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A STUDY ON THE INFLUENCE OF THERMAL STRESSES ON THE DYNAMIC BEHAVIOR OF PLATES

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Abstract. *In numerous situations, structures that work under vibratory loads are subject to temperature variations that cause dilatations or contractions which, once restrained, originate internal stresses. These latter provoke changes in the stiffness of the structure and, consequently, modifies its dynamic behavior. In this context, in the present work the authors develop physical and mathematical models of plate structures, aiming at characterizing the thermal influence on the natural frequencies values of rectangular plates. For this purpose, finite element analyses are carried-out in three phases: in the first, given the distribution of input heat and thermal boundary conditions, the temperature distributions are computed; in the second phase, static structural analysis is performed to compute the thermal stresses. Then, in the third phase, dynamic analysis is performed accounting for the presence of thermal stresses. After the presentation of the underlying theoretical aspects, the results of numerical simulations considering different boundary conditions and aspects ratios of the plate are presented and discussed in terms of the nature and magnitude of influence of temperature on the natural frequencies and mode shapes.*

Keywords: *vibrations, thermal analysis, dynamic behavior, plate models*

1. INTRODUCTION

In many circumstances, structures structural components operating in dynamic regime are subjected to considerable temperature gradients both with respect to time and space, originated from the environmental conditions or even from the structure operation itself. Notable cases are the space structures, subject to heating-cooling cycles due to the exposition to solar radiation, aeronautical structures, which typical temperature variations ranging from –60ºC to 60ºC, and many other types of industrial equipment, such as gas turbines, internal combustion engines, etc.

It has become widely known that non uniform temperature space distribution and constrained thermal expansions/contractions may generate additional stress fields which, in turn, influence the dynamic characteristics of structures such as rods, beams, plates, shells and portal frames (Mead 2003; Flores, 2004). Therefore, in numerous situations it of practical interest it becomes indispensable to consider this influence.

In this context, the present work is focused on numeric-computational modeling procedures of the thermo-mechanical coupling of plate structures. In the sequence, after the presentation of theoretical foundations, finite element simulations are described, putting the thermo-mechanical coupling in evidence and quantifying the degree of influence of the temperature fields on the natural frequencies of a rectangular plate, under uniform and non-uniform temperature distributions and different mechanical boundary conditions. Moreover, as opposed to previous research studies, in which the temperature fields are imposed arbitrarily, in this paper they are obtained from thermal analyses in which heat is prescribed. This procedure is adopted to better represent real situations.

2. THEORETICAL FOUNDATIONS

In this section, the interest is to put in evidence the influence of temperature on the dynamic bending behavior of rectangular plates.

Considering a thin rectangular plate of dimensions *a* and *b*, with thickness *h*, subjected to a temperature distribution $T\left(x,y\right)$, which is considered constant along the thickness of the plate. The temperature distribution generates a plane stress distribution, governed by an Airy stress function, $\phi(x,y)$, in such a way that the stress components are obtained according to (Johns, 1965):

$$\sigma_x\left(x,y\right)=\frac{\partial^2\varphi}{\partial x^2},\quad \sigma_y\left(x,y\right)=\frac{\partial^2\varphi}{\partial x^2},\quad \tau_{xy}\left(x,y\right)=\frac{\partial^2\varphi}{\partial x\partial y} \tag{1}$$

where, $\phi(x,y)$ is the solution of the following equation:

$$\nabla^4\phi(x,y)=-E\alpha\nabla^2T(x,y) \tag{2}$$

The operators appearing in Eq. (1), are defined as:

$$\nabla^2(.)=\partial^2(.)/\partial x^2+\partial^2(.)/\partial y^2 \tag{3}$$

$$\nabla^4(.)=\left[\partial^2(.)/\partial x^2+\partial^2(.)/\partial y^2\right]^2 \tag{4}$$

Moreover, E is the Young modulus and α is the coefficient of thermal dilatation of the material.

According to (Timoshenko, 1936) and (Mead, 2003) the Airy stress function must satisfy the boundary conditions in terms of stresses and displacements on the plate borders.

The modeling of the bending vibratory behavior based on variational principles, which includes the finite element method, requires the formulation of the kinetic and potential energies of the plate under the influence of the thermal stresses. Assuming Kirchhoff's thin plate theory hypotheses, those energies are expressed as (Gérardin and Rixen, 1997):

$$\mathrm{T}=\frac{\rho h}{2}\iint\left(\frac{\partial w}{\partial t}\right)^2 dxdy \tag{5}$$

$$\mathrm{U_e}=\frac{Eh^3}{24(1-\nu^2)}\iint\left\{\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right]^2-2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right\}dxdy \tag{6}$$

$$\mathrm{U_\sigma}=\frac{h}{2}\iint\left[\sigma_x\left(\frac{\partial w}{\partial x}\right)^2+2\sigma_{xy}\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)+\sigma_y\left(\frac{\partial w}{\partial y}\right)^2\right]dxdy \tag{7}$$

In the preceding expressions, T is the kinetic energy, U_e is the strain energy associated to the plate bending and U_σ is the potential energy associated to the thermal stresses. Also, $w(x,y)$ is the transversal displacement field, ρ is the volumetric mass density and ν is the Poisson ratio of the plate material.

These equations make clear that the presence of thermal stresses influences the potential energy and, consequently, the plate flexural stiffness.

3. FINITE ELEMENT MODELING

The finite element method allows modeling complex geometries and performing both thermal and structural analyses quite straightforwardly. For this reason, this modeling technique is used in this work for the characterization of the dynamic behavior of rectangular plates with prescribed thermal loads.

It is considered a family of rectangular plates, with constant thickness of 1mm and aspect ratios ranging from 1 to 2.5 (100×100mm to 100×250mm), made of ABNT 1020 steel. The designs studied are with all sides clamped and with only two opposite sides clamped. The temperatures considered are uniform distributions, ranging from 200 to 600K, with a step of 1K, adopting the reference environment temperature at 293K.

Also, a comparative analysis considered plates with the same geometrical and mechanical properties, subject to a heat flux input on a specific region and convection over the plate surfaces. The values of heat flux used ranged from 0 to 0.006 W/mm² applied on an elliptical area. Also, the convection coefficient takes the value 5 W/m²K (constant for all cases) with external temperature of 22°C. After the equilibrium temperature distribution is achieved, this distribution is used to compute pre-stresses.

For this case, three boundary conditions considered are: fully clamped plate, plate with two sides clamped and free plate. Also, the notation "*w1,w2,..*" indicates the vibration modes, and the "*relative temperature*" used in the simulations is the temperature variation relative to the environment temperature considered as being22°C.

Using these parameters, the model is built using the commercial software ANSYS Mechanical APDL®, making use of the shell element "SHELL281", which has 8 nodes (ANSYS, 2013), as seen in Fig. 1.

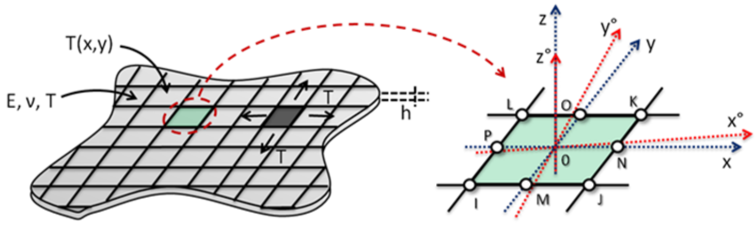

Figure 1. Illustration of the SHELL281 element.

Assuming linear behavior, the inertia and stiffness matrices are computed, taking into account the stress loads due to the imposed temperature. This originates two stiffness matrices: the elastic stiffness matrix and the geometric stiffness matrix. Following this procedure, the modal analysis problem is formulated, represented by an eigenvalue problem, involving the preceding stiffness matrices, as follows:

$$[\bar{K}_{el} - T_{ND}\bar{K}_{th} - \Omega^2\bar{M}]\{W_{jk}\} = 0 \tag{8}$$

Both problems are solved numerically, for the first ten natural frequencies and corresponding vibration modes. This procedure is repeated for each temperature considered, and for ten different aspect ratios, in the previously defined interval, for the cases of uniform temperature distributions. As for the transient analysis, four aspect ratios and four values of heat flux are considered, which are applied on an elliptical region, as shown in Fig. 2.

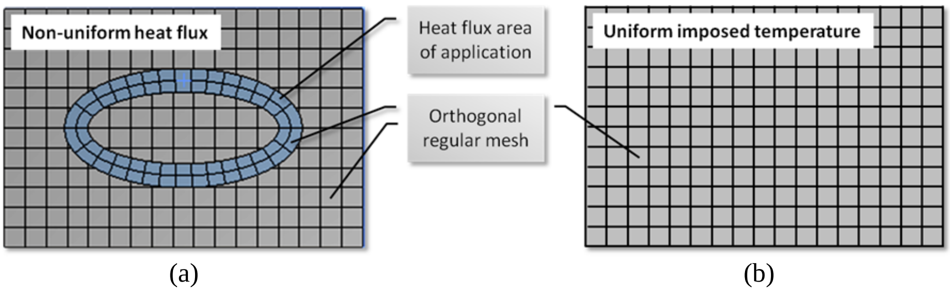

Figure 2. Thermal loads considered in the simulations: (a) non-uniform heat flux applied on a elliptical region; (b) uniform temperature distribution.

The elliptical heating region is defined as an ellipse that is 10mm wide and is distant 25mm from the margins of the plate. As the aspect ratio of the plate changes the ellipse is redefined accordingly.

Summarizing, the modeling procedure comprises the following steps: 1. Creation of the FE model, along with its external loads and conditions; 2. Solution of the thermal analysis; 3. Evaluation of the static problem, using the thermal induced stresses; 4. Solution of the modal eigenvalue problem, using the thermal induced stresses.

4. RESULTS

The simulations results for both cases, with non-uniform or uniform temperature distributions are depicted in surface graphics, and a cut section for a fixed aspect ratio is also presented for the case of uniform temperature distribution (see Figs. 3 to 9.

One can clearly notice that for both cases there is a continuous and monotonically decrease in values of the natural frequencies as the temperature increases. This can be explained by the tendency of expansion of the plate, which gives rise to predominant compression stresses. Also relevant is the tendency of the natural frequencies to vanish, for higher positive temperature increase. This characterizes the occurrence of plate buckling.

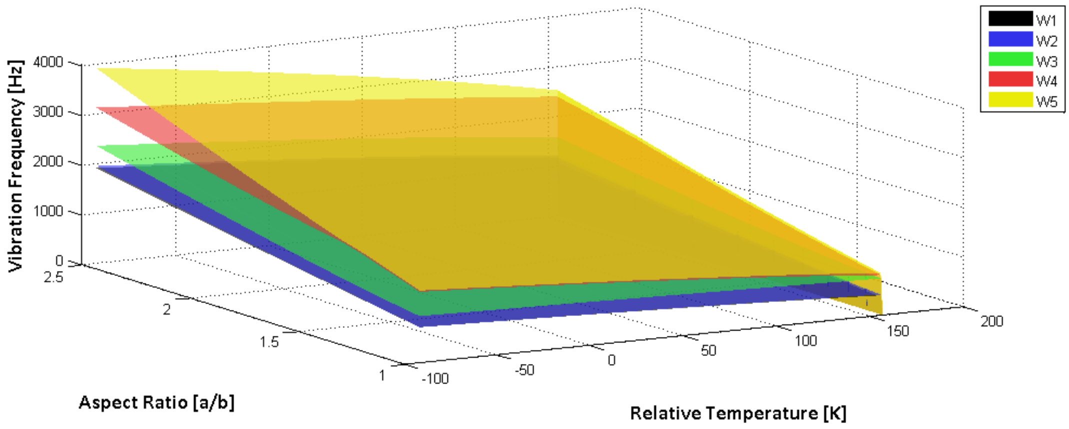

Figure 3. Variation of the natural frequencies of plate clamped on two sides, for uniform temperature distribution and different aspect ratios.

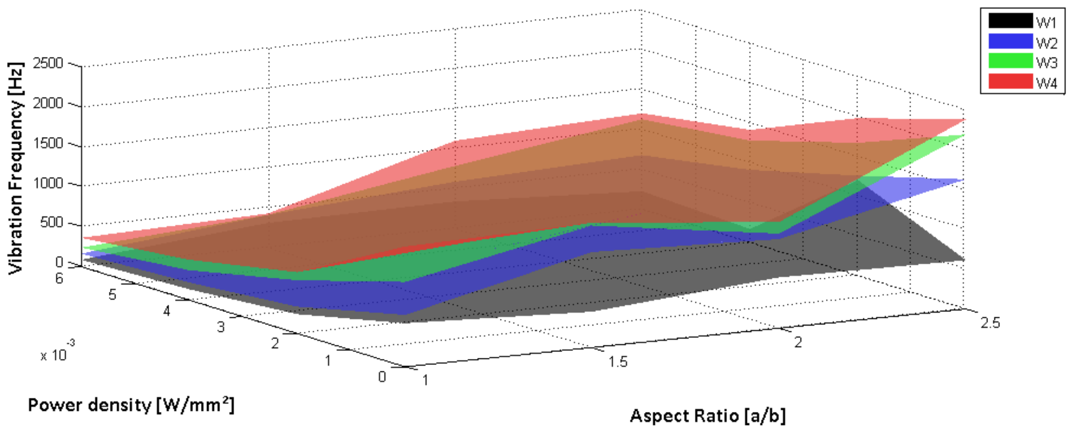

Figure 4. Variation of the natural frequencies of plate clamped on two sides, for non uniform temperature distribution and different aspect ratios.

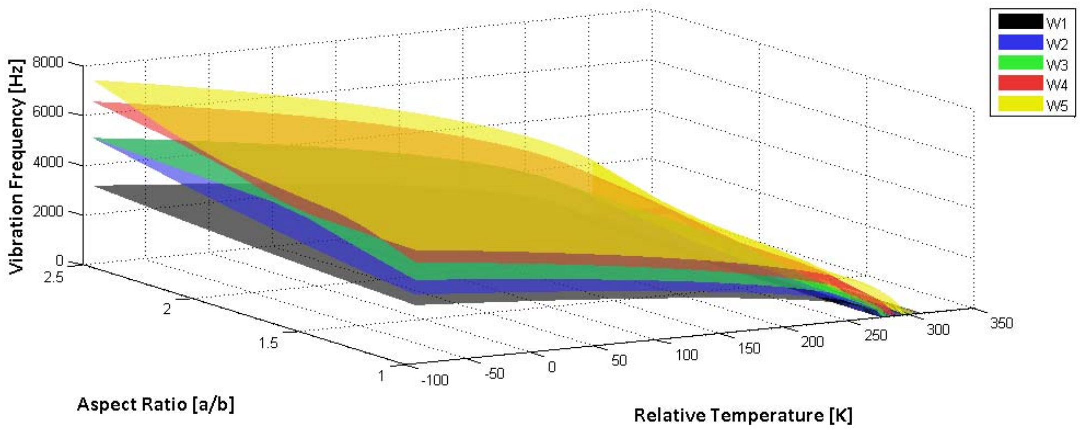

Figure 5. Variation of the natural frequencies of plate clamped on four sides, for uniform temperature distribution and different aspect ratios.

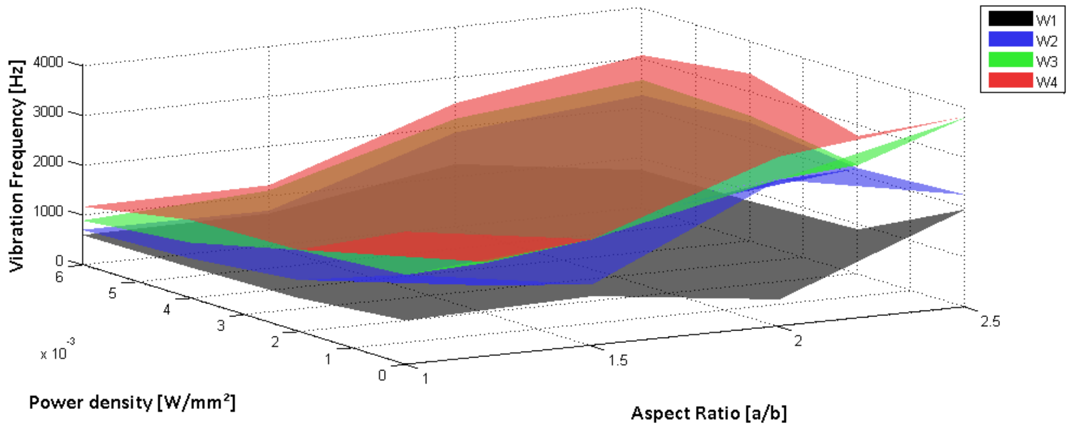

Figure 6. Figure 5. Variation of the natural frequencies of plate clamped on four sides, for non uniform temperature distribution and different aspect ratios.

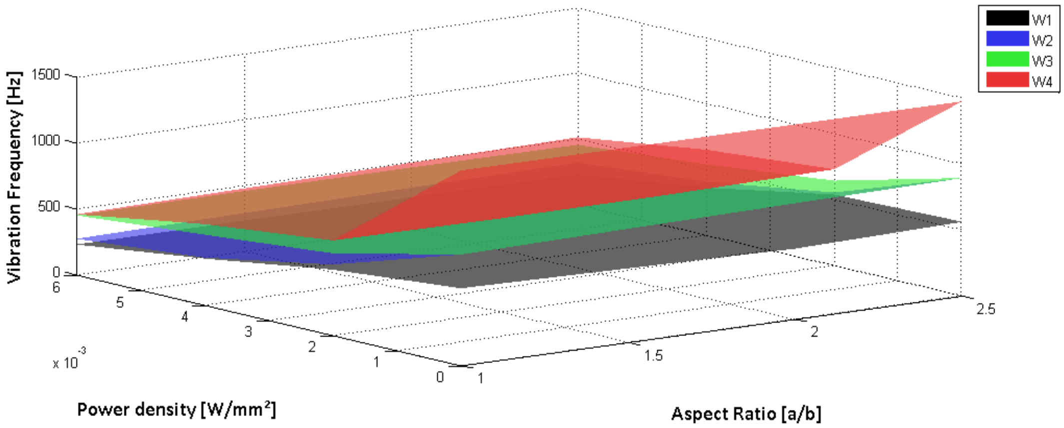

Figure 7. Figure 5. Variation of the natural frequencies of free plate, for non uniform temperature distribution and different aspect ratios.

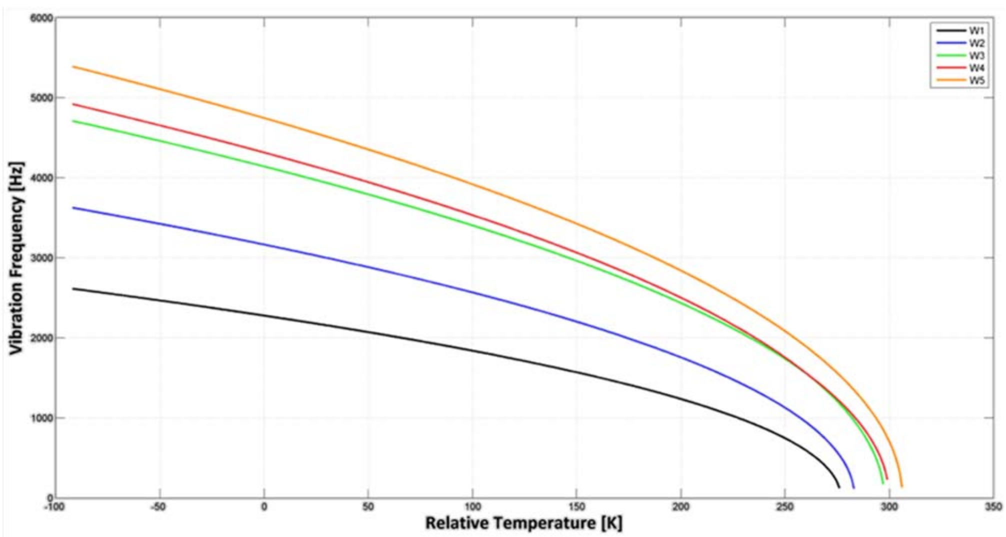

Figure 8. Figure 5. Variation of the natural frequencies of the free plate, for uniform temperature distribution and aspect ratio 1.5.

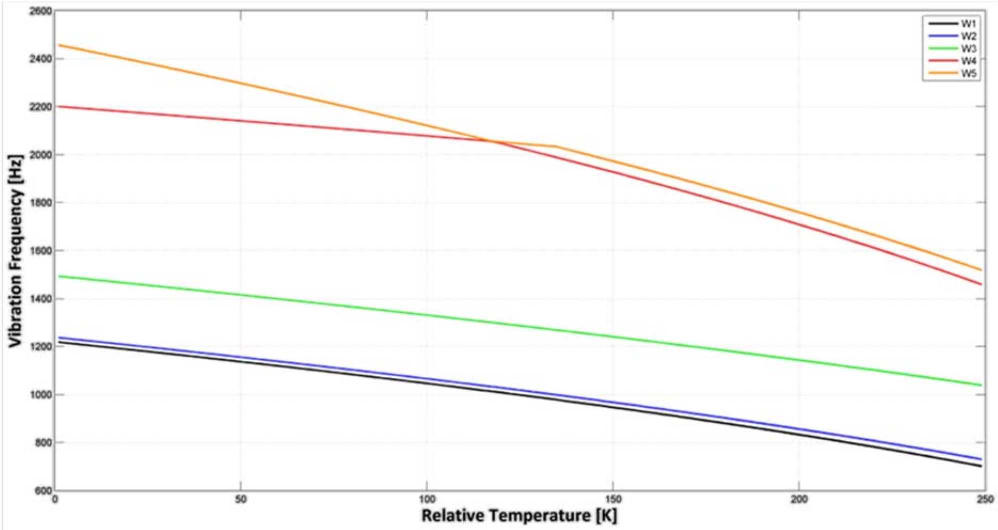

Figure 9. Figure 5. Variation of the natural frequencies of plate clamped on two sides, for uniform temperature distribution and aspect ratio fixed at 1.5.

The effect of the aspect ratio can be seen in all cases and have a strong, but quasi-linear effect over the natural frequencies values. It is worth noting the effect of the aspect ratio for the free plate, which is much weaker than in the scenarios, having little effect on the plate natural frequencies.

The first five vibration modes of interest are presented in Figs. 10 to 14, for the aspect ratio of 1.5 and temperature of 293K.

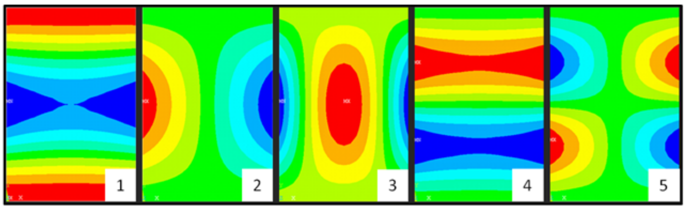

Figure 10. Vibration modes for AR=1.5 for the plate clamped on two sides, with uniform temperature distribution.

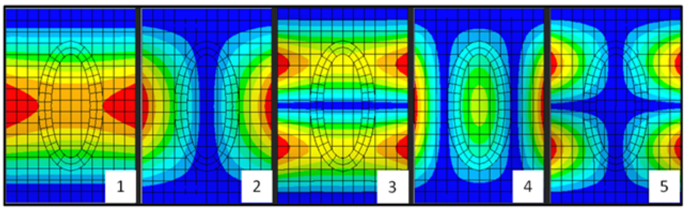

Figure 11. Vibration modes for AR=1.5 for the plate clamped on two sides, with non-uniform temperature distribution.

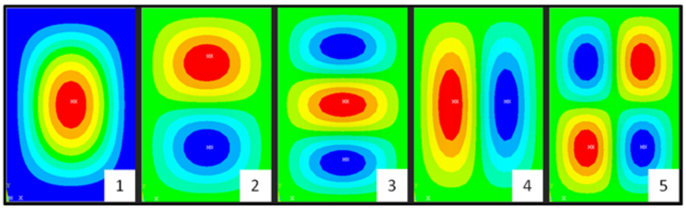

Figure 12. Vibration modes for AR=1.5 for the fully lamped plate, with uniform temperature distribution.

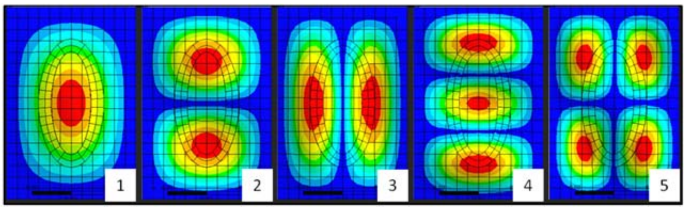

Figure 13. Vibration modes for AR=1.5 for the fully clamped plate, with non-uniform temperature distribution.

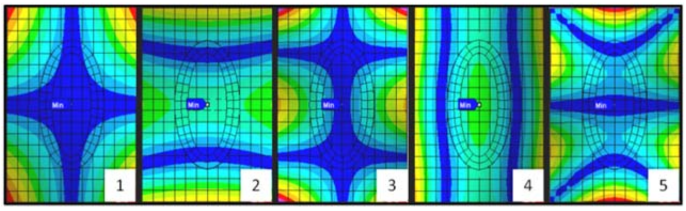

Figure 14. Vibration modes for AR=1.5 for the free plate, with non-uniform temperature distribution.

5. CONCLUSIONS

The performed simulations allowed characterizing the influence of temperature fields on the dynamic behavior of rectangular plates. The finite element model created was found to be very convenient as it enabled the thermal and structural analyses to be performed quite straightforwardly.

The results obtained have shown that the variation of the plate natural frequencies with the temperature can be very significant, and most of all, quasi-linear for a broad range of temperature values. Also, very interestingly, the plate buckling condition can be predicted, as the point in which the first natural frequency vanishes.

Comparing the effects of temperature on the mechanical behavior of plates, for the cases of uniform and non-uniform temperature distributions, it was found out that the trends for both cases are quite similar. Also, although the modes are likely to change their order when the temperature distribution varies, their shapes are still similar and their natural frequencies values are also close.

These results can be applied to large number of real engineering problems, such as the prediction and improvement of vibration levels in general machinery, which operates in a large range of temperatures.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

ANSYS, A. R. (2013). Mechanical APDL - Coupled Field Analysis Guide. ANSYS Inc.

Flores, J. E. (2004). Caracterização do efeito de enrijecimento por tensões e identificação de cargas em estruturas baseada em respostas dinâmicas. Uberlândia: FEMEC - UFU.

Gérardin, M., e Rixen, D. (1997). Mechanical Vibrations - Theory and Application to Structural Dynamics. U.K.: John Wiley & Sons.

Johns, D. J. (1965). Thermal Stress Analysis. Elsevier Science and Technology Books.

Mead, D. J. (2003). Vibration and buckling of flat free-free plates under non-uniform in-plane thermal stresses. Journal of Sound and Vibration, pp. 141-165.

Timoshenko, S. (1936). Theory of Elastic Stability. New York: McGraw-Hill.

8. RESPONSIBILITY NOTICE

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