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# SIDESLIP ANGLE ESTIMATION FOR GROUND VEHICLES

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***Abstract.*** *The design of a robotic vehicle of Autonomous Mobility Laboratory (LMA) at UNICAMP is developed in-vehicle platform FIAT-PUNTO. In addition to a set of sensors, actuators, mechanisms and components (hardware and software), new technologies should be developed to support the Automation, Control, Perception, Localization and Navigation. The purpose of this work is to present a non-linear model of the vehicle, a tire dynamics model and a stochastic estimation model for designing virtual sensors that estimate several critical parameters of the vehicle dynamics. The simulations are executed applying the extended Kalman filter (EKF), and the obtained data are evaluated with the acquired data by software simulation.*

***Keywords:*** *Sideslip Angle, Kalman Filter, Autonomous vehicle, Simulation.*

## 1. INTRODUCTION

Vehicular safety is a crucial item that needs attention from all governmental institutions around the world. Different researches have shown an increase in the number of accidents due to human errors (Waiselfisz, 2013) (NHTSA, 2015). However, many countries and vehicles manufacturers have invested a lot in the development of new systems that improve the vehicular safety.

Nowadays, autonomous vehicles are a promising option for the future of cars circulation. A new experience and challenge is opened: designing safe, comfortably and fast vehicles. But safety is the main goal to achieve before the decision of autonomous vehicles reach the current security standards for passengers and pedestrians.

Vehicular safety systems have improved and taken a great importance in recent years as a result of the evolution of techniques for acquiring variables involved in vehicular maneuverability, costs of installation in new vehicles and recent health and safety policies of governments around the world. The main function of vehicular parameters estimation is the development of advanced technologies that increase safety and driving performance, meanwhile increasing the cost efficiency (Piyabongkarn *et al.*, 2009). Vehicular parameter estimation allows the design of vehicles that are more safe with feasible prices for massive production. That goal is accomplished with a good designing of the states observer.

The selection of the states observer technique has been discussed widely. The literature describes many observers techniques. The Luenberger and the Extended Luenberger observers are parameter estimation techniques for linear and non-linear systems respectively, applied in vehicular systems (Stéphant *et al.*, 2004). The sliding mode observer is a robust controller technique used to estimate the vehicle sideslip angle (Zhang *et al.*, 2009) (Stéphant *et al.*, 2004). The particle filter is a technique for discrete approximation of non-linear dynamic systems with a non-Gaussian probability distribution using particles. It was applied to improve the road safety estimating the lateral forces acting on the vehicle (Wang *et al.*, 2012). The Kalman filter is a stochastic technique used to estimate vehicular parameters in several works (Dakhlallah *et al.*, 2008) (Venhovens and Naab, 1999) (Doumiati, 2009). In this work, an estimation methodology of the vehicle state using the extended Kalman filter (EKF) is considered. Furthermore, the model used for the observer includes a nonlinear Dugoff tire forces model. The measured variables are collected with a vehicle simulator developed in the LMA. The set of sensors of the intelligent vehicle VILMA measure the accelerations, the yaw rate, the steering and the longitudinal velocity of the vehicle. By gathering all these variables and applying the stochastic estimator, several critical vehicle variables are estimated: the global sideslip angle, the lateral forces on the contact patch of the tires and its relation with the vertical forces of the tires.

Figure 1 summarizes the work presented by this paper. The four-wheel vehicle model, the Dugoff tire model and the stochastic estimation method used in this work are presented in section 2. The results obtained are presented in section 3. Finally, the analysis and conclusions of the work are presented in section 4.

## 2. Mathematical Vehicle Model

Figure 2 shows the four-wheel vehicle diagram. The model used in this work is based on the methodology applied for the development of the single track model (Rajamani, 2012) and the four wheel vehicle model (FWVM) (Doumiati,

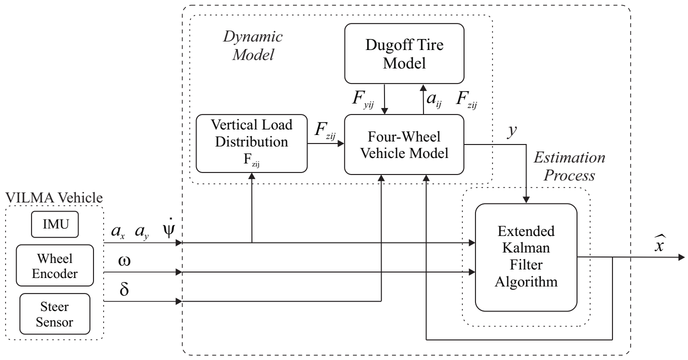


Figure 1. Block diagram

2009). The characteristics of the vehicle model are: the FWVM has 7 degrees of freedom; the longitudinal forces on the rear wheels, the dynamics of suspension, roll and pitch dynamics are neglected. The forces acting on the vehicle are represented by: $F_{zij}$ are the vertical tires forces, $F_{xij}$ are the longitudinal tires forces and the $F_{yij}$ are the lateral tires forces, where $i \in \{1, 2\}$ represents front or rear and $j \in \{1, 2\}$ represents left and right, respectively. The steering angle $\delta$ is simplified assuming $\delta_{11} \approx \delta_{12} \approx \delta$.

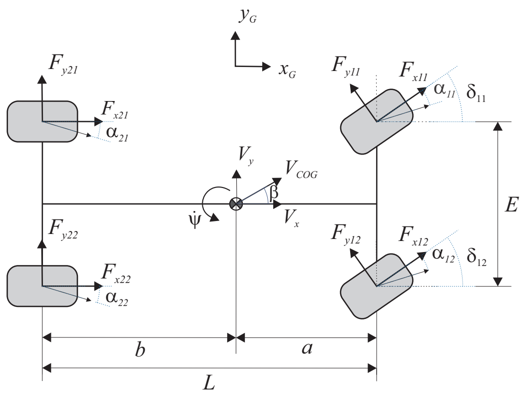


Figure 2. Planar four-wheel vehicle model (Doumiati, 2009)

## 2.1 Vertical Force Calculation

Assuming flat ground, the distribution of vertical forces acting on the vehicle tires are calculated using Eqs. (1)

$$\begin{aligned} F_{z11} &= m_v g \tfrac{b}{2L} - m_v \tfrac{h}{2L} a_x - m_v \tfrac{hb}{EL} a_y \\ F_{z12} &= m_v g \tfrac{b}{2L} - m_v \tfrac{h}{2L} a_x + m_v \tfrac{hb}{EL} a_y \\ F_{z21} &= m_v g \tfrac{a}{2L} + m_v \tfrac{h}{2L} a_x - m_v \tfrac{ha}{EL} a_y \\ F_{z22} &= m_v g \tfrac{a}{2L} + m_v \tfrac{h}{2L} a_x + m_v \tfrac{ha}{EL} a_y \end{aligned} \quad (1)$$

where $m_v$ represents the vehicle mass, $g$ is the gravitational acceleration, $a$ e $b$ are the distances from center of gravity (*COG*) to frontal and rear axle respectively, $L$ is the distance between axles, $E$ is the track width, $h$ is the height of the *COG*, $a_x$ and $a_y$ are the longitudinal and lateral accelerations of the vehicle respectively.

## 2.2 Four-Wheel Vehicle Model

The behavior of vehicle lateral dynamic are described by Eq. (2) (Doumiati, 2009)

$$\begin{aligned} \ddot{\psi} &= \tfrac{1}{Iz} \Sigma M_z \\ a_y &= \tfrac{1}{m_v} \Sigma F_{yij}, \\ a_x &= \tfrac{1}{m_v} \Sigma F_{xij}, \\ \dot{V}_x &= V_y \dot{\psi} + a_x, \\ \dot{V}_y &= -V_x \dot{\psi} + a_y, \end{aligned} \quad (2)$$

where $\psi$ is the yaw angle of the vehicle, $V_y$ and $V_x$ are lateral and longitudinal velocities of the vehicle, $Iz$ is the moment of inertia of the vehicle, $M_z$ is the aligning moment, $F_{yij}$ and $F_{xij}$ are the lateral and longitudinal forces at the contact patch of the wheel.

The tire sideslip angle $\alpha_{ij}$ is the angle between longitudinal axis of the tire and the direction of tire speed. It is calculated with the known and estimated parameters of the vehicle

$$\begin{aligned}
\alpha_{11} &= \delta - \arctan\left[\frac{V_y+a\dot{\psi}}{V_x-E\frac{\dot{\psi}}{2}}\right], \\
\alpha_{12} &= \delta - \arctan\left[\frac{V_y+a\dot{\psi}}{V_x+E\frac{\dot{\psi}}{2}}\right], \\
\alpha_{21} &= -\arctan\left[\frac{V_y-b\dot{\psi}}{V_x-E\frac{\dot{\psi}}{2}}\right], \\
\alpha_{22} &= -\arctan\left[\frac{V_y-b\dot{\psi}}{V_x+E\frac{\dot{\psi}}{2}}\right],
\end{aligned} \tag{3}$$

and the global sideslip angle of the vehicle COG $\beta$ is the angle between the longitudinal axis of the vehicle and the direction of vehicle speed. This parameter influence the lateral force acting on the vehicle (Dakhlallah *et al.*, 2008) and is determined by

$$\beta = \arctan\frac{V_y}{V_x}. \tag{4}$$

Equation (5) describes the linear longitudinal velocities of the rear tires $V_{wx2j}$. The longitudinal vehicle velocity approach is given by obtaining the media of the longitudinal velocities of the rear wheels (Doumiati, 2009)

$$V_{wx2} = R_w w_2 \ \text{ with, } w_2 = \tfrac{w_{x21}+w_{x22}}{2}, \tag{5}$$

where $R_w$ is the effective wheel radius, $w_2$ is the mean angular velocity of rear wheels.

### 2.3 Tire Model

The acting forces on the vehicle are on the contact patch of the tires. Modeling the tire behavior is the most important parameter for obtain safe driving. In the literature exists many models with different approach: the magic formula (Pacejka *et al.*, 1987), the Dugoff model (Dugoff *et al.*, 1970), the F-tire model and the linear model.

The Dugoff tire model is accurate for calculate the longitudinal and lateral tire forces. In comparison with the Pacejka Magic formula, the Dugoff model has a formulation with fewer parameters (Song *et al.*, 2014). The Dugoff tire model describes the tire longitudinal force

$$F_{x|dugoff} = C_\sigma \frac{\sigma_x}{1+\sigma_x} f(\lambda), \tag{6}$$

and the tire lateral force

$$F_{y|dugoff} = -C_\alpha \frac{\tan\alpha}{1+\sigma_x} f(\lambda), \tag{7}$$

with $f(\lambda)$

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda, & if \quad \lambda < 1 \\ 1, & if \quad \lambda \geq 1 \end{cases} \tag{8}$$

and

$$\lambda = \frac{\mu F_z(1+\sigma_x)}{2\left[(C_\sigma\sigma_x)^2 + (C_\alpha\tan\alpha)^2\right]^{\frac{1}{2}}}, \tag{9}$$

where $C_\sigma$ is the longitudinal stiffness, $C_\alpha$ is the cornering stiffness, $\sigma_x$ is the slip ratio and $\alpha$ is tire sideslip angle.

The dynamic behavior of the tire is described by a first order equation. Equation (10) is used for modeling the lateral tire force dynamics

$$\dot{F}_y = \frac{V_x}{\sigma}\left(-F_y + F_{y|dugoff}\right), \tag{10}$$

where $F_{y|dugoff}$ is the lateral force obtained with the Dugoff tire model, $V_x$ is the longitudinal velocity of the COG and $\sigma$ is the tire relaxation length.

### 2.4 Observer Design

The extended Kalman filter is the technique used to estimate the states of vehicular system in this work. Equation (11) describes the nonlinear discrete-time stochastic system, that is represented by the process model and measurement model equations

$$\begin{aligned}\mathbf{x}_{k+1} &= f(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k \\ \mathbf{y}_k &= h(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k\end{aligned} \tag{11}$$

where $\mathbf{x}_k$ represents the state vector, $\mathbf{u}_k$ is the input vector, $\mathbf{y}_k$ is the output vector, the random variables $\mathbf{w}_k$ and $\mathbf{v}_k$ represent the process and measurement noise respectively. The specific equations for time and measurement updates are presented by Kalman (1960) and Welch and Bishop (2001)
Time Update equations:

$$\begin{aligned}\hat{\mathbf{x}}_k^- &= f(\hat{\mathbf{x}}_{k-1}, \mathbf{u}_k) \\ \mathbf{P}_k^- &= \mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^T + \mathbf{Q}_{k-1}\end{aligned} \tag{12}$$

Measurement Update:

$$\begin{aligned}\mathbf{K}_k &= \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \\ \hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{y}_k - h(\hat{\mathbf{x}}_k^-)) \\ \mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-\end{aligned} \tag{13}$$

Where $\hat{\mathbf{x}}_k^-$ is the *a priori* vector states estimate, $\hat{\mathbf{x}}_k$ is the *a posteriori* vector states estimate, $\mathbf{Q}_k$ and $\mathbf{R}_k$ are the process noise and measurement noise covariance matrices respectively, $\mathbf{P}_k^-$ and $\mathbf{P}_k$ are the *a priori* and *a posteriori* estimate error covariance respectively, and $\mathbf{K}_k$ is the Kalman filter gain.

The matrix $\mathbf{A}_k$ is the Jacobian of the dynamics equations around the *a priori* estimate:

$$\mathbf{A}_k = \left[\frac{\partial f(x)}{\partial(x)}\right]_{x=\hat{\mathbf{x}}_k^-} \tag{14}$$

The matrix $\mathbf{H}_k$ is the Jacobian of the measurement equations around the *a priori* estimate:

$$\mathbf{H_k} = \left[\frac{\partial h(x)}{\partial(x)}\right]_{x=\hat{\mathbf{x}}_k^-} \tag{15}$$

The state vector $\mathbf{x}_{1\times n} \in \Re^n$ is

$$\mathbf{x}_k = \left[\ \dot{\psi}\ V_x\ V_y\ F_{y11}\ F_{y12}\ F_{y21}\ F_{y22}\ F_{x1}\ \right], \tag{16}$$

the input vector is

$$\mathbf{u_k} = [\ \delta\ F_{z11}\ F_{z12}\ F_{z21}\ F_{z22}\ ], \tag{17}$$

and the output variables are

$$\mathbf{y_k} = [\ \dot{\psi}\ V_x\ a_x\ a_y\ ]. \tag{18}$$

### 2.5 Sideslip angle estimation using the kinematic model

Leung *et al.* (2011) presents a method for calculate the sideslip angle using the position of the vehicle referenced to the global coordinates framework (G-frame) of the vehicle. In Fig. 3, the G-frame and the v-frame are shown in two-dimensional space with their corresponding subscripts. The heading angle $\psi$ represents the orientation of the vehicle v-frame about the G-frame. It is calculated through the quaternion acquired by the inertial sensor in the simulator. The track angle $\upsilon$ is the angle between the G-frame and the velocity vector of the vehicle's COG. It is calculated by deriving the vehicle's COG position in the G-frame and applying Eq. (19)

$$\upsilon = \arctan\left(\frac{\dot{Y}_G}{\dot{X}_G}\right) \tag{19}$$

The sideslip global angle is obtained applying Eq. (20)

$$\beta_{kinematic} = \upsilon - \psi \tag{20}$$

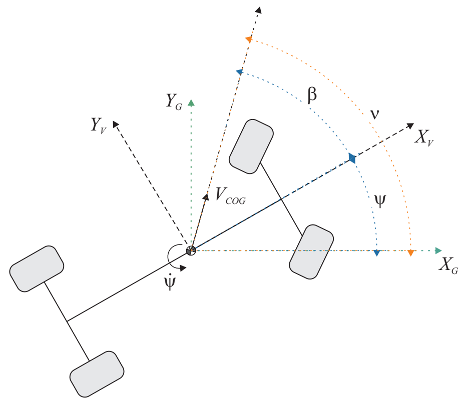



Figure 3. Planar vehicle model (Leung *et al.*, 2011)

## 3. Results

Table 1 summarizes the vehicle characteristics and numerical values of the vehicle platform FIAT PUNTO

Table 1. Vehicle Parameters

| Symbol | Parameter | Value | |
|---|---|---|---|
| $m_v$ | Vehicle mass | 1500 | $kg$ |
| $h$ | Height of the COG | 0.54 | $m$ |
| $a$ | Distance from COG to front axle | 1.3 | $m$ |
| $b$ | Distance from COG to rear axle | 1.5 | $m$ |
| $E$ | Track-width | 1.5 | $m$ |
| $g$ | Gravitational acceleration | 9.8 | $m/s^2$ |
| $C_{\alpha f}$ | Cornering stiffness of front axle | 52000 | $N/rad$ |
| $C_{\alpha r}$ | Cornering stiffness of rear axle | 55000 | $N/rad$ |
| $\sigma$ | Tire relaxation length | 0.3 | $m$ |
| $\mu$ | Tire/ground friction coefficient | 0.9 | - |
| $R_w$ | Wheel radius | 0.32 | $m$ |

The parameters $C_\alpha f$, $C_\alpha r$ are assumed constants, their values are adopted in the range of values for commercial vehicles (Dugoff *et al.*, 1970) (Pacejka *et al.*, 1987). The parameter $\mu$ is assumed constant in the simulated trajectory. The tire relaxation length $\sigma$ is the approximate distance needed to build up tire forces (Doumiati, 2009) (Rajamani, 2012).

### 3.1 Simulation

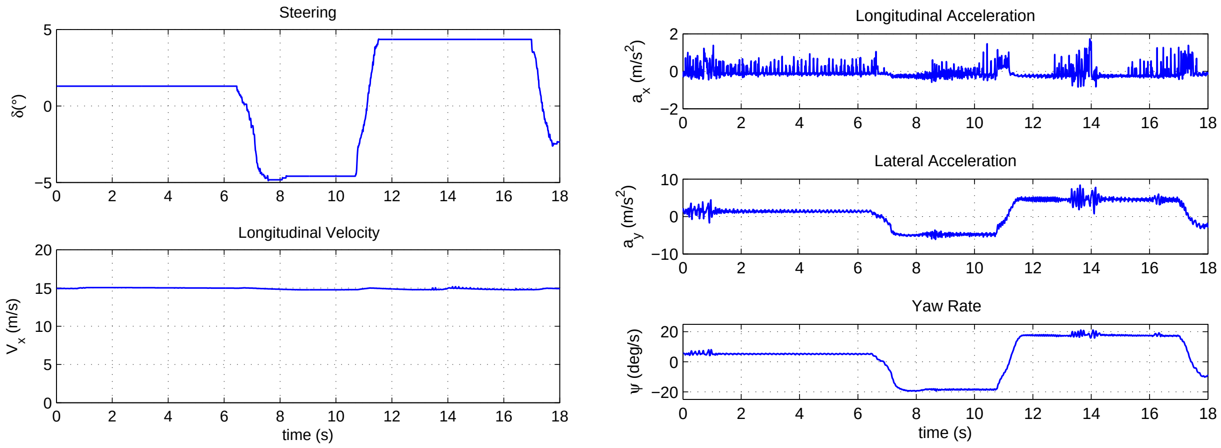
The mathematical model of the vehicle dynamics and the observer EKF was programmed in MATLAB. The acquisition data from sensors was simulated using the vehicle simulator VILMA implemented by the laboratory for autonomous mobility (LMA) in robotic operating system (ROS), interacting with the car model through GAZEBO. The sensors are assumed with gaussian noise, normal distribution, zero-mean value, and the values of standard deviation of each one as: the accelerometer $N \sim (0, 1m/s^2)$, the gyroscope $N \sim (0, 1°/s)$, and the encoder $N \sim (0, 0.1°)$.

Figure 4 shows the signals acquired with the vehicle sensors by VILMA. Those signals are embedded in the EKF algorithm where is computed *offline*. The responses of the systems states estimated are presented in the next figures.

The test maneuver performed by VILMA simulator is depicted in Fig. 5. The test is executed with a car velocity of $15m/s$ $(54km/h)$ and a steering angle range between 5° and $-5°$. Figure 6 shows the response of the tire vertical forces due to the behavior of sensed vehicle accelerations.

### 3.2 Longitudinal Forces Estimation

The front tires longitudinal force estimated $Fx1_{estimated}$, presented in Fig. 7, is compared by longitudinal force $Fx_{model}$, that is determined by the longitudinal acceleration measured $a_x$ times vehicle mass $m$.




(a) (b)

Figure 4. Sensor signals acquired: (a) Vehicle maneuver, (b) Sensors measurements

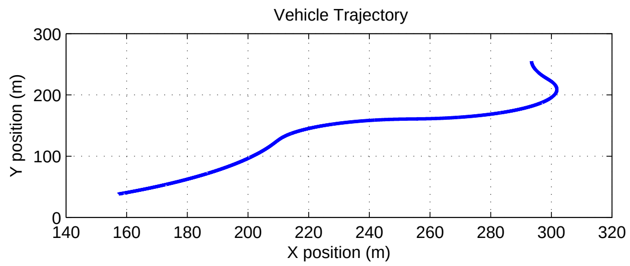



Figure 5. Trajectory of the vehicle. (X,Y) represents the vehicle's COG coordinates during movement

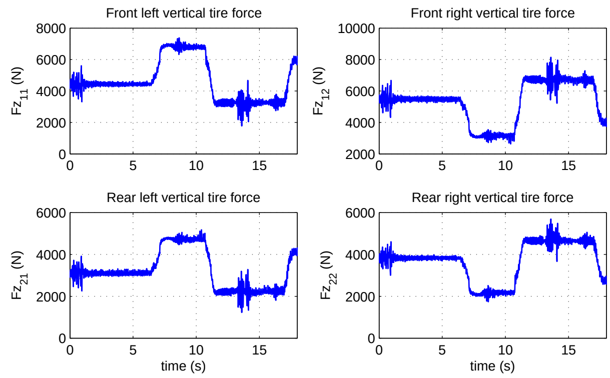
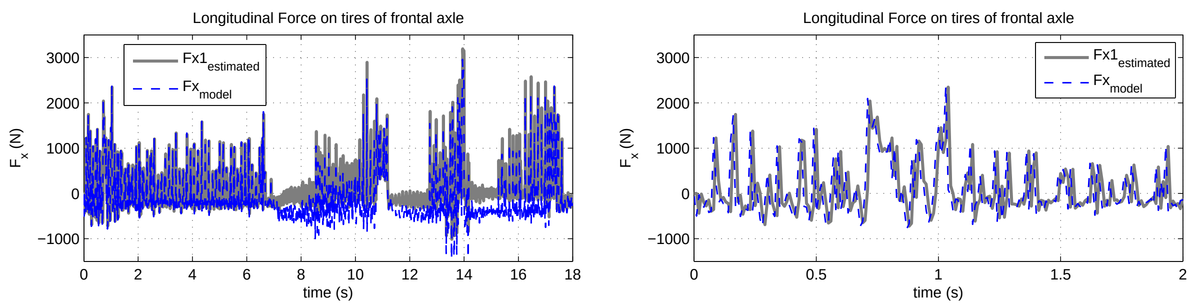



Figure 6. Tires vertical force

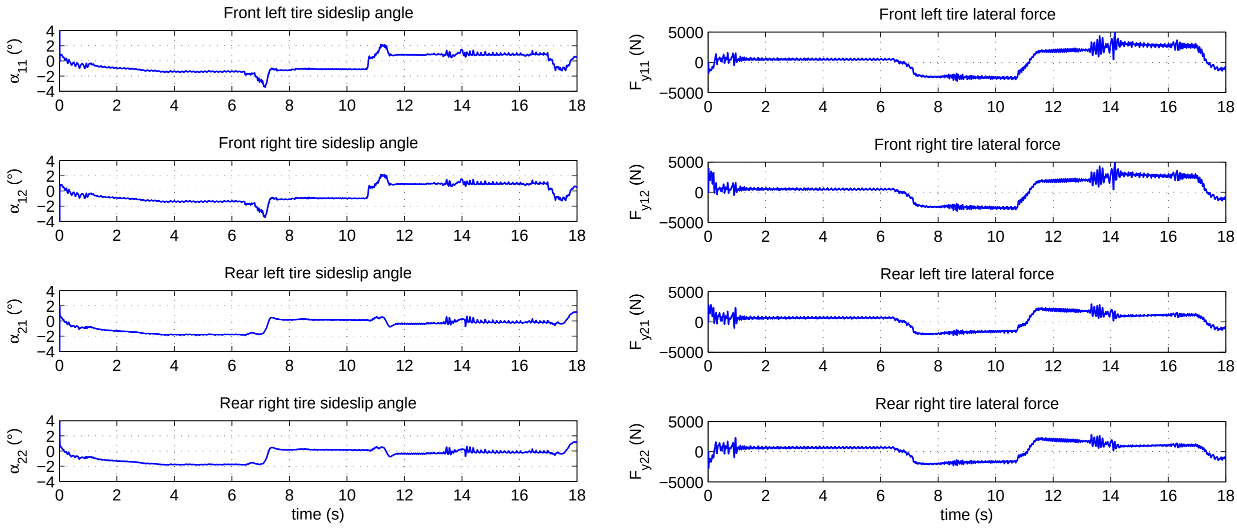



(a) (b)

Figure 7. Longitudinal forces estimation: (a) Front tires longitudinal force estimate, (b) Time interval (0-2s)

### 3.3 Global Sideslip Angle Estimation

Figure 8(a) shows the tire sideslip angle calculated by Eq. (3). Figure 8(b) presents the lateral force estimated in each wheel of the vehicle.




(a) (b)

Figure 8. Sensor signals acquired: (a) Tires sideslip angle estimated, (b) Tires Lateral Force estimated

Figure 9 shows the assessment of the global sideslip angle. The estimated value $\beta_{estimated}$ is calculated by Eq. (4) and the value $\beta_{kinematic}$ is obtained by Eq. (20).

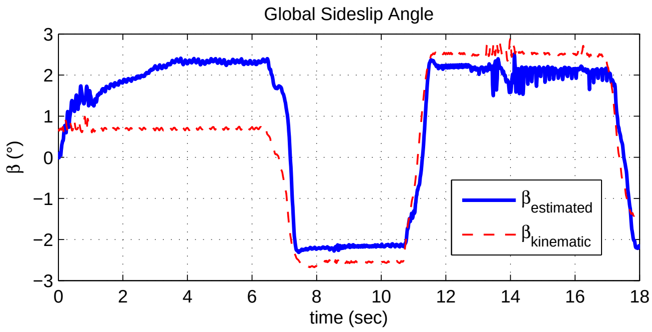



Figure 9. Global sideslip angle

### 3.4 Lateral Friction

The lateral friction coefficient is the ratio between the tire lateral force and the tire vertical force, in the presence of sideslip

$$\mu_{yij} = \frac{F_{yij}}{F_{zij}} \tag{21}$$

Figure 10 presents the relation between the lateral and vertical forces acting on the vehicle's tires.

## 4. Conclusion

The simulation allows interacting with the software and the programmed environment. The main goal of the vehicle dynamics simulation is observing, analyzing and assessing the vehicle behavior under limits of safety conditions.

The extended Kalman filter is used in order to estimate the vehicle critical parameters as the global sideslip angle and the vehicle's tires forces. The estimation of these parameters is relevant for improving the efficiency cost of active safety systems of the vehicle (Dakhlallah *et al.*, 2008). Results shows that estimation of friction coefficient give good results when sideslip angle is significant as previsible. Future work will concern with the validation of the obtained results through simulation. Experiments are required to validate the proposed methodology.

## 5. ACKNOWLEDGEMENTS

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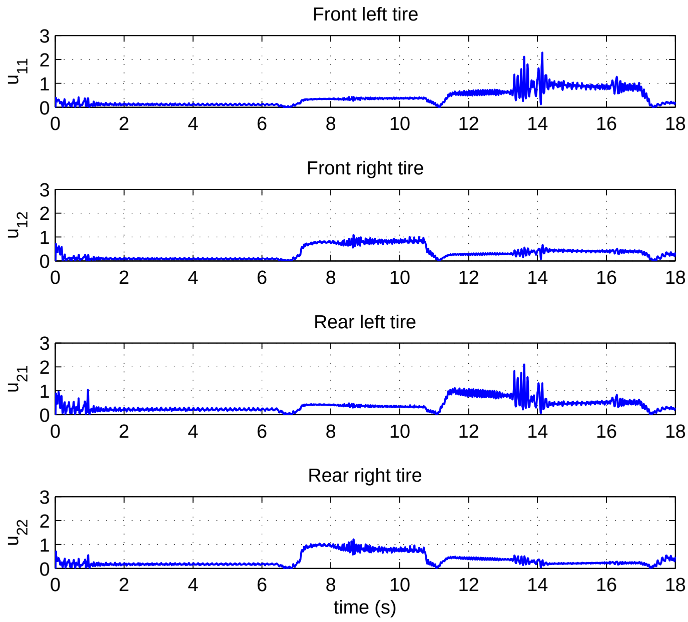


Figure 10. Forces relation $F_{yij}/F_{zij}$

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