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# STATE ESTIMATION TECHNIQUES FOR ROTOR UNBALANCE IDENTIFICATION

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***Abstract.*** *Global warming has been an issue for the international community since the beginning of 90s. As a result the scientific community focused the attention on the research of green house gases (GHG) emission reduction. Recently, new light has been shed as well on energy efficiency solutions. On one hand this means looking for strategies to exploit at best new energy sources for electric energy production. On the other hand energy efficiency is enhanced by monitoring techniques able to check and ensure the reliability of the energy plant. In this framework, the concept of virtual sensing plays a key role. Several physical quantities are relevant to determine the amount of input energy that is dissipated and not conveyed into useful output power. Unfortunately, these quantities cannot be directly measured in many practical applications. As a consequence, state estimation techniques come out as an extremely helpful tool. The combination of the available measurement data with the information provided by the system dynamic model allows the estimation of different unmeasurable system quantities. In the current research a Kalman Filter based algorithm is applied to investigate and assess the health condition of a rotor system with particular attention to the unbalance estimation.*

***Keywords:*** *Kalman Filter, Rotordynamics, Unbalance, Virtual Sensing*

## 1 Introduction

Energy efficiency and energy management are currently two of the key topics in engineering research. The common goal is to achieve high energy efficiency reducing the environmental impact and the energy costs. New solutions for energy storage and energy harvesting are under development to reduce the ecological footprint of machines and vehicles. An important role is played as well by the continuous improvements in the renewable energy systems. Renewable energy sources are primarily those which are inexhaustible in nature. Obvious examples are hydroelectric power, solar energy, wind power and also renewable waste and biomass. Other sources as geothermal energy and ocean gradient energy are often included in the "renewable energy" group. These kinds of energy sources are derived from the thermal energy stored in the crust of the earth and will not be completely sustainable in the very long term scenario (Evans, 2007). Whichever primary energy source we consider it should flow through an energy conversion chain in order to provide energy to the final user. An intuitive example is the car. While driving our car, we use the engine in order to convert the gasoline energy into mechanical work to power the wheels. The primary energy source is in this case the petrol that undergoes several chemical transformations to become gasoline that can be considered as an energy carrier. The same applies for renewable energy sources. Solar power or wind power travels through a conversion chain in order to be converted into electricity. The global efficiency of an energy plant depends on the efficiency of this first conversion. The less energy losses are present in this phase the more electricity will reach the final user. Many renewable energy plants base the first conversion step on a technology that is typical of conventional energy plants: steam or gas turbine power generation. Turbine based powertrains have been extensively studied during past years (Muszynska, 2005; Korpela, 2012). The attention has been focused on several aspects, among others on fault detection (Pennacchi and Vania, 2005). This last aspect is of crucial importance from the powertrain efficiency point of view. The most common malfunctions that can affect a turbine powertrain are as well source of vibrations. The energy the system loses in vibration will not be converted into useful energy. This paper focuses on the first source of vibrations for a powertrain, unbalance, and on the application of a virtual sensing technique for the early stage unbalance detection and evaluation. The first section will provide some details about the adopted dynamic model while the virtual sensing technique will be presented in the second section. The third section focuses on the obtained numerical results and final conclusions are reported at the end of the paper.

## 2 Rotordynamics modeling

Rotors are fundamental components in several industrial applications. Disregarding the specific application, the health of a rotor power train plays a key role for the plant efficiency. Among the different kinds of malfunction that can affect a rotor power train unbalance is one of the most important and the sooner it can be detected the better. Unfortunately, unbalance cannot be directly measured. Current diagnostics techniques are based mainly on the post-processing of the induced vibrations. This research work instead fits in a virtual sensing framework. Applying state estimation algorithms it is possible to detect the presence of unbalance and/or misalignment and extract the unbalance location and/or magnitude. The system modeling phase plays a key role for the functioning of the virtual sensing algorithm. The basic concept of this approach is indeed to add knowledge to the available measurements through a dedicated modeling technique being then able to extract more information from the experimental data. Several mathematical models have been proposed during past years to describe the dynamics of a shaft-rotor system (Muszynska, 2005). The current paper relies on a model available in literature: the isotropic rotor model.

The adopted model is a basic rotor model (Childs, 1993). The mass is concentrated in the rotor and the shafts are modeled as slender beams. The motion of the rotor consists of two lateral transversal modes: vertical and horizontal. The two motions are not coupled and the result is a rotor lateral circular orbital motion. Lateral modes are important for rotor dynamics since there always exists a potential exciting force for those modes: the unbalance. Unbalance can be described as rotor uneven mass distribution. From the modeling point of view, this effect can be simulated placing the rotor center of gravity (COG) eccentrically with respect to the axis of rotation (Fig. 1).

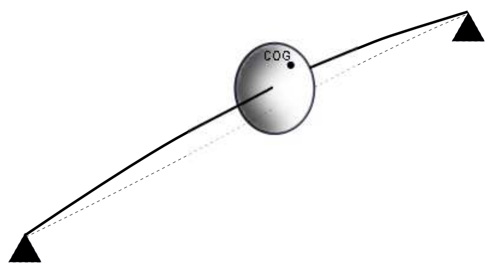


Figure 1. Isotropic rotor model.

The eccentricity of the COG results in an external exciting force with rotating synchronous character, i.e. the force frequency is the rotor rotating frequency. Different assumptions have been made in the rotor modeling process, the most important being (Muszynska, 2005):

- the rotor is rotating at a constant rotational speed $\Omega$
- the gyroscopic effect is neglected
- the model is linear: all the considered forces are linear either in time or in displacement, velocity or acceleration
- supports are considered elastic in both x and y direction

The governing equations of the model of Fig. 1 can be obtained starting from the expressions of potential, dissipative and kinetic energy through the Lagrangian approach:

$$\begin{aligned} M\ddot{x} + D\dot{x} + Kx &= F\cos(\Omega t + \delta) \\ M\ddot{y} + D\dot{y} + Ky &= F\sin(\Omega t + \delta) \end{aligned} \tag{1}$$

The symbols *M*, *D*, and *K* in Eq. (1) are respectively the rotor mass, the damping associated with the shafts and the stiffness resulting from the series of flexible supports and shafts. The dot accent symbols the derivative with respect to time. The force *F*, projected on the two axes x and y, originates from the unbalance and is a function of the unbalance radius *r* and the square of the constant rotational speed $\Omega$ (Eq. 2) .

$$F = Mr\Omega^2 \tag{2}$$

This simple model can be extended to a more generic case of n rotors connected in series (Fig. 2). The model foresees two rotors, rigidly coupled, with flexible supports. As in the previous model, the shafts are considered massless and all the mass is concentrated in the rotors.

This second model represents a more realistic case study and is as well more interesting from the virtual sensing point of view. In section 4, we will show how virtual sensing allows to reduce the total number of sensors requested to detect unbalance. In the following section instead few details about the adopted state estimation algorithm will be provided.

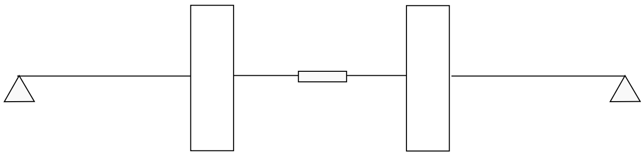

Figure 2. Series of connected rotors.

## 3 Virtual sensing algorithm

The adopted virtual sensing algorithm is based on the Kalman Filter (KF) theory (Simon, 2006). The KF is an iterative solution to the problem of filtering experimental data through a linear filter: it is very effective in evaluating the state of a dynamic system affected by noise with Gaussian distribution and null mean value. KF can be used as well as a state observer to predict unmeasured or unmeasurable system quantities. The estimation performed by the KF consists of a prediction and a correction step. The former is based on the numerical model of the system, usually described in terms of state variables, the latter on a suitable weighting of the difference between the predicted and the collected measurement data. In the discrete time domain, at time step $k$, the system dynamics can be expressed as:

$$\begin{aligned} \mathbf{z}_k &= \mathbf{F}_{k-1}\mathbf{z}_{k-1} + \mathbf{G}_{k-1}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \\ \mathbf{y}_k &= \mathbf{H}\mathbf{z}_k + \mathbf{v}_k \end{aligned} \tag{3}$$

where:

- $\mathbf{z}$ is the state variables vector
- $\mathbf{F}$ is the discrete time state matrix
- $\mathbf{G}$ is the discrete time input matrix
- $\mathbf{u}$ is the input force vector
- $\mathbf{w}$ is the process noise
- $\mathbf{y}$ is the measurement vector
- $\mathbf{H}$ is the observation matrix
- $\mathbf{v}$ is the measurement noise

The process and measurement noise, $\boldsymbol{w}$ and $\boldsymbol{v}$, are white, zero-mean, uncorrelated with known covariance matrices $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$ respectively.

The KF algorithm is a two step procedure. The first step, the prediction or the a priori state estimate step, is based on the system model:

$$\hat{\mathbf{z}}_k^- = \mathbf{F}_{k-1}\hat{\mathbf{z}}_{k-1}^+ + \mathbf{G}_{k-1}\mathbf{u}_{k-1} \tag{4}$$

The second step, the correction or the a posteriori state estimate step, starts from the a priori estimate (4) and corrects it according to the difference between the predicted and the available measurements:

$$\hat{\mathbf{z}}_k^+ = \hat{\mathbf{z}}_k^- + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{z}}_k^-) \tag{5}$$

The matrix $\boldsymbol{K}_k$ in (5) is the Kalman gain matrix that is a function of the noises covariances, the system matrices and the covariance of the state estimation error.

Starting from the basic KF formulation (4, 5) different extensions of the KF have been made (Simon, 2006). Among them the Augmented Kalman Filter (AKF) allows to estimate the unknown forces acting on the system including them in the state vector. This particular extension of the KF is the one that will be applied to the systems of Fig. 1 and Fig. 2 to estimate the investigated system malfunction: the unbalance. The prediction - correction steps involved in the AKF are exactly the same as presented in Eq. (4) and (5) for the original KF formulation. The differences arise in the expression of the state vector and state matrix. The first equation in Eq. (3) becomes:

$$\mathbf{z}_{\mathbf{a}k} = \begin{bmatrix} \mathbf{F}_{k-1} & \mathbf{G}_{k-1} \\ 0 & 0 \end{bmatrix} \mathbf{z}_{\mathbf{a}k-1} + \begin{Bmatrix} \mathbf{w}_{k-1} \\ \mathbf{w}_{u,k-1} \end{Bmatrix} \quad \mathbf{z}_{\mathbf{a}k} = \begin{Bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{Bmatrix} \tag{6}$$

Table 1. Simulation parameters.

| Parameter | Value |
|---|---|
| Rotor mass | 10 kg |
| Shaft stiffness | 4000 N/m |
| Support stiffness | 500 N/m |
| Eccentricity | 0.005 m |
| Rotational speed | 62.832 rad/s |

In Eq. (6) $\boldsymbol{w}_{u,k-1}$ is the noise associated with the unknown input. $\boldsymbol{w}_{u,k-1}$ satisfies the same hypothesis as $w$ and $v$. In absence of a better modeling available, the updating equation for the unknown input can be expressed according to the random walk model as follows:

$$\mathbf{u}_k = \mathbf{u}_{k-1} + \mathbf{w}_{u,k-1} \tag{7}$$

The methodology described in the current section has been applied to the models of section 2 to estimate the unbalance forces deriving from the COG eccentric position based on the measurements of the displacement of the shaft.It is indeed common practice in rotordynamics (Muszynska, 2005) to measure the displacements of the shaft centerline using proximity probes in order to analyze the structural vibrations and diagnose the malfunctions that are determining them. The obtained numerical results are presented in the following section.

## 4 Numerical results

The current section presents the numerical results obtained for the unbalance force estimation in the two modeling cases presented in section 2.

### 4.1 Input estimation on a single rotor

The first results are related to the model of Fig. 1. The model parameters are listed in Tab. 1.

The Newmark - Beta method has been used to integrate the system equations Eq. (1) and obtain the shaft centerline displacement, velocity and acceleration in presence of unbalance. The integration time step has been set to $10^{-4}$ s and the selected integrator parameter are $\gamma$= 0.5 and $\beta$= 0.25. The simulation lasts for 5 seconds and in Fig. 3 the last second is presented. The value of the rotational speed is relatively low with respect to the average regimes of powertrains. This doesn't affect the validity of the reported results since the aim is to validate the proposed virtual sensing methodology. The expected force is sinusoidal and, according to the parameters listed in Tab. 1, characterized by a peak to peak value of 400N and a frequency of 10Hz. The Root Mean Square (RMS) of the error computed according to Eq. (8), absolute value of the difference between the actual force $F$ and the estimated one $F_e$ has been chosen as indicator of the estimation quality.

$$Error = |(F - F_e)| \tag{8}$$

The obtained X and Y displacements have been used as input for the virtual sensing algorithm. A fictious gaussian noise has been added to the input signal to simulate the measurement noise in order to obtain a Singal to Noise Ratio (SNR) of 30 dB. In Fig. 4 the comparison among the actual input forces $F$ and the estimated ones $F_e$ is displayed. The developed virtual sensing algorithm allows to identify the existing unbalance forces with high accuracy. The RMS of the error between the estimated and actual forces acting on the system, defined according to Eq. (8), is in this first analyzed case about 1.3 N on a peak to peak value of the force of 400 N.

### 4.2 Input estimation on the two rotors system

The approach described in section 4.1 has been applied to the two rotors system of Fig. 2. Both shaft-rotor systems are characterized by the parameters listed in Tab. 1. The results reported in Fig.5 shows the effectiveness of the proposed methodology as well in this second case study. Due to the symmetry of the system, the unbalance forces are the same for both rotors. According to the parameters listed in Tab. 1, the forces acting on the system are characterized by a peak to peak value of 400N and a frequency of 10Hz. As in the previous case the input signal is characterized by a SNR of 30dB. The RMS of the error, computed according to Eq. (8), sets around 0.94 N on a peak to peak value of the force of 400 N.

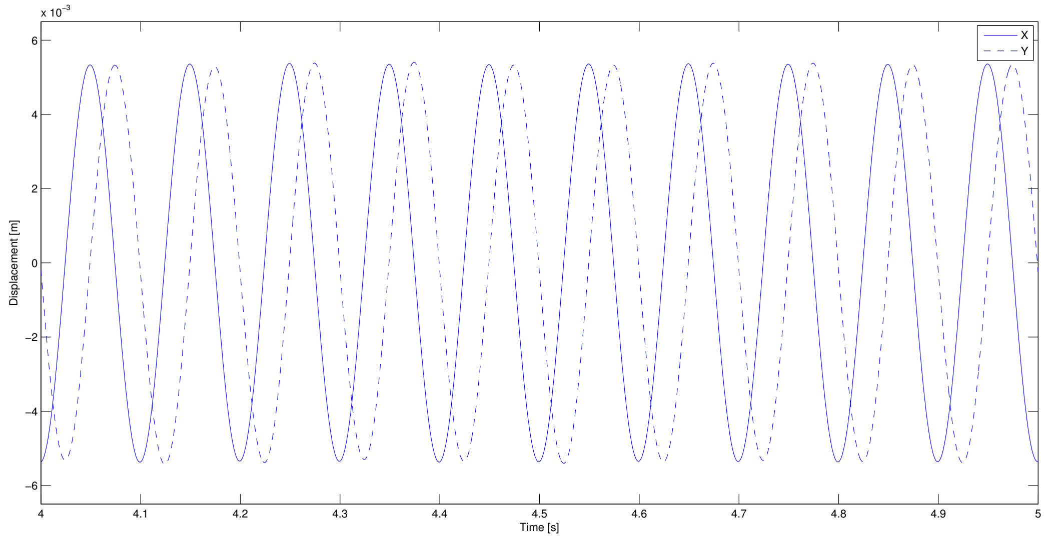


Figure 3. Shaft centerline displacements.

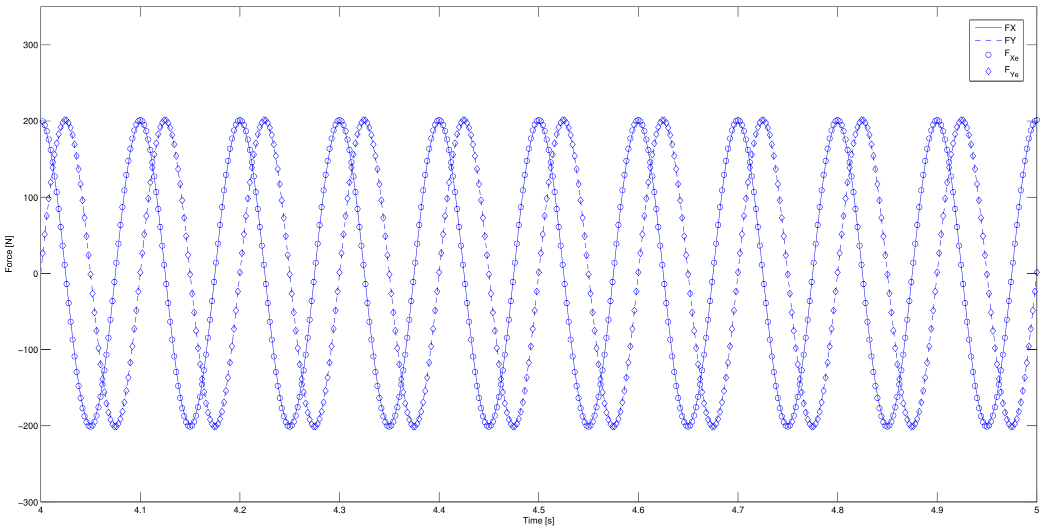


Figure 4. Unbalance force estimation on a single rotor.

Two additional realistic cases have been simulated considering the system of Fig. 2:

- Variable rotation speed
- Constant rotation speed and variable unbalance

The virtual sensing algorithm has been applied in both cases considering as input measurements the shaft centerline displacement of the first rotor. The results are presented in the following figures Fig. 6 and Fig. 7 respectively. The proposed algorithm appears to be effective as well in these two particular cases. In the first case the rotational frequency varies between 0 Hz and 10 Hz during the simulation time of 5 s. Due to the symmetry of the system, the unbalance forces are the same for both rotors and in Fig. 6 only the forces acting on the first rotor are displayed. In the second case, instead, the rotational frequency is fixed at 10 Hz. During the last 0.5 s of simulation the unbalance doubles on the first rotor and quadruplies on the second rotor. This second case study underlines the reactivity of the algorithm to sudden changes in the estimated quantity.

In the simulation of the two rotor system under variable rotational speed the addition of fictious noise (30dB SNR) to

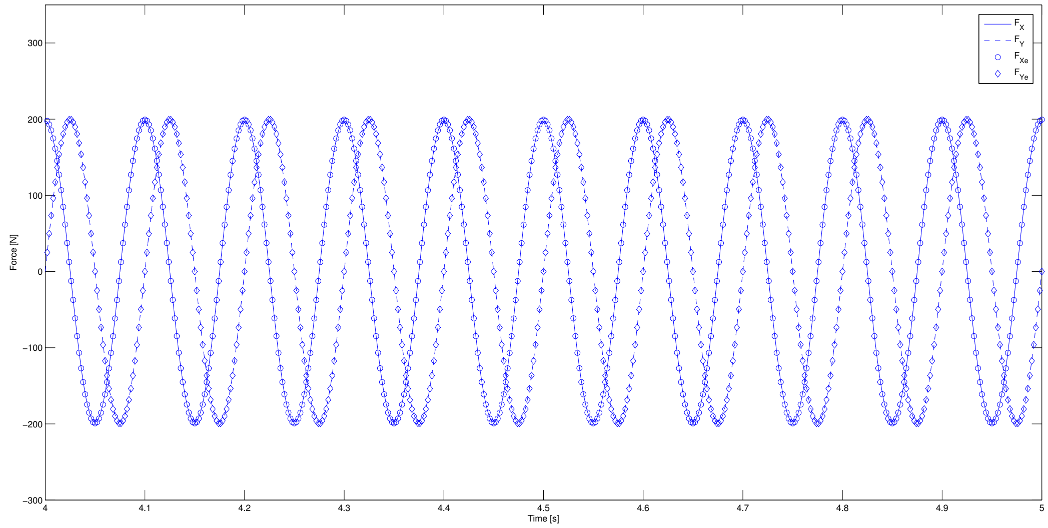


Figure 5. Unbalance force estimation on a two rotor system.

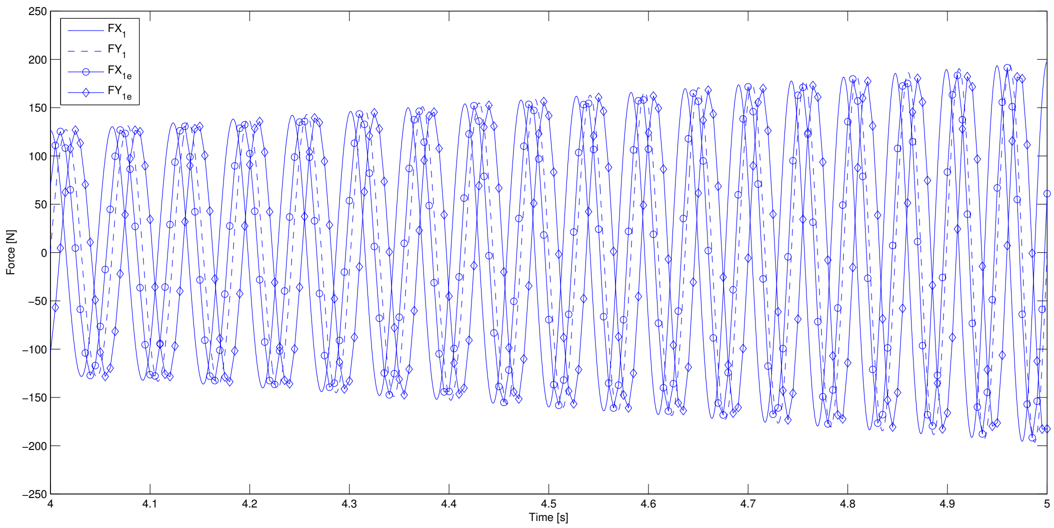


Figure 6. Unbalance force estimation on a two rotor system under variable rotational speed.

the input data leads to an accurate estimation characterized by a time lag of 0.01s. The RMS of the error is equal to 2.5 N.

In the second case study, constant rotational speed and variable unbalance, different SNRs have been considered for the input signal (respectively equal to 30, 10 and 3 dB). The simulations have been run without re adapting the AKF covariance matrices and in all the simulated cases the estimation presents no time lag as in Fig. 7, the RMS of the error, computed according to Eq. (8), is in all the simulated cases lower than 7.5%. A fine tuning of the AKF parameters may lead to lower error RMS values. Anyhow, the current results aim to show the algorithm reliability to changes in the quality of the input signal.

## 5 Conclusions

The current research deals with the application of virtual sensing techniques in an industrial framework. Rotor powertrains are the core component of several energy conversion plants. The efficiency of these plants is directly connected to the good functioning of the powertrain itself. Powertrain malfunctions induce structural vibrations. Given a certain amount of available input energy the machine wastes part of it in vibrations and the global system efficiency decreases. As a consequence, the sooner it is possible to detect the malfunctions the better. The idea behind this paper is to combine measurements with a state estimation algorithm in order to be able to detect and identify unbalance forces minimizing

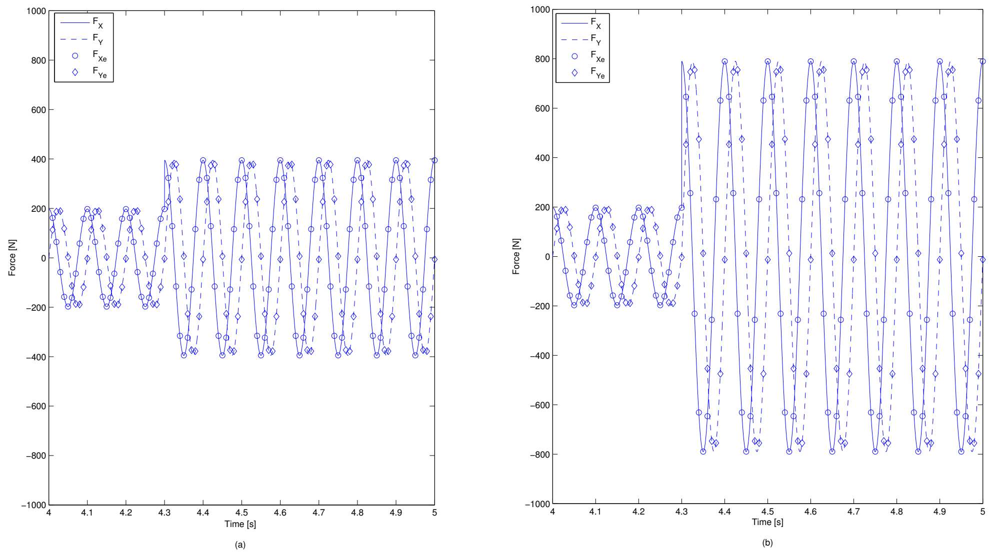


Figure 7. Unbalance force estimation on a two rotor system with variable unbalance: (a) first rotor, (b) second rotor.

as well the number of sensors needed. The presented results show how the virtual sensing procedure allows to quantify the unbalance forces starting from displacement measurements. In summary, the interaction between elements of classic rotordynamics theory and further development of the Kalman Filter, leads to a diagnostic system able to enhance the efficiency of the investigated powertrain. A special novel test rig has been designed and just built and the authors are planning to validate and estimate the effectiveness of the method in the very close future.

## 6 Acknowledgements

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