region of state space in which the posteriori probability density function has significant values does not overlap with the priori probability density function. This means that if all of our a priori particles are distributed according to priori probability density function, and compute the posteriori probability density function to resample the particles, only a few particles will be resampled to become a posteriori particles. The application uses the real measurement data for orbit and attitude of the CBERS-2 (China Brazil Earth Resources Satellite). The attitude dynamical model is described by nonlinear equations involving the Euler angles. The attitude sensors available are two DSS (Digital Sun Sensors), two IRES (Infra-Red Earth Sensor), and one triad of mechanical gyros. The results in this work show that one can reach accuracies in attitude determination within the prescribed requirements, besides providing estimates of the gyro drifts which can be further used to enhance the gyro error model.*

Keywords: *Regularized Particle Filter, Roughening Technique, Unscented Kalman Filter, nonlinear system, attitude determination*

1. INTRODUCTION

Many spacecraft missions require precise pointing of their sensors and demand a real time precise attitude determination, which involves estimating both sattelite attitude and gyros bias, from output information of the attitude measurements system. The importance of determining the attitude is related not only to the performance of the attitude control system but also to the precise usage of information obtained by payload experiments performed by the satellite.

Attitude estimation is a process of determining the orientation of a satellite with respect to an inertial reference system by processing data from attitude sensors. Given a reference vector, the attitude sensor measures the orientation of this vector with respect to the satellite system. Then, it is possible to estimate the orientation of the satellite processing computationally these vectors using attitude estimation methods. The bias can be defined as a output component not related to input to which the sensor is subjected and its components have features deterministic and stochastic. Therefore, you need to know and to characterize it, consequently set the method for estimating (Crassidis and Junkins, 2011).

The attitude stabilization here is done in three axes namely geo-targeted, and can be described in relation to the orbital system. In this frame, the movement around the direction of the orbital speed is named *roll* (ϕ), the movement around the normal direction to the orbit is called *pitch* (θ), and finally the movement around the Zenith/Nadir direction is called *yaw*

(ψ).

In this work the attitude is represented by Euler angles, due to its easy geometrical interpretation, with state estimation performed by the Regularized Particle Filter (RPF) compared to the Unscented Kalman Filter (UKF) (Garcia *et al.*, 2012), used with reference. This methods is capable of estimating nonlinear systems states from data obtained from different sensors of attitude. It was considered real data supplied by gyroscopes, infrared Earth sensors and digital solar sensors. These sensors are on board the CBERS-2 satellite (China-Brazil Earth Resources Satellite), and the measurements were recorded by the Satellite Control Center of INPE (Brazilian National Institute for Space Research).

2. ATTITUDE REPRESENTATION BY EULER ANGLES

As previously mentioned, the movement around the direction of the orbital speed is named *roll* (ϕ), the movement around the direction normal to the orbit is called *pitch* (θ), and the movement around the Zenith/Nadir direction is called *yaw* (ψ). See Fig. 1.

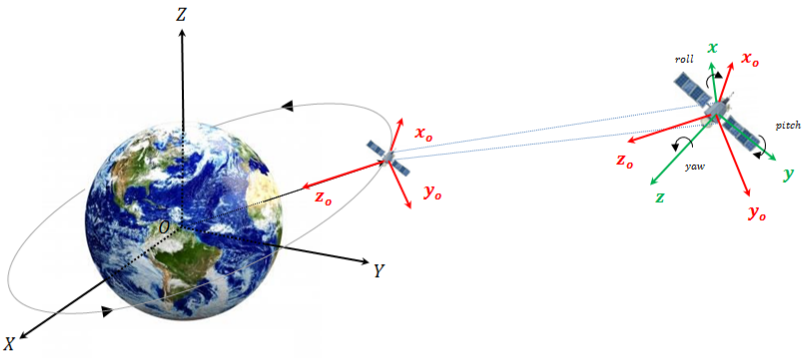

Figura 1: The rotation sequence adopted (ϕ,θ,ψ)

The transformation matrix R, which relates the fixed coordinate system in the satellite's body ($\mathbf{x}, \mathbf{y}, \mathbf{z}$) with the orbital coordinate system ($\mathbf{x}_o, \mathbf{y}_o, \mathbf{z}_o$), has its elements described in terms of Euler angles (ϕ, θ, ψ). The rotation sequence adopted in this work was the 3-2-1, and the following sequences of rotations are used (Fuming and Kuga, 1999)

- 1st rotation of an angle ψ (*yaw* angle) around the $\mathbf{z}_o$ axis.
- 2nd rotation of an angle θ (*pitch* angle) around an intermediate $\mathbf{y}'$ axis.
- 3rd rotation of an angle ϕ (*roll* angle) around the $\mathbf{x}$ axis.

Thus, we have that:

$$R = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} \tag{1}$$

In turn, the kinematic equations of Euler angles are given by (Fuming and Kuga, 1999; Wertz, 1978):

$$\begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{2}$$

where, ω_x, ω_y and ω_z are the components of the angular velocity of the satellite in *roll*, *pitch* e *yaw* directions.

Defining the state vector composed by: Euler angles (ϕ, θ, ψ) and the components of the gyros bias ($\varepsilon_x, \varepsilon_y, \varepsilon_x$); the differential equations of state for attitude and bias of the gyros are modeled as follows:

$$\begin{aligned} \dot\phi &= \hat\omega_x + \hat\omega_y\sin\phi\tan\theta + \hat\omega_z\cos\phi\tan\theta + \omega_0\left(\cos\phi\sin\psi + \sin\theta\sin\psi\tan\theta\right) \\ \dot\theta &= \hat\omega_y\cos\phi - \hat\omega_z\sin\phi + \omega_0\cos\psi \\ \dot\psi &= \hat\omega_y\sin\phi\sec\theta + \hat\omega_z\cos\phi\sec\theta + \omega_0\sin\theta\sin\psi\sec\theta \\ \dot{\varepsilon_x} &= 0 \\ \dot{\varepsilon_y} &= 0 \\ \dot{\varepsilon_z} &= 0 \end{aligned} \tag{3}$$

where ϕ, θ and ψ are the attitude angles obtained by some estimation process, and $\hat\omega_i$ are the gyros measurements.

3. THE PARTICLE FILTER

The initial concepts of Particulate Filter emerged in decade 40 present in Metropolis and Ulam (1949), something very similar to the Particle Filter was suggesting in Wiener (1956). The particle filter, which was invented for implementing a numerical Bayesian estimator is a statistical approximation by brute force, for estimation problems that are difficult to solve with the conventional Kalman filter, or highly nonlinear systems (Simon, 2006).

Suppose a non-linear system described by the equation

$$\begin{aligned} \boldsymbol{x}_{k+1} &= f_k(\boldsymbol{x}_k, \boldsymbol{w}_k) \\ \boldsymbol{y}_k &= h_k(\boldsymbol{x}_k, \boldsymbol{v}_k) \end{aligned} \tag{4}$$

where k is the time index, $\boldsymbol{x}_k$ is the state and $\boldsymbol{w}_k$ is the process noise, $\boldsymbol{y}_k$ are the measures and $\boldsymbol{v}_k$ is the noise measure. The functions $f_k(.)$ and $h_k(.)$ is a nonlinear time-varying system and the equation measures respectively. The noises $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are considered independent and whites with known probability density function.

To start the estimation of the problem, we generate randomly a given number N of vector states for the initial condition of the probability density function $p(\boldsymbol{x}_0)$, which is assumed known. These vectors are called states of particles and are represented as $\boldsymbol{x}_{0,i}^{+}$ $(i = 1, 2, ..., N)$.

$$\boldsymbol{x}_{0,i}^{+} \sim p(\boldsymbol{x}_{0,i}) \qquad (i = 1, 2, ..., N) \tag{5}$$

At each step $k = 1, 2, ...$, the particles propagate to the next time step using the process dynamics of the equation $f(.)$. This step is known as sampling.

$$\boldsymbol{x}_{k,i}^{-} = f_{k-1}(\boldsymbol{x}_{k-1,i}^{+}, \boldsymbol{w}_{k-1}^{i}) \qquad (i = 1, 2, ..., N) \tag{6}$$

where, each noise vector $\boldsymbol{w}_{k-1}^{i}$ is generated randomly based on the known probability density function of $\boldsymbol{w}_{k-1}$.

After getting all the measures in time k, we compute the conditional relative likelihood of each particle $\boldsymbol{x}_{k,i}^{-}$ assessed by probability density function $p(\boldsymbol{y}_k \mid \boldsymbol{x}_{k,i}^{-})$ obtained from equation measures $h(.)$ and noise probability density function of the measurements $\boldsymbol{v}_k$ (Simon, 2006).

$$\begin{aligned} q_i &= p(\boldsymbol{y}_k \mid \boldsymbol{x}_k^{-}) \\ q_i &\sim \frac{1}{(2\pi)^{m/2}\left|\boldsymbol{R}\right|^{1/2}} exp\left(\frac{-(\boldsymbol{y}^{*} - h(\boldsymbol{x}_{k,i}^{-}))^{T}\boldsymbol{R}^{-1}(\boldsymbol{y}^{*} - h(\boldsymbol{x}_{k,i}^{-}))}{2}\right) \end{aligned} \tag{7}$$

Normalizing the relative likelihoods obtained by Eq. (7) as:

$$\tilde{q}_i = \frac{q_i}{\sum_{j=1}^{N} q_j} \tag{8}$$

this ensures that the sum of all the probability is equal to 1.

The next step is to find a new dataset of $\boldsymbol{x}_{k,i}^{+}$ that is randomly generated based on the probability relative q_i. This ideal resampling is formally justified in Smith and Gelfand (1992), where is shown that the set functions probability density of new particle $\boldsymbol{x}_{k,i}^{+}$ tends to function probability density of $p(\boldsymbol{x}_k \mid \boldsymbol{y}_k)$ with a number of samples N next to infinity.

Thus, the resampling can be summarized as:

$$\boldsymbol{x}_{k,i}^{+} = \sum_{i=1}^{N} \tilde{q}_i \, \boldsymbol{x}_{k,i}^{-} \tag{9}$$

Thus, a set of particles $\boldsymbol{x}_{k,i}^{+}$ can be distributed according to the probability density function $p(\boldsymbol{x}_k \mid \boldsymbol{y}_k)$ and you can compute any desired statistical measure of this probability density function.

Finally, if we want to compute the expected value of $E(\boldsymbol{x}_k \mid \boldsymbol{y}_k)$ can approach there the average algebraic sum of the particles

$$E(\boldsymbol{x}_k \mid \boldsymbol{y}_k) \approx \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{x}_{k,i}^{+} \tag{10}$$

4. SAMPLE IMPROVERISHMENT

Sample improverishment occurs when the region of state space in probability density function $p(\boldsymbol{y}_k \mid \boldsymbol{x}_k)$ has significant values does not overlap with the $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{k-1})$. This means that if all of our *a-priori* particles are distributed according to $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{k-1})$, and we then use the computed probability density function $p(\boldsymbol{y}_k \mid \boldsymbol{x}_k)$ to resample the particles, only a few particles will be resampled to become *a-posteriori* particles. This is because only a few of the *a-priori* particles will be in a region of state space where the computed pdf $p(\boldsymbol{y}_k \mid \boldsymbol{x}_k)$ has significant value. This means that the resampling process will select only a few distinct *a-priori* particles to become a *a-posteriori* particles. Eventually, all of the particle will collapse to the same value[1]. This problem will be exarcerbated if the measurements are not consistent with the process model (modelling errors). This can be overcome by a brute-force method of simply increasing the number of particles N, but this can quickly lead to unreasonable computational demands and often simply delays the inevitable sample impoverishment. Other more intelligent ways of dealing with this problem can be used in Arulampalam *et al.* (2002) and Gordon *et al.* (1993).

4.1 Roughening

Roughening can be used to prevent sample improverishment, as shown in Gordon *et al.* (1993). In this method, random noise is added to each particle after the resampling process. This is similar to adding artificial process noise to the Kalman Filter. In the roughening approach, the *a-posteriori* particles are modified as follows (Simon, 2006):

$$\begin{aligned} &\boldsymbol{x}^{+}_{k,i}(m) = \boldsymbol{x}^{+}_{k,i}(m) + \Delta\boldsymbol{x}(m) \qquad (m = 1, ..., n) \\ &\Delta\boldsymbol{x}(m) \sim (0, K\boldsymbol{M}(m)N^{-1/n}) \end{aligned} \tag{11}$$

where $\Delta\boldsymbol{x}(m)$ is a zero-mean random variable (usually Gaussian). K is a scalar tuning parameter that specifies the amount of jitter that is added to each particle, N is the number of particle, n is the dimension of the state space and M is a vector containing the maximum difference between the particle elements before roughening. The mth element of the $\boldsymbol{M}$ vector is given as

$$\boldsymbol{M}(m) = \max_{i,j} \left| \boldsymbol{x}^{+}_{k,i}(m) - \boldsymbol{x}^{+}_{k,j}(m) \right| \quad (m = 1, ..., n) \tag{12}$$

where k is the time step, and i and j are particle numbers.

4.2 Regularized Particle Filter

Another way of preventing sample impoverishment is through the use of the Regularized Particle Filter (RPF) (Doucet *et al.*, 2001; Ristic *et al.*, 2004; Simon, 2006). This performs resampling from a continuous approximation of the pdf $p(\boldsymbol{y}_k \mid \boldsymbol{x}^{-}_{k,i})$ rather than from he discrete probability density function samples used thus far. Recall in our resampling step in Eq. (7) that we used the probability

$$q_i = p((\boldsymbol{y}_k = \boldsymbol{y}^*) \mid (\boldsymbol{x}_k = \boldsymbol{x}^{-}_{k,i})) \tag{13}$$

to determine the likelihood of selecting an *a-priori* particle to be an *a-posteriori* particle. Instead, we can use the pdf $p(\boldsymbol{x}_k \mid \boldsymbol{y}_k)$ to perform resampling. That is, the probability of selecting the particle $\boldsymbol{x}^{-}_{k,i}$ to be an *a-posteriori* particle is proportional to the pdf $p(\boldsymbol{x}_k \mid \boldsymbol{y}_k)$ evaluted at $\boldsymbol{x}_k = \boldsymbol{x}^{-}_{k,i}$. In the RPF, this probability density function is approximated as

$$\hat{p}(\boldsymbol{x}_k \mid \boldsymbol{y}_k) = \sum_{i=1}^{N} q_{k,i} K_h(\boldsymbol{x}_k - \boldsymbol{x}^{-}_{k,i}) \tag{14}$$

where $q_{k,i}$ are the weights that are used in the approximation. Later on, we will see that these weights should be set equal to the q_i probabilities that were computed in Eq. (7). Thus, $K_h(.)$ is given as

$$K_h(\boldsymbol{x}) = h^n K(\boldsymbol{x}/h) \tag{15}$$

where h is the positive scalar Kernel bandwidth, and n is the dimension of the state vector. $K(.)$ is a kernel density that is a symmetric probability density function that satisfies

$$\begin{aligned} &\int \boldsymbol{x}K(\boldsymbol{x})d\boldsymbol{x} = 0 \\ &\int \|\boldsymbol{x}\|_2^2 \, K(\boldsymbol{x})d\boldsymbol{x} < \infty \end{aligned} \tag{16}$$

[1]This is called the black hole of particle filtering

The Kernel $K(.)$ and the bandwidth h are chosen to minimize a measure of the error between the assumed true density $p(\boldsymbol{x}_k \mid \boldsymbol{y}_k)$ and the approximate density $\hat{p}(\boldsymbol{x}_k \mid \boldsymbol{y}_k)$:

$$\{K(\boldsymbol{x}), h\} = argmin \int \left[\hat{p}(\boldsymbol{x}_k \mid \boldsymbol{y}_k) - p(\boldsymbol{x}_k \mid \boldsymbol{y}_k)\right]^2 d\boldsymbol{x} \tag{17}$$

In the classic case of equal weights ($q_{k,i} = 1/N$ for $i = 1, ..., N$) the optimal kernel is given as

$$K(\boldsymbol{x}) = \begin{cases} \dfrac{n+2}{2V_n}\left(1 - \|\boldsymbol{x}\|_2^2\right) & if\ \|\boldsymbol{x}\|_2^2 < 1 \\ 0 & otherwise \end{cases} \tag{18}$$

where V_n is the volume of the n-dimensional unit hypersphere. $K(\boldsymbol{x})$ is called the Epanechnikov Kernel (Doucet *et al.*, 2001).

An n-dimensional unit hypersphere is a volume in n dimensions in which all points are one unit from the origin (Coxeter, 1973). In one dimensions, the unit hypersphere is a line with a length of two and volume $V_1 = 2$. In two dimensions, the unit hypersphere is a circle with a radius of one and volume $V_2 = \pi$. In three dimensions, the unit hypersphere is a sphere with a radius of one and de volume $V_3 = \frac{4\pi}{3}$. In the n dimensions the unit hypersphere has a volume $V_n = \frac{2\pi V_{n-2}}{n}$.

If $p(\boldsymbol{x} \mid \boldsymbol{y}_k)$ is Gaussian with an identity covariance matrix then the optimal bandwidth is given as

$$h^* = \left[8V_n^{-1}(n+4)(2\sqrt{\pi})^n\right]^{\frac{1}{n+4}} N^{-\frac{1}{n+4}} \tag{19}$$

In order to handle the case of multimodal probability density functions[2], we should use $h = \frac{h^*}{2}$ (Doucet *et al.*, 2001; Silverman, 1986). These choice for the Kernel and the bandwidth are optimal only for the case of equal weights and a Gausian pdf, but they still are often used in other situations for obtain good particle filtering results. Instead of selecting *a-priori* particles to become *a-posteriori* particles using the probabilities of Eq. (13), we instead select *a-posteriori* particles based in a pdf approximation given in Eq. (14). This allows more diversity as we perform the update from the *a-priori* particles to a *a-posteriori* particles. In general, we should set the $q_{k,i}$ weights in Eq. (14) equal to the q_i probabilities shown in Eq. (13).

Since this procedure assumes that the true density $p(\boldsymbol{x} \mid \boldsymbol{y}_k)$ has a unity covariance matrix, we numerically compute the covariance of the $\boldsymbol{x}_{k,i}^-$ at each time step. Suppose that this covariance is computed as $\boldsymbol{S}$ (an $n \times n$ matrix). Then we compute the matrix square root of $\boldsymbol{S}$, denoted as A, such that $\boldsymbol{A}\boldsymbol{A}^T = \boldsymbol{S}$ (*e.g.*, we can use Cholesky decomposition for this computation). Then we compute the kernel as

$$K_h(\boldsymbol{x}) = (\det \boldsymbol{A})^{-1} h_{-n} K\left(\frac{\boldsymbol{A}^{-1}\boldsymbol{x}}{h}\right) \tag{20}$$

5. COMPUTER SIMULATION AND RESULTS

The nonlinear system that represents the process and measurements equations of the problem is given by (Silva *et al.*, 2014):

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\varepsilon_x} \\ \dot{\varepsilon_y} \\ \dot{\varepsilon_z} \end{bmatrix} = \begin{bmatrix} \hat{\omega}_x + \hat{\omega}_y \sin\phi \tan\theta + \hat{\omega}_z \cos\phi \tan\theta + \omega_0 \left(\cos\phi \sin\psi + \sin\theta \sin\psi \tan\theta\right) \\ \hat{\omega}_y \cos\phi - \hat{\omega}_z \sin\phi + \omega_0 \cos\psi \\ \left(\dfrac{\hat{\omega}_y \sin\phi + \hat{\omega}_z \cos\phi + \omega_0 \sin\theta \sin\psi}{\cos\theta}\right) \\ 0 \\ 0 \\ 0 \end{bmatrix} + \boldsymbol{w} \tag{21}$$

$$\boldsymbol{y}_k = \begin{bmatrix} \arctan\left(\dfrac{-\left(S_{0y} - \psi S_{0x} + \phi S_{0z}\right)}{\left(S_{0x} + \psi S_{0y} - \theta S_{0z}\right)\cos 60^\circ + \left(S_{0z} - \phi S_{0y} - \theta S_{0z}\right)\cos 150^\circ}\right) \\ 24^\circ + \arctan\left(\dfrac{S_{0x} + \psi S_{0y} - \theta S_{0z}}{S_{0z} - \phi S_{0y} - \theta S_{0z}}\right) \\ \phi \\ \theta \end{bmatrix} + \boldsymbol{v}_k \tag{22}$$

Remembering that the state vector is composed by the attitude angles ϕ, θ, ψ, and by gyros bias ε_x, ε_y, ε_z present in $\hat{\omega}_x$, $\hat{\omega}_x$ and $\hat{\omega}_x$ respectively; the term ω_0 is an angular velocity of the satellite in relationship to Earth. The matrices

[2] A multimodal probability density functions have more than one local maxima

$\boldsymbol{w} = \left[\begin{array}{cccccc} w_\phi & w_\theta & w_\psi & w_{\varepsilon_x} & w_{\varepsilon_y} & w_{\varepsilon_z} \end{array} \right]^T$ and $\boldsymbol{v}_k = \left[\begin{array}{cccc} \upsilon_{\alpha_\psi} & \upsilon_{\alpha_\theta} & \upsilon_{\phi_H} & \upsilon_{\theta_H} \end{array} \right]^T$ are the process and measurements noises, respectively.

To validate and analyze the performance of estimators, real data of CBERS-2 satellite were used. The CBERS-2 satellite was launched on October 21, 2003. The measures are for the day April 21, 2006, being collected by the ground system at a sampling rate of about 8.56s for about 10 min of observation

In fact, the ACS (Attitude Control System) on board the satellite has full access to sensor measurements sampled at a rate of 4Hz for the three gyros, aligned with axes x, y and z of the satellite; 1Hz for the two Infrared Earth Sensors to the angle ϕ (*roll*) and θ (*pitch*); and 0.25Hz for both Digital Solar Sensors, related to the angles of *pitch* (α_θ) and *yaw* (α_ψ). However, due to the limited telemetry the system can only recover telemetries of sensors at 9s sampling only when the satellite passes over the tracking station. This means that the ground system does not have the full set of measurements that are available to the ACS on board (Lopes and Kuga, 2005).

Figures 2 and 3, present the attitude angles and gyros bias estimation using the RPF for $N = 500$ particles comparated with Unscented Kalman Filter (UKF), this results for the UKF are presents in Garcia *et al.* (2012).

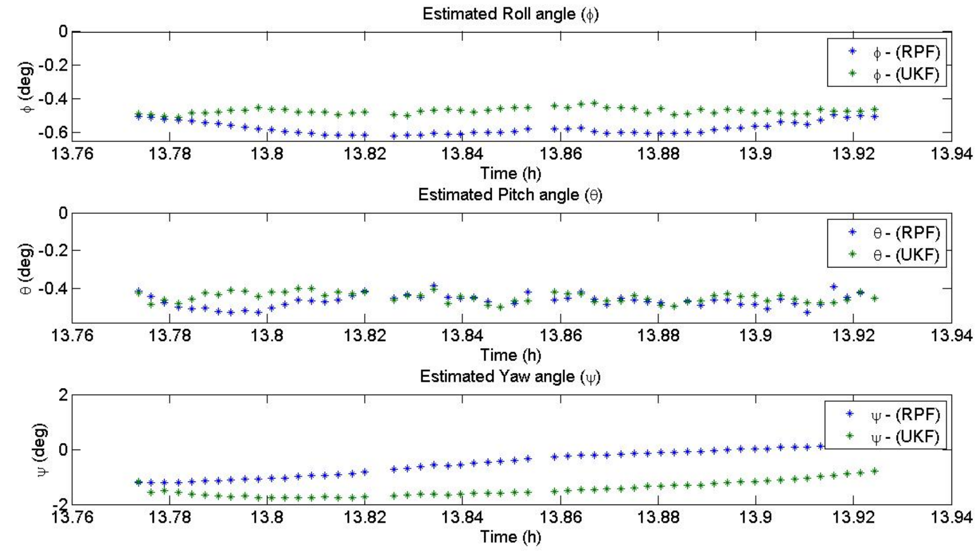

Figura 2: Estimated *roll*, *pitch* and *yaw* angles respectively

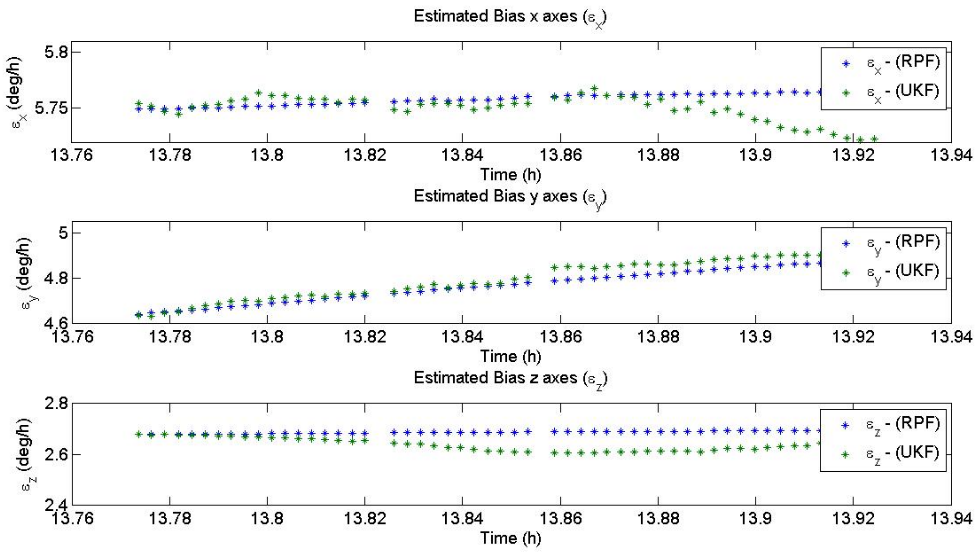

Figura 3: Estimated gyros bias around the **x**, **y** and **z** axes respectively

Before analyzing the accuracy of the filters in question, it is important to analyze their convergence done through configuration of residual for the two DSS and for the two IRES, in the Fig. (4) and Fig. (5) below shows the residual frequency for DDS and IRES for each of the filters in the analysis (RPF and UKF), presenting characteristics make a Gaussian.

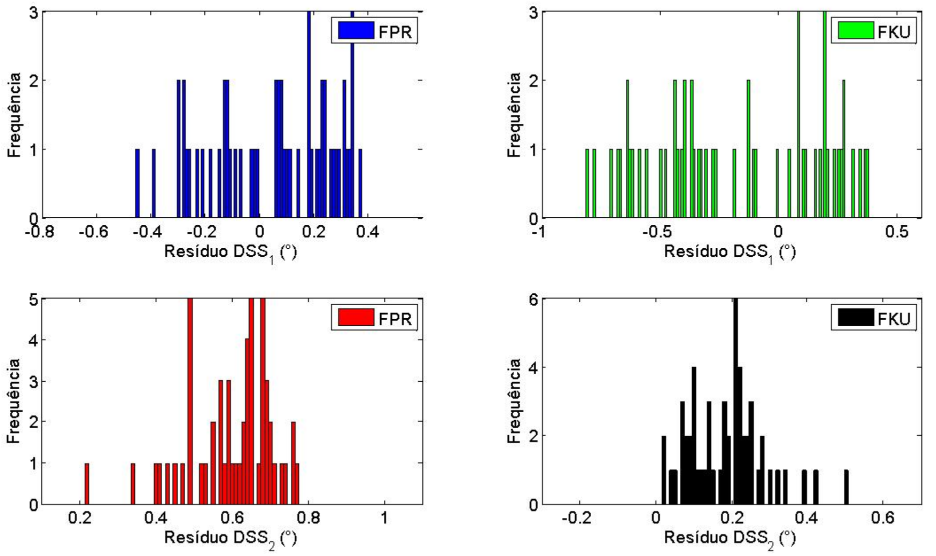

Figura 4: Frequency Residuals for the DSS on board the CBERS-2 satellite

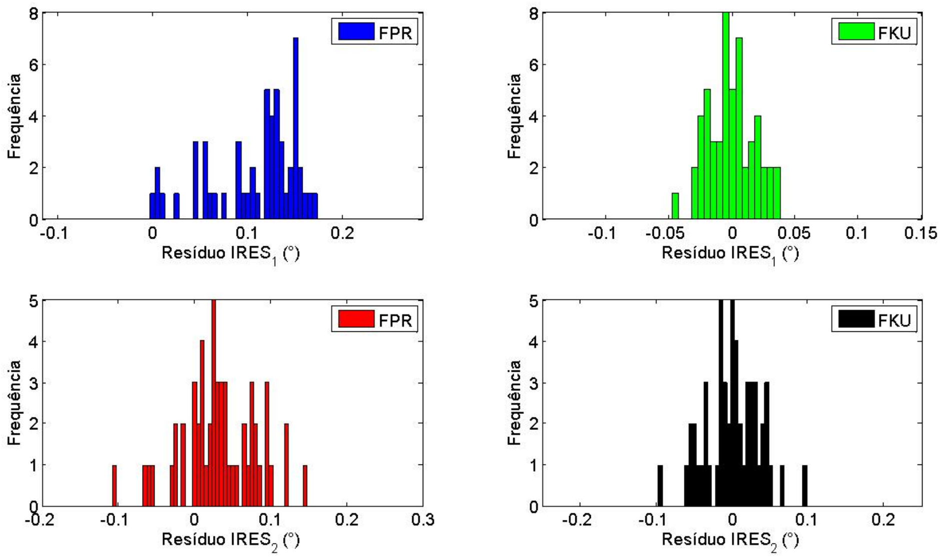

Figura 5: Frequency Residuals for the IRES on board the CBERS-2 satellite

It is said that a Filter is converging when your residual is close to zero average the Tab. 1 shows the average value and the standard deviation of the DSS and IRES residuals for each of the filters presented in Fig. (4) and Fig. (5)

It can be seen that the attitude and gyros bias estimation converge and the results for RPF are in close agreement with the UKF, although the results for the UKF appear to be more accurate. This accuracy for the RPF It is achieved when greater the number of particles is used, but this leads in a larger processing time, as can be checked in Tab. 2

6. CONCLUSIONS

The main objective of this study was to estimate the attitude of a CBERS-2 like satellite, using real data provided by sensors that are on board the satellite. To verify the consistency of the estimator, the attitude was estimated by least squares method. The usage of real data from on-board attitude sensors, poses difficulties like mismodelling, mismatch of sizes, misalignments, unforeseen systematic errors and post-launch calibration errors.

In general terms, for nonlinear system, the Kalman filtering can be used for state estimation, but the Particle filtering may give better results at the price of additional computational effort. In system that has non-Gaussian noise, the Kalman filtering is optimal linear filter, but again the Particle filtering may perform better. The Unscented Kalman Filter provides a balance between the low computational effort of te Kalmam filtering and the high performance of the Particle filtering (Simon, 2006).

It should be added that there is some similarity between the filters, the UKF transform a set of points via known nonlinear equations and combines the results to estimate the mean and covariance of the state. However, in the Particle

Tabela 1: Mean and standard deviation statistics of the DSS and IRES Residuals

	RPF	UKF
DSS_1 Res.(deg)	-0.0118 ± 0.2651	-0.1627 ± 0.3550
DSS_2 Res.(deg)	0.6888 ± 0.1433	0.1753 ± 0.0873
$IRES_1$ Res.(deg)	0.0599 ± 0.0844	0.0015 ± 0.0057
$IRES_2$ Res.(deg)	0.0831 ± 0.0631	-0.0022 ± 0.0081

Tabela 2: Comparing processing time cost

	RPF for $N = 500$ particles	UKF
Average CPU time	$76.1696s$	$0.2883s$

Filtering (and the RPF used in this work) the points (the cloud of particles) are chosen randomly, whereas in the UKF the points (the cloud of sigma points) are chosen on the basis of a specific algorithm.

Because of this, the number of points used in a Particle Filter generally needs to be much greater than the number of points in a Unscented Kalman Filter and, consequently, with this large number of particles for the Particle Filtering the computational effort will increase

Finally, this work provide a kinematic attitude solution besides estimating biases (gyro drifts) and it can be concluded that the algorithm of the RPF converges and the results are in close agreement with the results in previou work (Garcia *et al.*, 2012), which used the UKF for estimation of attitude and gyros bias.

7. ACKNOWLEDGEMENTS

The authors would like to thank the financial support received by CAPES, FAPESP 2012/ 21023-6, CNPQ 303119/ 2010-1, and the partial support from project SIA-DCTA-INPE under contract FINEP 0.1.06.1177.03

8. REFERÊNCIAS

Arulampalam, M., Maskell, S., Gordon, N. and Clapp, T., 2002. "A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking". *IEEE Transactions on Signal Processing*, Vol. 50, No. 2, pp. 174 – 188.

Coxeter, H., 1973. *Regular Polytopes*. Dover, New York.

Crassidis, J.L. and Junkins, J.L., 2011. *Optimal Estimation of Dynamic Systems*. Chapman and Hall/CRC Applied Mathematics and Nonlinear Science, New York.

Doucet, A., de Freitas, N. and Gordon, N., 2001. *Sequential Monte Carlo Methods in Practice*. Springer-Verlag, New York.

Fuming, H. and Kuga, H.K., 1999. "Cbers simulator mathematical models". *CBTT Project, CBTT/ 2000/ MM/ 001*.

Garcia, R.V., Kuga, H.K. and Zanardi, M.C., 2012. "Unscented kalman filter applied to the spacecraft attitude estimation with euler angles". *Mathematical Problems in Engineering*, Vol. 2012, pp. 1–12.

Gordon, N., Salmond, D. and Smith, A., 1993. "Novel approach to nonlinear/non-gaussian bayesian state estimation". *IEEE Proceedings-F*, Vol. 140, No. 2, pp. 107 – 1813.

Lopes, R.V.F. and Kuga, H.K., 2005. "Cbers-2: On ground attitude determination from telemetry data". *INPE, Internal Report C-ITRP*.

Metropolis, N. and Ulam, S., 1949. "The monte carlo method". *Journal of the American Statistical Association*, Vol. 44, No. 247, pp. 335 – 341.

Ristic, B., Arulampalam, S. and Gordon, N., 2004. *Beyound the Kalman Filter: Particle Filters for Tracking Applications*. Artech House, Norwell, Massachusetts.

Silva, W.R., Kuga, H.K. and Zanardi, M.C., 2014. "Application fo the extended h_∞ filter for attitude determination and gyro calibration". *Advances in the Astronautical Sciences*, pp. 1501 – 1515.

Silverman, B., 1986. *Density Estimation for Statistics and Data Analysis*. Chapman and Hall, London.

Simon, D., 2006. *Optimal State Estimation: Kalman, H_∞, and Nonlinear Approaches*. Wiley, New York.

Smith, A. and Gelfand, A., 1992. "Bayesian statistics without tears: A sampling resampling perpective". *The American Statistician*, Vol. 46, No. 2, pp. 84 – 88.

Wertz, J.R., 1978. *Spacecraft attitude determination and control*. D. Reidel, Dordrecht, Holanda.

Wiener, N., 1956. *I am a Mathematician*. MIT Pres, Cambridge, Massachesetts.

9. RESPONSIBILITY NOTICE

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