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# UNCERTAINTY IN REMAINING USEFUL LIFE PREDICTION

**Stephen Ekwaro-Osire**
**Haileyesus B. Endeshaw**
Texas Tech University
Department of Mechanical Engineering
Lubbock, TX 79409, USA
stephen ekwaro-osire@ttu.edu
haile.endeshaw@ttu.edu

**Duc H. Pham**
Texas Tech University
Department of Mechanical Engineering
Lubbock, TX 79409, USA
stephen ekwaro-osire@ttu.edu
duc.pham@ttu.edu

**Fisseha M. Alemayehu**
Penn State Hazleton
Department of Engineering
Hazleton, PA 18202, USA
fma12@psu.edu

***Abstract.*** *Often wind turbine gearboxes do not meet the expected operating lifetime due to premature failure in the drive-train system. The gearboxes fail after three to seven years of operation although the expected lifetime of the wind turbine is twenty years. The estimation of the remaining useful life (RUL) has emerged as an important tool to schedule critical maintenance and repairs to prevent failure of gearboxes. The RUL depends on the observed condition monitoring, operational environment, and the current state of the gearbox. It has been shown that understanding and accurately quantifying the impact of uncertainty are critical in estimating RUL of gearboxes. Existing methods for quantifying uncertainty in prognostics and RUL can be classified as testing-based and condition-based prognostics. Methods for testing-based prognostics are based on rigorous testing before and/or after operating an engineering system, whereas methods for condition-based prognostics are based on monitoring the performance of the system during operation. In the condition-based prognostics, the uncertainty in RUL is subjective. In this context, RUL is a random variable whose probability distribution is sought by casting the task as an uncertainty propagation problem. Several statistical methods have been proposed to solve this uncertainty propagation problem. However, each method introduces its own set of challenges. For this study, the following research question was developed: Does Monte Carlo method improve the quantification of uncertainty in RUL prediction? To answer this research question, Monte Carlo method was used in RUL prediction. The results of this study showed that the Monte Carlo method improves the quantification of uncertainty in RUL prediction. Moreover, the probabilistic sensitivity analysis showed random variables that had the largest contribution to the RUL. The impact of the proposed framework on the uncertainty in RUL of wind turbine gearboxes is highlighted and advocated.*

***Keywords:*** *condition monitoring, prognostics, remaining useful life, uncertainty, wind turbine gearbox*

## 1. INTRODUCTION

The remaining useful life (RUL) of a component is the time left at a certain operation. RUL estimation is one of the key factors in condition based maintenance (CBM) and prognostics and healthy management (PHM). The RUL of a component is a random variable and it depends on the current age of the component, the operation environment, and the observed condition monitoring (CM) or health information. Often wind turbine gearboxes do not meet the expected operating lifetime due to premature failure in the drive-train system. The gearboxes fail after three to seven years of operation although the expected lifetime of the wind turbine is twenty years (F. M. Alemayehu & Ekwaro-Osire, 2015). The estimation of the remaining useful life (RUL) has emerged as an important tool to schedule critical maintenance and repairs to prevent failure of gearboxes. The RUL depends on the observed condition monitoring, operational environment, and the current state of the gearbox. It has been shown that understanding and accurately quantifying the impact of uncertainty are critical in estimating RUL of gearboxes.

Several probability methods have been used to determine the RUL in the term of "probability of failure" $P_f$ or "reliability index" $\beta = -\Phi^{-1}(P_f)$ of a given structural component, where $\Phi()$ is the standard normal cumulative distribution. In the case of fatigue, which is a degradation process, $P_f$ and/or $\beta$ is usually expressed as a function of time.

In order to formulate a probabilistic model, it is necessary to define a limit state function, $G(X_i)$, where $X_i$ are probabilistic parameters, such as the "load" or the "resistance" of the structural component. $G(X_i)$ is defined such that $G(X_i) > 0$, which means the limit state is satisfied, whereas $G(X_i) \leq 0$ implies "failure." $G(X_i) = 0$ represents the failure surface. Certain criteria may be considered to define "failure." In fracture mechanics methods, these may include the exceedance of a critical crack size, critical number of cycles, or yield stress. The generic formulation of the limit state function of a structural mechanics problem, where the "load," $S$ on the structural component and its "resistance," $R$ are fully independent, takes the following form (Ibrahim, 2015)

$$G(X_i)= R(X_i)|_{i=1,\ldots,j} - S(X_i)|_{i=j+1\ldots,k}\,. \tag{1}$$

The limit state function for a probabilistic model based on the fracture mechanics method with failure criteria defined as the exceedance of a critical number of cycles, $N_c$ may be considered as follows:

$$G(z_i)= N_c - N, \tag{2}$$

where the first term on the right-hand side can be equivalent to the measure of the "resistance" of a structural component, $R$, and the second term, $N$ is a measure of the "load," S or the number of cycles.

## 2. METHODOLOGY

A material subjected fatigue loading fails at a stress level far below its yield point (Juvinall & Marshek, 2000). The stress level, where the material is able to sustain during fatigue, is called the endurance limit. In this section, the methodology used for calculating the RUL of a material subjected to fatigue stresses is described. First, the framework used to calculate the *S-N* plots and the RUL prediction will be described. Next, the uncertainty considered in the RUL prediction will be explained.

### 2.1 S-N plots and RUL prediction

Depending on the type of stress the material is subjected to, its *S-N* plots could be different. Juvinal and Marshek (Juvinall & Marshek, 2000) used three *S-N* lines for bending, axial loading and torsion with their peak alternating stress capabilities of $0.9S_{UT}$, $0.75S_{UT}$ and $0.72S_{UT}$, respectively, where $S_{UT}$ is the ultimate strength of a material. In this paper, fatigue due to reversed bending stresses is considered for the purpose of illustrating RUL prediction. The stress versus number of cycle (*S-N*) plots are widely used to predict fatigue life of a material (Singh, Thakur, & Sullivan, 2005). The fatigue life of a material depends on many factors including ultimate strength of the material, size, temperature, etc. Considering a specimen with height, $H$, base, $B$ at a temperature of $T$, the endurance limit, $S_e$, can be given by (Juvinall & Marshek, 2000),

$$S_e = S_n C_L C_G C_S C_T C_R \tag{3}$$

where $S_n = 0.5S_{UT}$, $C_L$ is load factor, $C_G$ is size factor, $C_S$ is surface factor, $C_T$ is temperature factor and $C_R$ is reliability factor. The consideration of these factors takes into account different scenarios and reduces the $S_e$. The temperature factor for instance accounts for the effect of change in temperature in the strength of the material. The reliability of the calculated $S_e$ was considered using the reliability factor. The estimated mean stress $S_m$ at $10^3$ cycles is given by $0.9S_{UT}$.

The reversed stress ($\sigma_a$) versus number of cycle ($N$) relationship could be given by Budynas, R., & Nisbett, K. (2014),

$$\sigma_a = aN^b \tag{4}$$

where

$$b = -\frac{1}{3}\log\left(\frac{S_m}{S_e}\right) \tag{5}$$

$$a = 10^{(\log(S_m)-3b)}. \tag{6}$$

If a part is subjected to various loadings for different number of cycles, the endurance limit deteriorates, which is commonly referred to as cumulative fatigue damage (CFD). Two methods are widely used to consider CFD in fatigue life analysis: Palmgren-Miner's method and Manson's method. In this study, Manson's method, which is a preferred approach, was used Budynas, R., & Nisbett, K. (2014). Depending on $N$ and $\sigma_a$, RUL of a component could follow different trajectories. Thus, RUL versus $N$ plots were obtained to illustrate the changes in RUL until failure. To illustrate the RUL calculation, various loading conditions were considered such as, constant and variable stresses. After the cumulative fatigue damage was calculated, the new *S-N* line was included to depict the decrease in life cycle. For each *S-N* line, a corresponding RUL plot was obtained. Plots of deterministic approach were obtained using MATLAB.

The RUL line has $m$ segments designating the $m$-1 number of cyclic loading conditions. The additional segment shows the RUL after the last cyclic loading was applied. The RUL could be obtained by subtracting the number of cycles in each segment from the total number of cycles obtained from the *S-N* line. In general, for any number of segments the RUL can be expressed as,

$$RUL = \begin{cases} N_1 - n & n = [0, n_1] \\ N_2 - n & n = [n_1, n_2] \\ \vdots & \\ N_m - n & n = [n_{m-1}, n_m] \end{cases} \tag{7}$$

where $n$ is the number of cycles of each segment, $m$. But from Eq. (4), $N$ could be written as $N = (\sigma_a/a)^{1/b}$. Substituting in Eq. (7), the RUL is given by,

$$RUL = \begin{cases} \left(\frac{\sigma_a}{a_1}\right)^{\frac{1}{b_1}} - n & n = [0, n_1] \\ \left(\frac{\sigma_a}{a_2}\right)^{\frac{1}{b_2}} - n & n = [n_1, n_2] \\ \vdots & \\ \left(\frac{\sigma_a}{a_m}\right)^{\frac{1}{b_m}} - n & n = [n_{m-1}, n_m] \end{cases} \tag{8}$$

RUL is, thus, a function of parameters including $S_{UT}$, $B$, $H$ and $T$ since from Eq. (4), constants $a$ and $b$ are functions of those parameters.

**2.2 Uncertainty in RUL prediction**

It is important to note that uncertainties are prevalent in design and manufacturing (F. Alemayehu & Ekwaro-Osire, 2013). This could be illustrated by different outcomes of repeated experiments (Haldar & Mahadevan, 2000). Hence, it is preferable to determine the reliability of a part probabilistically rather than simply relying on outcomes of deterministic analyses. In this study, uncertainties are considered in estimating the RUL of a part (Sankararaman, Daigle, & Goebel, 2014), (Sankararaman & Goebel, 2012). Physical parameters which are selected to be random are temperature, $T$, ultimate strength of the material, $S_{UT}$, width of part, $B$, and height of part, $H$. The mean of the random variables is given in Table 1. A normal distribution was used with a CoV of 0.1 for $S_{UT}$, 0.01 for $B$ and $H$ and 0.04 for $T$.

Table 1. Random variables (Norton, 2011).

| Random Variable | Mean, μ |
|---|---|
| $S_{UT}$ (MPa) | 600 |
| T (K) | 500 |
| B (mm) | 150 |
| H (mm) | 150 |

The framework developed in MATLAB to find the *S-N* lines and RUL plots was also used here with a few modifications described as follows. In order to perform probabilistic analysis, the probabilistic software NESSUS was interfaced with MATLAB for co-analysis. The random variables were listed in NESSUS. A probabilistic analysis entails running the co-analysis from NESSUS, which communicates with MATLAB to change the values of the random variables according to their distribution. The RUL is then calculated for each instance of the random variables. In this study a Monte Carlo method with 1000 samples was used. The results of the probabilistic analysis were then obtained from NESSUS as cumulative distribution function (CDF) and sensitivity plots. The CDF shows the reliability for each RUL value in the range whereas the sensitivity plots depict the random variables that cause the highest change in the RUL.

The performance function, $Z(X)$, where X represents the random variables, $X = [S_{UT}, B, H, T]^T$, is given by,

$$Z(X) = \text{RUL}(X) = \text{RUL}(S_{UT}, B, H, T) \tag{9}$$

At a certain $n_i$, the RUL could be calculated from Eq.(8). The RUL could be further expressed in terms of the random variables by using,

$$RUL(X) = \left(\frac{\sigma_a}{a}\right)^{\frac{1}{b}} - n \tag{10}$$

where $b$ and $a$ are as given in Eq. (5) and Eq. (6), respectively.

It can be seen from Eq. (10) how the random variable $S_{UT}$ influences the RUL. The rest of the random variables, i.e. the dimensional values ($B$ and $H$) and the temperature $T$, are reflected in the size factor $C_G$ and the temperature factor $C_T$, respectively.

## 3. RESULTS AND DISCUSSION

In this section, a steel material undergoing a completely reversed fatigue stress was used to illustrate the purpose of the methodology developed for RUL prediction. First, deterministic results of the *S-N* curve and the RUL are presented for different loading scenarios. Second, probabilistic results, which play an important role in RUL prediction, are included to demonstrate the effect of inherent uncertainty of physical parameters in RUL prediction. The difference in the results of the two approaches is then discussed.

### 3.1 Deterministic approach

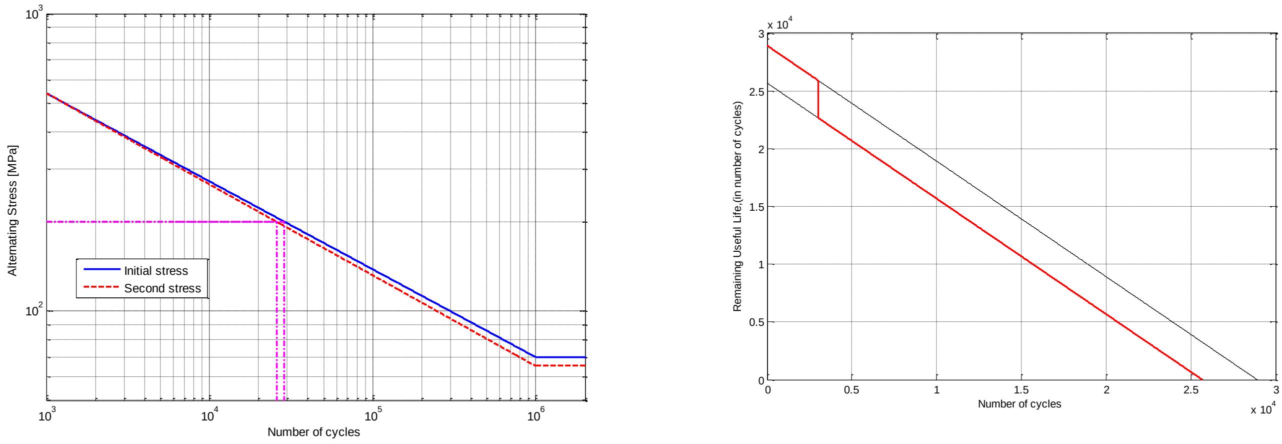
Considering an alternating stress, $\sigma_a$, of 200 MPa applied for 3000 cycles, the life of the component was determined from *S-N* plot. Using the factors CL =1, CG = 0.7466, CS = 0.5841, CT = 0.71and CR = 0.753, the endurance limit of the specimen was determined to be 70 MPa. As can be seen from Fig. 1(a), the life cycle of the component for the given loading condition is 2.89 x $10^4$ cycles. However, after the component has been subjected to 200 MPa for 3000 cycles, the endurance limit of the material deteriorates and hence a new *S-N* line with a lower endurance limit must be used. The change in the endurance limit arises from the cumulative fatigue damage (CFD), according to Manson's method. It could then be deduced from Fig. 1(a) that if the same amount of stress is applied once again, one needs to use the new *S-N* plot to predict the life cycle. Therefore, due to CFD, the new life cycle was reduced by 3242 cycles. It is therefore advisable to consider CFD while estimating RUL of a component.

Once the life of the component was predicted, a separate RUL graph was plotted (see Fig. 1(b)). The plot shows the behavior of the RUL as a function of the number of cycles until the component fails. The black lines correspond to the two life cycles obtained from the *S-N* curve. As the number of cycles increases, the RUL decreases linearly. The line above corresponds to the original life cycle prediction whereas the lower one corresponds to the life prediction after the first 3000 cycles. The red line on the other hand shows the real trajectory of the RUL. Figure 1(b) depicts that after the first 3000 cycles, the life of the component was decreased.

From Fig. 1, for a stress of 200 MPa applied for 3000 cycles, the deterministic approach predicts that the part will be safe and will work for 2.57 x $10^4$ cycles more. For instance, one can assume that the part has no risk of failure if it operates for an extra $N$ = 2 x $10^4$. This risk of failure at this $N$ value is discussed in Sec. 3.2. The *S-N* lines were compared with those used in (Norton, 2011).

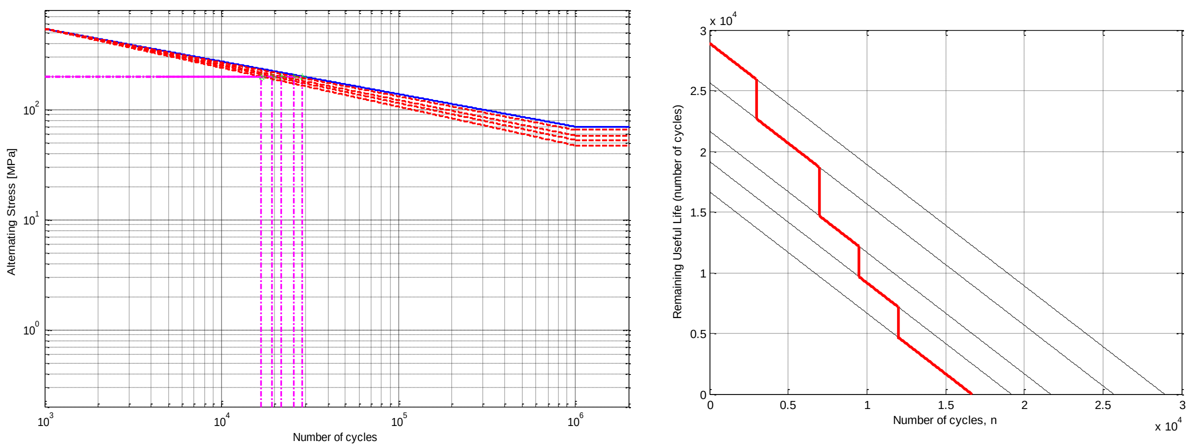
The second loading scenario considers a completely reversed stress of amplitude 200 MPa applied for five intervals in the following order: 3000, 4000, 2500, 2500 and 5000 cycles. Figure 2(a) shows the *S-N* plot. It can be seen that the endurance limit of the material is steadily decreasing with each application of cyclic load compelling the specimen to follow the new *S-N* line.

Figure 2(b) shows the RUL of the specimen until failure. The thin black lines represent the original RUL plots for each cycle time whereas the bold red zigzag line shows the actual trajectory of the RUL. It can be seen that initially the specimen has about 2.98 x $10^4$ life cycles remaining and it steps down to almost 2.25 x $10^4$ after the first 3000 cycles. After following 4000 cycles on the RUL plot, it then steps down to the next trajectory and so on. Eventually the material fails after 1.7 x $10^4$ cycles. This value is 43% less than the originally estimated value of 2.98 x $10^4$. Although the stress level is the same, i.e. 200 MPa, the CFD has caused the RUL to decrease. It is therefore, strongly recommended to consider CFD while calculating RUL.




(a) (b)

Figure 1. Effect of cumulative fatigue damage (a) *S-N* plot (b) remaining useful life (RUL) plot




(a) (b)

Figure 2. Application of load multiple cycles (a) S-N plot (b) RUL plot

The last case considered in the deterministic approach was the case of variable loading. Variable loading scenario is prevalent in various machine parts such as gearbox of wind turbine due to the varying input wind speed. Therefore, it is important to study the effect of variable loading conditions. In this case, completely reversed stresses of amplitude 200Mpa, 170 MPa, 190 MPa and 180 MPa are applied for 3000 cycles, 4000 cycles, 2500 cycles and 2500 cycles, respectively. Figure 4(b) shows the stress amplitudes. Note that the figure is intended to show only the amplitude variation for illustration, not the actual number of cycles for each amplitude of stress or loading.

Figure 3 illustrates the *S-N* curve for each load cycle. The order of the lifecycle is labeled as $N_1$ through $N_4$. Depending on the stress level, the lifecycle of the specimen could increase or decrease. After 200 MPa load application, the load on the specimen decreases to 170 MPa. Although, there is a decrease in the endurance limit, the lifecycle was increased to $N_2$. Next the load was increased to 190 MPa. Now, due to an increase in load as well as a decrease in the endurance limit, the lifecycle shows a big reduction to $N_3$. Finally, a decreased stress of 180 causes $N_4$ which is a slightly higher lifecycle than $N_3$.

Figure 4(a) shows the RUL plot for the S-N lines in Fig. 3. Each segment of the RUL corresponds to each loading cycle of the component. After the initial 3000 cycles, the RUL increases as depicted in Fig. 3 and follows the top trajectory. After 4000 cycles of a stress of 170 MPa, the increase in stress to 190 MPa causes the RUL to fall. Finally a decrease in stress to 180 MPa results in a higher RUL value. After 2500 cycles of the final load of 180 MPa, the RUL is a little over $10^4$. However, in order to complete the RUL plot shown in Fig. 4(a), the final stress value was applied until failure.

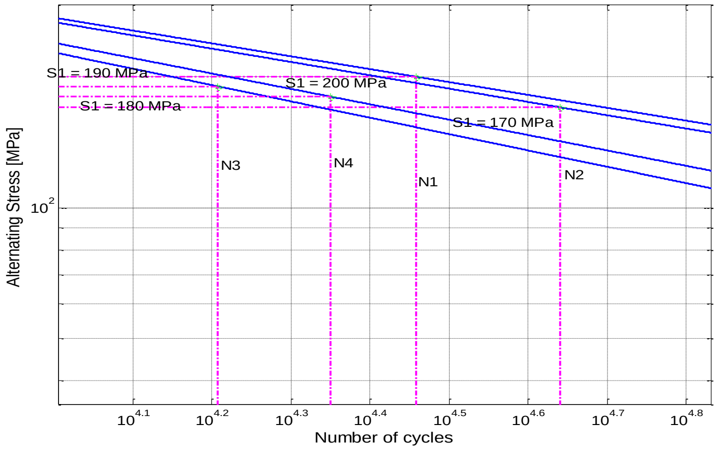
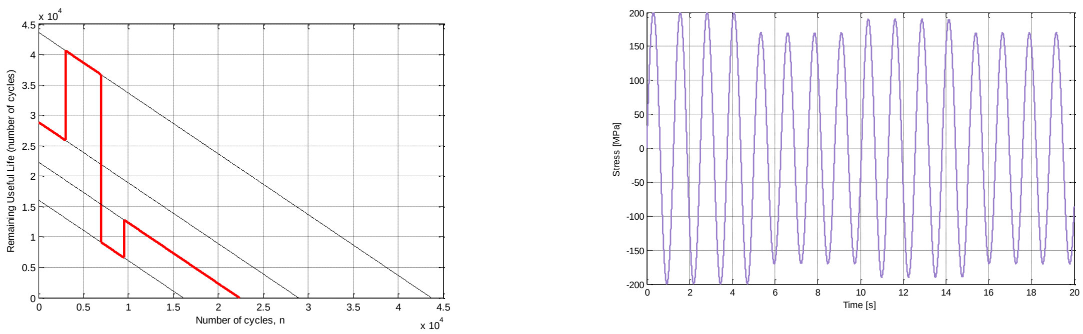



Figure 3. *S-N* plot for various stress values considering cumulative fatigue damage (CFD)




(a) (b)

Figure 4. (a) RUL plot for variable loading (b) Input alternating stress

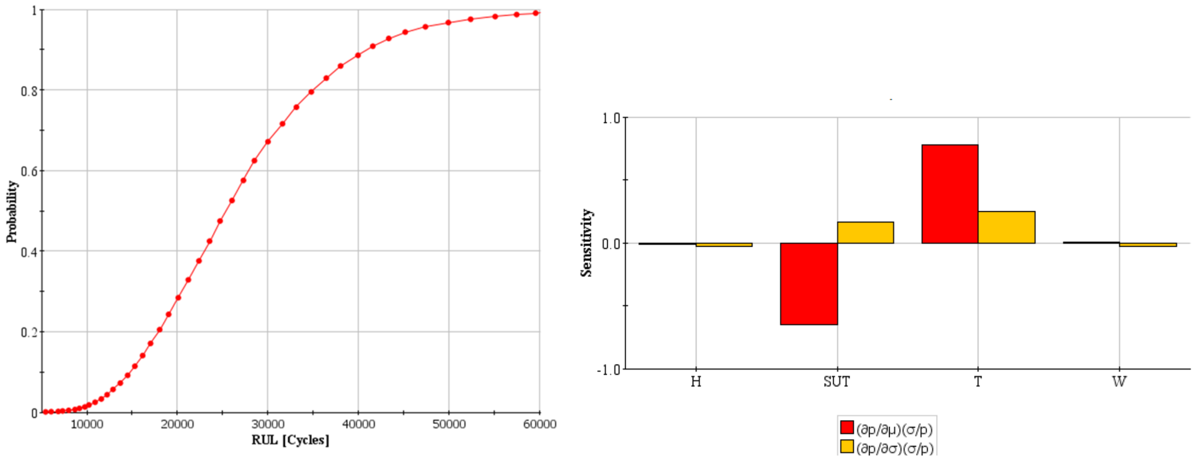
## 3.2 Probabilistic approach

This section focuses on the effects of inherent uncertainty of a component and working environment in prediction of its RUL. The random variables considered were the ultimate strength of the material ($S_{UT}$), operating temperature ($T$), width, $B$ and height, $H$ of the component, respectively. Assuming a CoV of 0.10, and a normal distribution for the random variables, the CDF of the RUL was obtained.

Using the same loading conditions as shown in Sec. 3.1, Monte Carlo method with 10,000 samples were used to perform the probabilistic analysis for RUL. The CDF in Fig. 5(a) shows the probability that the RUL is $N$ number of cycles or less. The mean RUL is the one that corresponds to a probability of 0.5. Although the expected lifetime of a component based on the deterministic analysis is 2 x $10^4$ cycles, the probabilistic analysis suggests that the RUL could vary as is shown in Fig. 5(a). For instance, there is a probability of 28% that the RUL could be 2 x $10^4$ cycles or less. It could be deduced, therefore, that it is not possible to be 100% certain about the predicted RUL because each prediction is inherently associated with the underlying uncertainty of component and working environment.

Figure 5(b) shows the probabilistic sensitivity levels of the four random variables used. The bars in red show the effect of changes in the mean of each random variable to the probability of RUL whereas those in yellow depict the effect of change of their standard deviation. Considering only the mean values, it can be observed that the RUL is most sensitive to the mean of the random variable, $T$, temperature, followed by ultimate strength, $S_{UT}$. It could be observed from the figure that an increase in the mean value of $T$ increases the probability of failure whereas an increase in the mean of $S_{UT}$ decreases the probability of failure, i.e. increases reliability. Therefore, it is recommended to pay special attention to these most sensitive parameters while designing for fatigue.

Figure 6 shows the sensitivity factors of the random variables to the RUL. It is the sensitivity at different levels of RUL. The main influential parameters are the temperature, $T$, and the ultimate strength, $S_{UT}$, which have positive and negative influences on the probability, respectively. Therefore, increasing $T$ from its current mean value will result in increasing the probability that the RUL will be the nominal value or less; whereas increasing $S_{UT}$ will decrease the probability. The cusp in the sensitivity curves shows the nominal RUL value where the sensitivity is minimal.




(a) (b)

Figure 5. (a) Cumulative Distribution Function (CDF) of the probability of occurrence of the performance function Z. (i.e. the measure of probability of the part to sustain N cycles) (b) Sensitivity levels of the random variables

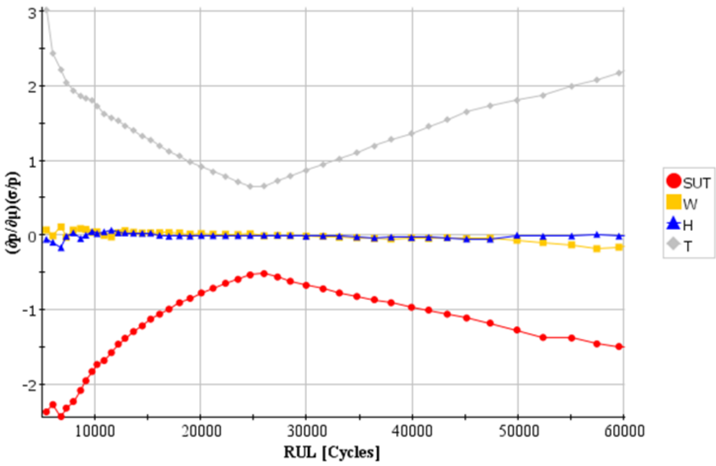



Figure 6. Sensitivity factors

## 4. CONCLUSIONS

While designing for fatigue, the remaining useful life (RUL) of a component can be depicted by S-N curves. However, inherent uncertainties could cause RUL predictions depicting significant deviation from the actual or observed RUL. Erroneous prediction of RUL could cause two problems. First, if the RUL cycles were predicted too high, unsuspected failure occurs in gearboxes which results in unintended downtime or damage of parts. If, on the other hand, the RUL cycles were predicted too low, the component may be replaced too early, causing in unnecessary cost. This study develops a methodology that considers uncertainty in the prediction of RUL. The methodology predicts RUL as well as the reliability of the predicted RUL. Simplified examples of a machine component which undergo similar variable loading conditions such as gearbox components were considered to illustrate the RUL prediction. Results show that considering uncertainty will be safer to predict RUL more accurately. It also shows that the temperature and the ultimate strength of the material have a high influence on the RUL and special attention must be given these parameters.

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