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AUTOREGRESSIVE MODELS FOR DAMAGE PROGNOSIS IN SMART STRUCTURES

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Abstract: *Damage Prognosis is the estimation of the remaining useful life of a structure leading to economic and life-safety benefits. This estimation is based on the quantification of damage present in a structure through the development of predictive models to track structural integrity. Usually, after a certain level of damage is detected, its evolution is evaluated in terms of failure mechanics and a damage law. However, the novelty in the present work is based on the parametrization of the structure's dynamic behavior under different structural conditions using autoregressive models. Coefficient extrapolation is employed to enable the creation of a predictive autoregressive model capable to forecast structural behavior for a more severe damaged condition. The advantage is to enable damage prognosis without the need of intricate physics or mathematical based models. This methodology was tested with signals measured on an aluminum plate coupled with piezoelectric patches subjected to subsequent levels of localized mass loss resulting in a successful prediction of the last damage scenario. The results have shown the ability to detect and to predict correctly the structural state of the system.*
Keywords: *damage prognosis, structural health monitoring, time series, predictive models*

1. INTRODUCTION

Damage Prognosis is the estimate of the remaining useful life of a damaged system (Farrar, 2003). Damage will adversely affect its current and future performance occurring in two distinct time scales (Farrar and Worden, 2007). Damage can be nucleated at the material level and its propagation will depend upon the operational and environmental conditions (e.g. load and temperature variations) leading to a catastrophic fault. On the other hand, damage can also occur on a much shorter scale of time due to discrete events, such as aircraft land, as presented by Inman *et al.* (2005). Because of the important dependence of damage on the operational and environmental conditions, the process of measuring such parameters for damage prognosis is known as Usage Monitoring (Farrar and Lieven, 2007).

Damage Prognosis capability is related to the evolution of simpler Structural Health Monitoring (SHM) techniques that only detect or localize damage (Farrar and Lieven, 2007; Worden *et al.*, 2007). This estimate relies on the creation of prediction models whose output together with Usage Monitoring information will attempt to forecast system performance (Farrar and Lieven, 2007). The multidisciplinary nature of Damage Prognosis is evidenced by Chattopadhyay *et al.* (2013) whose report emphasizes mutual efforts of material, sensor and data processing areas to create reliable prognosis strategies. The major interests for the development of methods to predict structural integrity are to improve the potential economic and life-safety benefits supplied by SHM.

As it can be noticed, the creation of predictive models is the main key for damage prognosis. After the current damage level is characterized by a SHM technique, it is evaluated in terms of failure mechanics and a damage law (Kulkarni and Achenbach, 2008). Since crack propagation is mainly caused by fatigue, different methods have been used to characterize and predict loading (Ling and Mahadevan, 2012). A different approach is to attempt to model the structure's response under different levels of damage. This was performed by Albakri and Tarazaga (2014) who crated electromechanical impedance signatures based on spectral element method. The prediction behavior of the model was verified by subsequent local stiffness reductions.

Another approach is to use statistical models to track structural changes. This enables the development of prognosis strategies without the need of intricate physical models. This fact was verified by Silva *et al.* (2008) who compared a traditional method for damage detection using electromechanical impedance signatures with a more sophisticated one based on the estimation of autoregressive moving average model with exogenous input (ARMAX). These models predicted temporal responses of the structure for different structural conditions and were associated to statistical process control in order to enable a robust damage detection minimizing the occurrence of false alarming. In addition, according to the authors the latter methodology is very suitable for damage prognosis, which is developed in the present work in a similar fashion. Similarly, Pavlopoulou *et al.* (2013) used Lamb wave excitation to monitor composite panels under different loading conditions applying outlier analysis and both linear and nonlinear principal component analysis. The robustness of these processing methods was ensured as they were able to represent the correct structural status even when fed with inadequate training data set.

In this sense, the present work aims to achieve the prediction of damage without the use of physics or mathematical based models, e.g. finite or boundary element methods. This is achieved by the employment of time series associated to autoregressive models using well-known results from system identification and signal processing areas (Sohn *et al.*, 2000; Lu and Gao, 2005; Silva *et al.*, 2011). Because these models are basically discrete filters they do not require a great processing capability which enables their applicability in on-board wireless hardware. This approach also presents advantages over traditional damage detection techniques (e.g. electromechanical impedance) since it is not dependent on important estimation parameters and on the frequency range adopted (Sohn *et al.*, 2001; Fassois and Sakellariou, 2007; Silva *et al.*, 2008; Figueiredo *et al.*, 2011). Thereby, the novelty in this work is the modeling of the structure's response as each new structural condition is identified enabling the extrapolation of its future behavior.

The paper is organized as follows. Firstly, the experimental setup is showed and the measured signals are presented. Next, the damage detection scheme is detailed followed by the explanation of the damage prognosis methodology and the discussion of its results and effectiveness. Finally, the conclusions point out the advantages and drawbacks found.

2. EXPERIMENTAL SETUP

The present experimental tests were carried in CPqD, Laboratory of Mechanical Tests, on a 1,6 mm thick aluminum plate coupled with six piezoelectric (PZT) patches. The structure was clamped at both ends in order to reduce measurement interference and kept at controlled room temperature (23 ± 3 °C) (Fig. 1). A pitch-catch measurement mode was chosen with PZT 5 as actuator and PZT 2 as sensor. Data generation and acquisition was performed by a National Instruments PXI module and controlled by LabVIEW software with a sampling rate of 10 MHz and an acquisition time window of 1.5 ms. The structure was excited by a five peak Gaussian tone-burst with 10 V_{pp} of amplitude and actuation frequency of 150 kHz to guarantee damage detectability. This frequency generates an A0 packet of 930 m/s with around 6 mm of wavelength, which is located at the time window from 200 to 230 μs (Fig. 2).

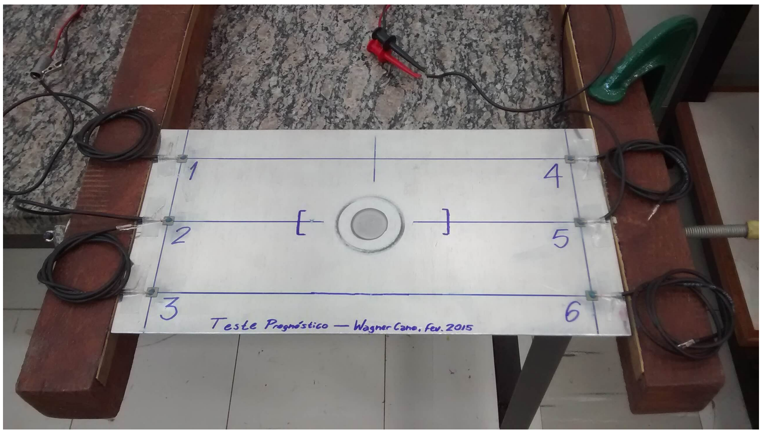


Figure 1: **Aluminum plate coupled with six PZT patches (circular damage in the center of the plate).**

Two series of 10 signals were measured in distinct periods for each case studied for repeatability evaluation, as presented in Tab. 1, with each acquired signal being a mean of 10 signals. The response signals showed in Fig. 4 (a) were normalized to produce zero mean and unitary standard deviation as a requirement for the following statistical methodology developed for damage detection.

Damage was simulated through subsequent electrolytic polishing procedures resulting in localized mass loss of 22 mm in diameter taking into account the rule that this dimension must be greater than half size of the wavelength generated (Lee and Staszewski, 2003). This procedure was carried in the same face as the PZTs were bonded to the plate in an attempt to reproduce the effects of corrosion, which often affects aircraft structures and is hard to detect in inaccessible areas (Fig. 3).

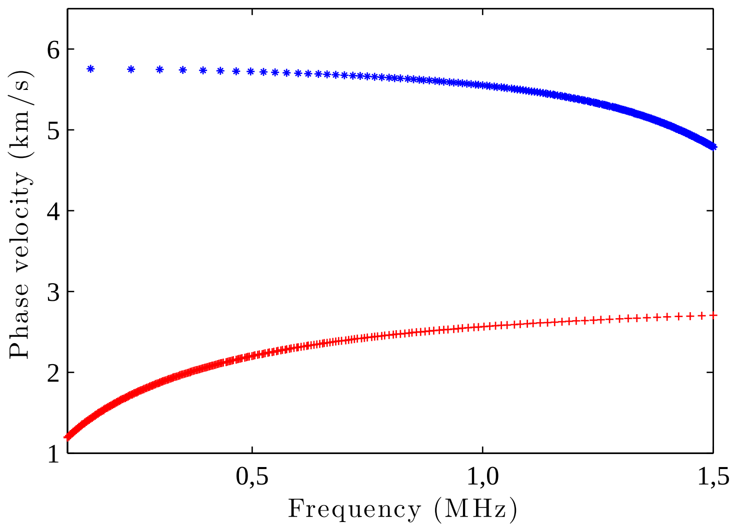


Figure 2: **Dispersion curves for a 1,6 mm thick aluminum plate: $*$ − S0 mode; $+$ − A0 mode.**

Table 1: **Sequence of measurements.**

Condition tested	Test number	Damage depth (mm)	Thickness reduction (%)
Reference	1 - 20	0	0
Damage 1	21 - 40	0,10	6,25
Damage 2	41 - 60	0,20	12,50
Damage 3	61 - 80	0,30	18,75
Damage 4	81 - 100	0,40	25,00

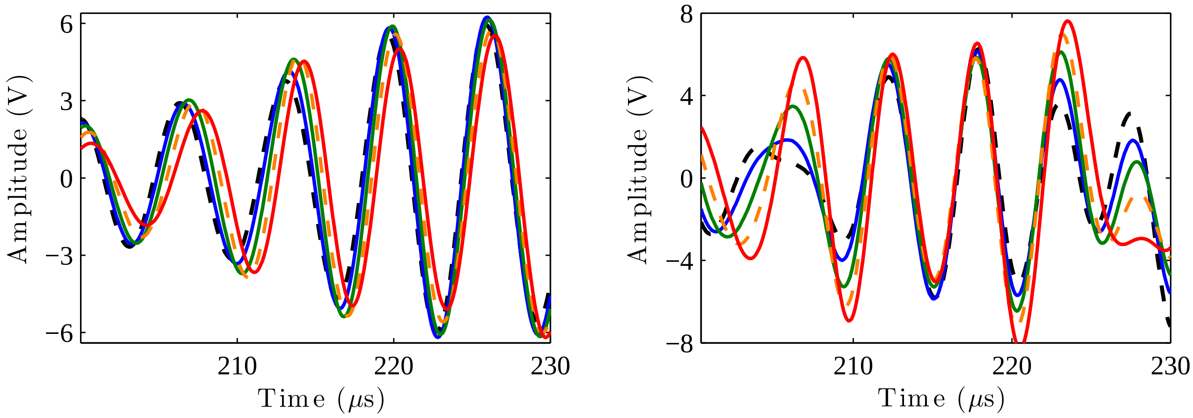
Figure 3: **Electrolytic polisher for the creation of a controlled mass loss with progressive depth.**

2.1 DAMAGE DETECTION

Initially an AR model was created in order to provide a parametrization of the undamaged condition using the first response signal measured. This model is defined by:

$$A(q)y(k) = e(k) \tag{1}$$

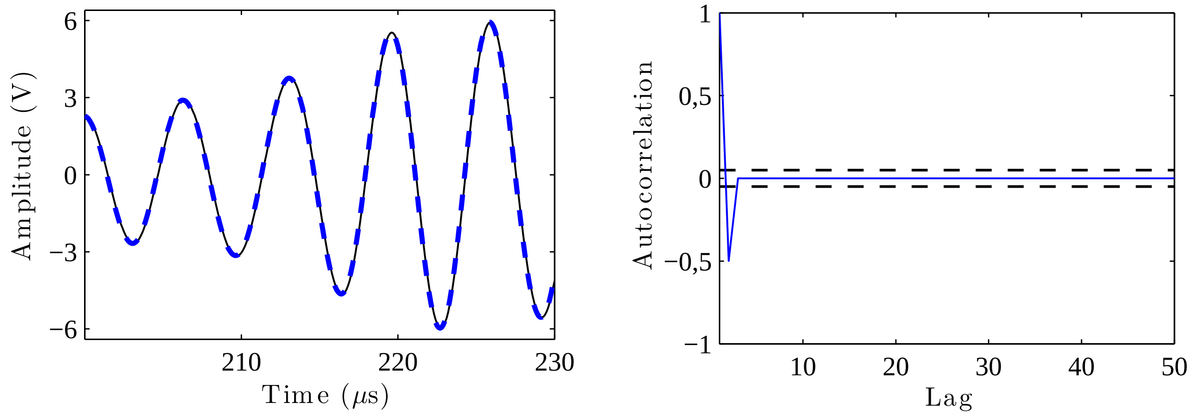
where $y(k)$ is the estimated response signal in terms of discrete samples k. $A(q)$ is a polynomial function which can be computed by many methods, such as ordinary least squares procedures or the Yule-Walker equation (Box *et al.*, 1994). This polynomial is function of the shift operator q of order n_a which is unknown *a priori* but can be defined using a minimization method. The Akaike's information criterion (AIC) (Ljung, 1998) was used to guide the order selection together with the desire that the resulting prediction errors $e(k)$ would be able to carry information about damage enabling the detection. A suitable value for the reference model order was 2 leading to a satisfactory validation using the second response signal measured (Fig. 5).



(a) Normalized resposnse signals.

(b) Prediction errors from reference AR model.

Figure 4: **Signals in the time window from 200 to 230 μs corresponding to the first antisymmetric wave mode (test number: — — 1, —— 21, —— 41, — — 61, —— 81).**



(a) Direct comparison between the certification signal (solid black line ——) and the adjusted signal (dashed blue line — —) with a fit percentage of 99.9.

(b) Residue evaluation of adjusted response converging to the confidence interval of 10% (— —) as the past samples or lags increase.

Figure 5: **Validation of the reference AR model for damage detection.**

The γ index was used as the damage index due to its better sensitivity to variations in amplitude since data dispersion is expected in the prediction errors when response signals of damaged conditions are filtered by the reference AR model. This index is defined as (Lu and Gao, 2005):

$$\gamma_N = \frac{\sigma(e_N)}{\sigma(e_{N_{ref}})} \tag{2}$$

where N is the test number and the subscript ref refers to the position of the reference indices with $N_{ref} = 1, 2, 3, \ldots, 10$ in this case generating 10 indices per test number, as presented in Fig. 6(a).

The thresholds for damage occurrence are defined by the computation of the control chart X-bar using the mean γ index value denoted by $\bar{\gamma}$ (Fig. 6(b)). To do so, only first 10 indices calculated were employed – the other 10 signals (test numbers 11-20 of the reference condition, for example) were kept to verify measurement variability and the occurrence of false indications of damage. The first step is the rearrangement of the indices in m subgroups of size n, normally chosen to be 4 or 5 (Montgomery, 1996):

$$[\bar{\gamma}]_{i,j} = \begin{matrix} \bar{\gamma}_{1,1} & \bar{\gamma}_{1,2} & \cdots & \bar{\gamma}_{1,n} \\ \bar{\gamma}_{2,1} & \bar{\gamma}_{2,2} & \cdots & \bar{\gamma}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\gamma}_{m,1} & \bar{\gamma}_{m,2} & \cdots & \bar{\gamma}_{m,n} \end{matrix} \tag{3}$$

The motivation for the use of subgroups is that the distribution of the subgroup mean can be reasonable approximated by a normal distribution as a result of the central limit theorem (Sohn *et al.*, 2000). Thus, the mean and the standard deviation of each subgroup is calculated for $i = 1, 2, \ldots, m$:

$$\overline{X}_i = \mu(\bar{\gamma}_{i,j}) \qquad S_i = \sigma(\bar{\gamma}_{i,j}) \tag{4}$$

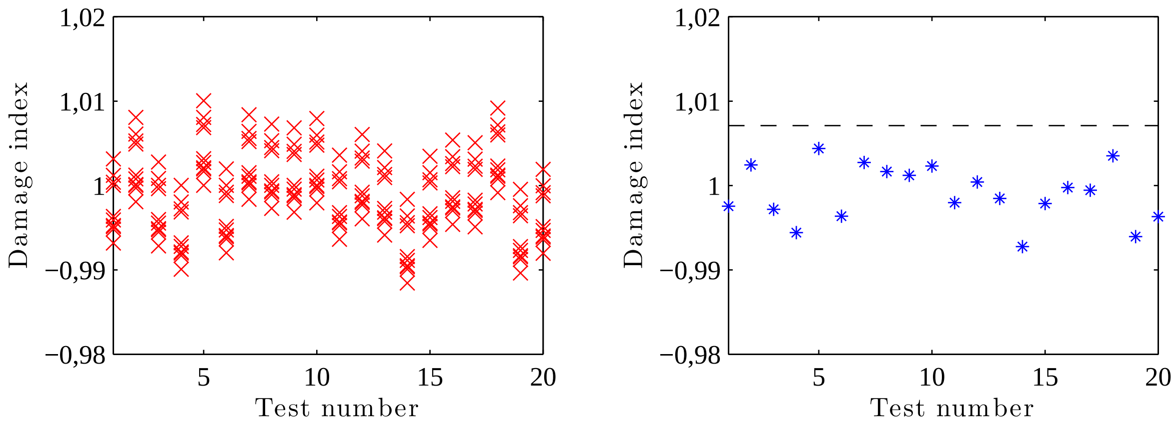
with $\mu(*)$ and $\sigma(*)$ being mean and standard deviation, respectively. Next, the central line (CL) is obtained with the following expression:

$$CL = \mu(\overline{X}_i) \tag{5}$$

Since data dispersion is expected for damaged conditions, only the upper control limit (UCL) is used as the threshold for damage detection:

$$UCL = CL + Z_{\alpha/2}\frac{S}{\sqrt{n}} \tag{6}$$

where $S = \mu(S_i)$. Due to measurement variability, the number of subgroups n was defined as 2 resulting in a subgroups of 5 elements. The term $Z_{\alpha/2}$ is the point in a normal distribution of zero mean and unitary standard deviation such that $P(z > Z_{\alpha/2}) = \alpha/2$. In the present work a confidence interval of 99.73% was chosen leading to $Z_{\alpha/2} = 3$. Therewith, damage is detected when damage index value becomes greater than the threshold.



(a) 10 γ inidces obtained for each test number.

(b) Mean γ indices ($*$) calculated for each test number and the respective threshold (— —).

Figure 6: **Damage indices for the reference condition.**

When such fact is observed, a new threshold value is defined by Eq. (6) to enable the detection of more severe damage. Figure 7 presents the damage indices computed for all signals measured whose behavior demonstrated the expected increase of data dispersion as damage became more severe. It is possible to obverse, for example, that damage index of test number 21 is located above the first threshold. At this moment damage is detected and a new threshold is obtained with the damage indices calculated with response signals from test numbers 21-30. These two identified conditions could enable the prediction of a third one as long as damage location and mass loss progression remain the same employing an adequate prognosis methodology.

3. DAMAGE PROGNOSIS

After the identification of distinct structural conditions the prognosis is attempted. An approach similar to the one for damage detection is performed but using autoregressive models with exogenous inputs (ARX). This choice was guided due to the subtle changes verified in the response signals for the different conditions tested – predominantly phase shifts. In this sense, an ARX model is defined as:

$$A(q)y(k) = B(q)u(k) + e(k) \tag{7}$$

where $u(k)$ is the input signal which is multiplied by the polynomial $B(q)$. Because of the dependence of the input signal, its expected that this kind of model will be able to better capture structural behavior exclusively for the condition they it is created. This premise is crucial to ensure an adequate and unique identification of each structural condition tested enabling the prognosis of the structure's future behavior by coefficient extrapolation.

The following procedure aims to forecast the structure's behavior for a future condition taking into account past modeled conditions. Firstly, the input signal and the A0 packet of the response signals of test numbers 1-10 were used to create the first series of structural identification models. Model validation was performed with signals from test number 11-20, so model 1 was validated with signals from test number 11 and so on. New of these series are created for the other damaged conditions detected as well, similarly to what was performed for the thresholds.

The compromise to generate structural identification models "dedicated" only to the structural condition that they were created led to a order choice of $n_a = 4$ and $n_b = 3$. The first ARX model was created with the signals

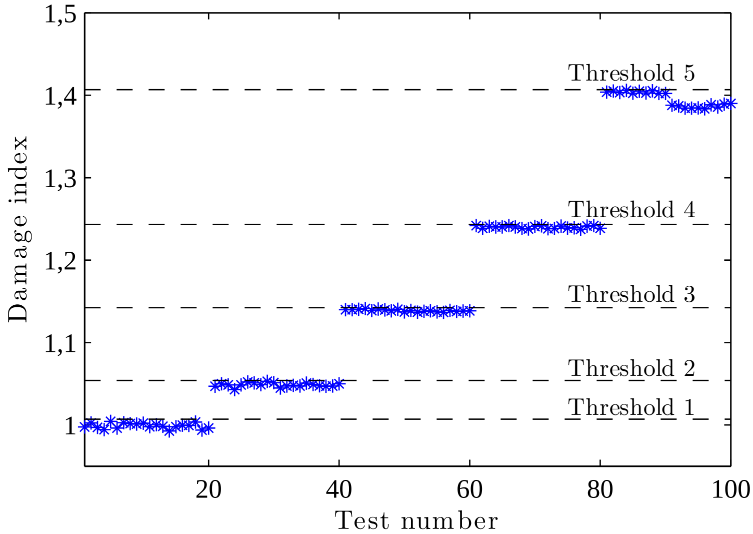


Figure 7: **Damage indices (γ index) and thresholds for damage detection.**

from test 1 and validated with the response signal from test 11 showing an excellent fit percentage higher than 99% and a minimum level of residues in the response similarly to the validation of the AR model used for damage detection. All other structural identification models obtained for reference and damaged conditions presented similar results. Such identification capability was evaluated by the data dispersion in the prediction errors since this parameter should be minimized when both model and signals are related to the same structural condition, as discussed before. Each group of 10 structural identification models generated a set of prediction errors whose standard deviation was calculated. Their mean value and dispersion is presented in Fig. 8 evidencing the desired characteristic.

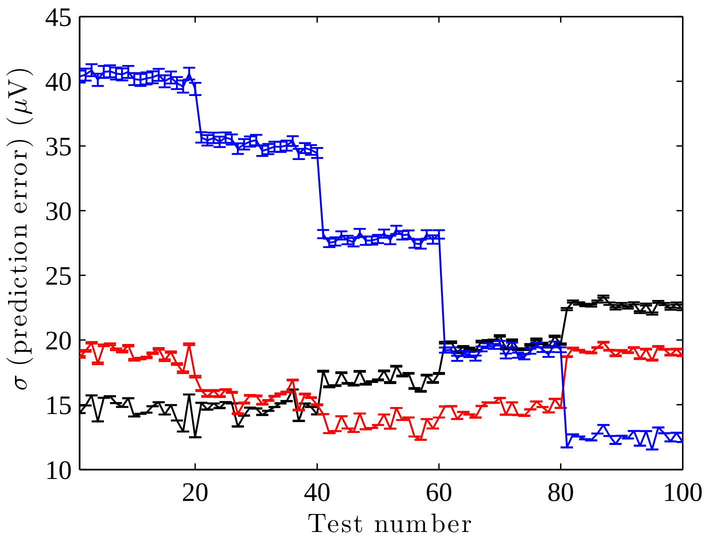
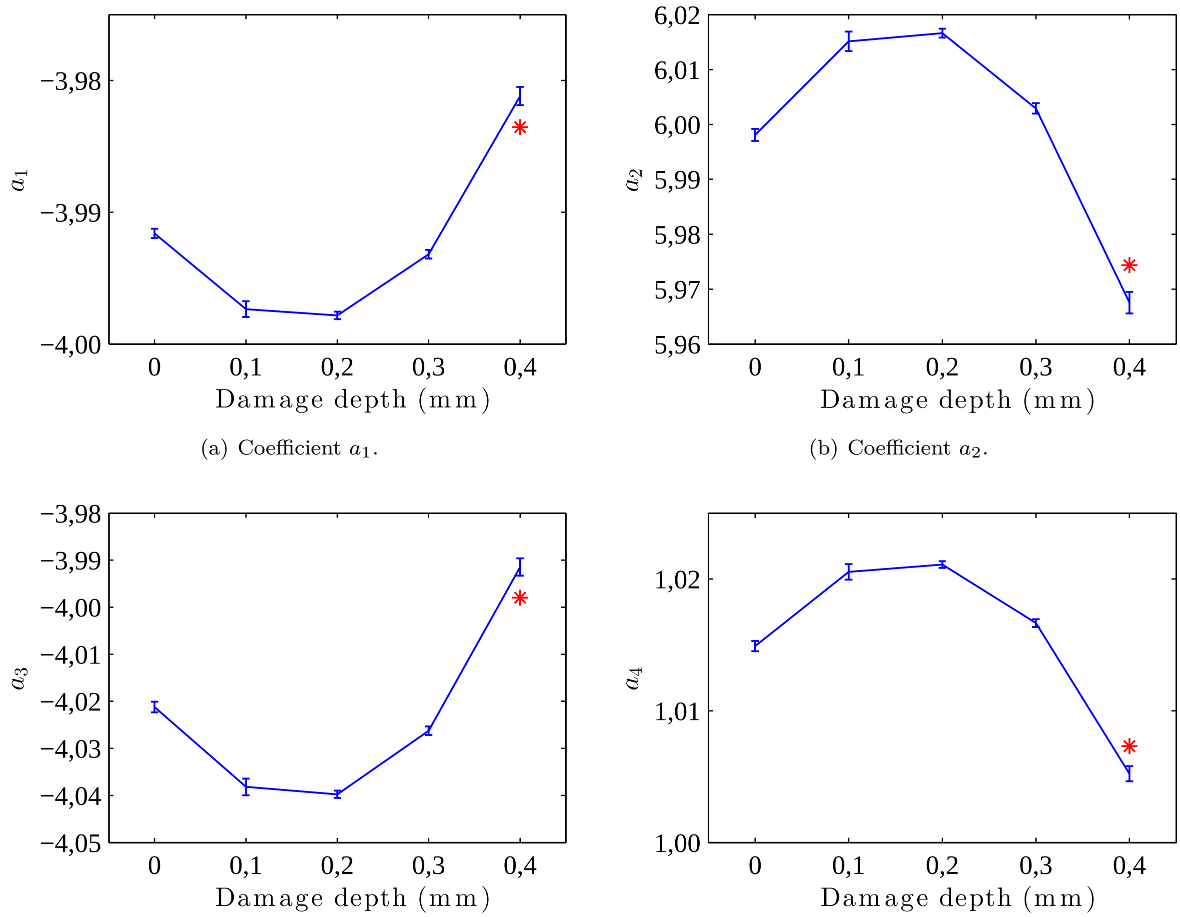


Figure 8: **Mean value and dispersion of standard deviations obtained from prediction errors generated by structural identification models of: ——— reference condition; ——— second damaged condition; ——— fourth damaged condition.**

The prognosis is accomplished by the extrapolation of ARX model coefficients for the construction of theoretical predictive models. The basic assumption is that the damage evolution follows the same position and type so any new damage is inserted in the prediction. Since extrapolation aims to provide the value of a variable out of the range of known values, the trend of the preceding data is taken into account. However, the accuracy expected in the extrapolated coefficients is directly related to their behavior. A visual analysis evidences the complex evolution of coefficient values for the conditions modeled. Thereby, a preliminary approach is to only attempt to predict the response signal of the last damaged condition (0,4 mm of damage depth) taking into account all previous condition tested, as seen in Fig. 9 and Fig. 10. The best extrapolation result was obtained using the spline interpolation method.

The computation of these theoretical models follows the opposite way of the autoregressive model estimation.

Thus, the extrapolated coefficients form the polynomials $A(q)$ and $B(q)$ and the predictive response can be estimated. The success of the forecasting procedure is evidenced by Fig. 11 since the general decreasing behavior of the prediction errors' standard deviation with the increase of damage severity was reproduced. Also, the predictive model generated prediction errors with levels of data dispersion similar to the structural identification models, as well. Therefore, this methodology was able to produce a model to reasonable forecast structural behavior employing only measured data without the need of any physics or mathematical based models.



(a) Coefficient a_1.

(b) Coefficient a_2.

(c) Coefficient a_3.

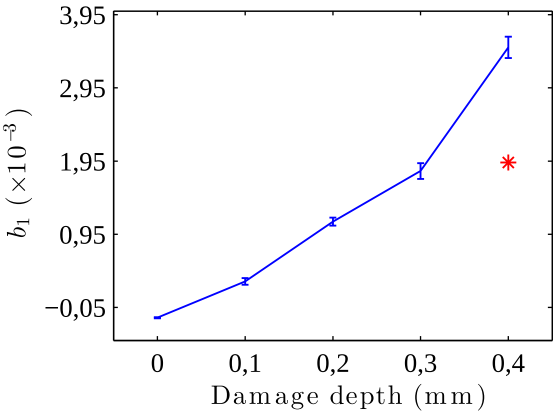
(d) Coefficient a_4.

Figure 9: **Coefficient comparison for polynomial $A(q)$: —— from structural identification models; $*$ − for theoretical predictive model (extrapolated).**

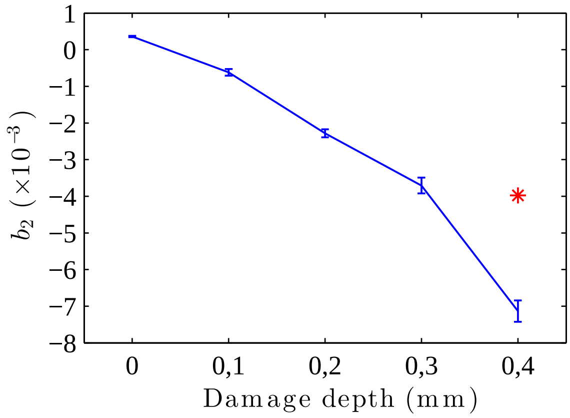
Despite of this promising preliminary result in modeling a future damaged condition, it is important to note that only a limited damage scenario was considered. Since wave propagation is ruled by many factors, the right shift of A0 wave packet with damage progression could be interrupted beyond a thickness reduction. This eventual break of pattern could compromise the predictive ability of these theoretical models because of the dependence on coefficient extrapolation. Moreover, damage location and progression remained the same throughout the experiment, what would hardly happen in a real world application. Therefore, the occurrence of a new type of damage or in a different location has to be studied in order to improve prognosis robustness. Finally, the associated inverse problem is not trivial because of the black box nature of ARX models or, in other words, because of their exclusive dependence on the system's response without a clear physical link.

4. FINAL REMARKS

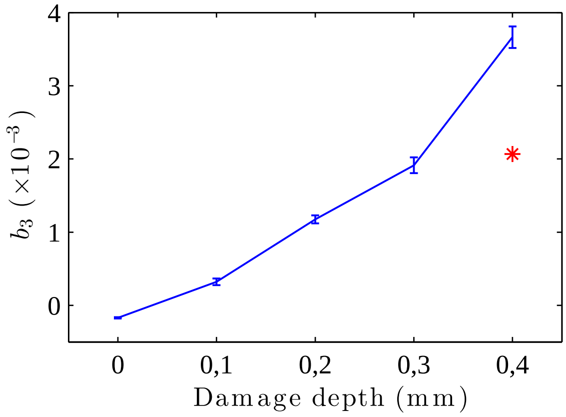
This chapter presented the achievements of a damage detection routine based on statistical process control with the ability to detect discrete levels of a progressive damage as small as 0,1 mm of depth (6,25% in thickness reduction) with statistical confidence. However, it is important to highlight that a high confidence interval of 99.73% had to be used in order to avoid false indications of damage. Also, despite of the caution to keep test conditions constant, some measurement variability was observed throughout the tests which contributed to the increase of the confidence interval as well (temperature variations evidenced by the clear change of damage index value between test numbers 81-90 and 91-100 – last damaged condition). Finally, the statistical distribution of the damage indices could be evaluated to ensure that normal distribution assumption is accepted or the use of other types of data distribution is justified.



(a) Coefficient b_1.



(b) Coefficient b_2.



(c) Coefficient b_3.

Figure 10: **Coefficient comparison for polynomial $B(q)$: ——— from structural identification models; $*$ – for theoretical predictive model (extrapolated).**

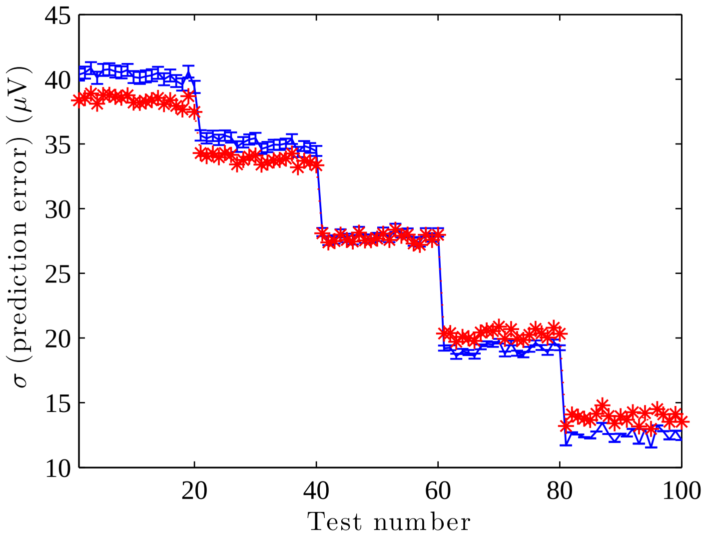


Figure 11: **Comparison of the mean value and dispersion of standard deviations obtained from prediction errors generated by the structural identification models for the fourth damaged condition (——— 0.4 mm of damage depth, test numbers 81-100) and the absolute value of this parameter generated by the theoretical predictive model ($*$).**

Moreover, because of the complex behavior of the identified ARX models' coefficients, a preliminary damage prognosis technique was attempted in order to forecast the structure's behavior. This strategy satisfactorily predicted the structure's dynamic response for a damage of 0,4 mm o depth (25% of thickness reduction) taking into account four previous structural conditions and using the data dispersion verified in the prediction errors as the parameter of evaluation.

The drawbacks of this predictive procedure can be associated with the limited damage scenario considered. To make this methodology more suitable to real world applications, it is desirable to ensure the prognosis of more severe thickness reductions. Also, the occurrence of different types of damage in different locations in a changing environment has to be evaluated. A suggestion to overcome these drawbacks will be to associate an analytical wave propagation model of a plate by spectral elements (SEM) with the coefficients of the ARX model. In this scenario, it would be possible to predict complex damages caused by multiple discontinuities. The idea is to choose a SEM wave model validated with the autoregressive model identified. Future research in this sense is welcome.

5. ACKNOWLEDGEMENTS

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7. AUTHORIAL RESPONSIBILITY

The authors are the only responsible for the content of this work.