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ANALYTICAL NON-INTRUSIVE TECHNIQUE TO ESTIMATE THERMAL CONTACT CONDUCTANCES IN DOUBLE LAYERED MATERIALS BY COMBINING THE RECIPROCITY FUNCTIONAL AND THE INTEGRAL TRANSFORM APPROACHES

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Abstract. *The increasingly interest in using composite materials in engineering application requires the proper knowledge of the interaction that occurs between its layers. One of these interactions is related to the heat flow resistance trough the interface of different materials, known as thermal contact resistance, or its reciprocal, the thermal contact conductance. Methods to estimate thermal contact resistance usually require temperature measurements taken inside the sample (test body) and complicated experimental arrangements. In this work we propose an analytical and non-intrusive method to solve an inverse heat transfer problem in order to estimate a steady and one-dimensional distribution of the thermal contact conductance, combining the reciprocity functional and the classical integral transform techniques (CITT). The method was applied to some test cases using simulated measurements and results were compared with the exact solution. Although the results do not match perfectly the exact solution, estimates were done in a fraction of second, using only non-intrusive measurements, which are the main advantages of such method. A derived application of this method is the detection of flaws in the interface of composite materials. Further investigations must be performed on aiming to improve the accuracy of the method.*

Keywords: *Reciprocity functional, inverse problems, thermal contact conductance, CITT, non-intrusive method.*

1. INTRODUCTION

Many processes in engineering and related sciences requires the complete knowledge of the interaction that occurs in the interface of materials or living bodies, or even in mixed boundaries, such as a material body and a living organism that are placed in contact. The physical characteristic of this interface is the existence of a gap between the materials. One of these interactions is related to heat flow process in the contacting interface of two bodies where a temperature discontinuity exists. This temperature jump between the surfaces of two bodies in contact is the result of a thermal resistance to the heat flow in the interface, been called thermal contact resistance, or its reciprocal, thermal contact conductance-TCC.

In many industrial applications it is essential the previous knowledge of the TCC value for the design and proper operation of systems and equipment, such as in aerospace (Campos Velho, 2008), electronic (Xuan *et al.*, 2014), nanoparticles (Goodarzi *et al.*, 2014) or refrigeration (Bahrami *et al.*, 2014).

Most of existing procedures to estimate TCC values are experimental and require very complex techniques and complicated arrangements. In addition, many of these techniques also require the knowledge of surface profiles of the materials in contact, such as its roughness, and temperature measurements taken inside the materials, close to the contact interface.

Physically, the interface of two materials is characterized by an irregular distribution of micro regions. Only in some parts of these micro regions the materials are in perfect contact. When the materials are submitted to a heat flow, in micro regions where the materials are in contact, the temperature across the interface presents continuity, while in micro regions without a perfect contact a temperature discontinuity between its surfaces is observed. Considering this, TCC (h_c) is defined as the ratio of the heat flow (q) to the temperature drop (ΔT) at the interface between the materials:

$$h_c = \frac{q}{\Delta T} \tag{1}$$

From this definition, for regions with perfect contact between the materials, where no discontinuity in temperature is verified, TCC is infinity. On the other hand, in micro regions where the materials are not in contact, presenting a temperature discontinuity across the interface, TCC has a finite value. In the limiting case, where a perfect thermal insulation interface is present, TCC is null.

TCC can also be analyzed by using two approaches, one local and other lumped or global. The local one is related to some physical and geometric irregularities in the micro regions between the materials. In these cases, TCC is locally defined, with a distribution over the entire interface. The other approach, the lumped one, is related with the general characteristics of the surfaces present in the interface. In this case, TCC is taken as a constant over the entire interface, considering the average rate of heat flow to temperature drop in all micro regions along the interface.

Heat transfer in the interface between two materials is a complex process and it is affected by many factors (Gilmore, 2002), the main ones being:

(a) Pressure;
(b) Mean distance between the surfaces in the interface;
(c) Thermal conductivity of the materials (k);
(d) Surfaces properties: hardness and roughness;
(e) Material properties: yield strength and modulus of elasticity; and
(f) Average temperature in the interface.

This large number of parameters and the microscopic characteristics of the thermal process in the interface result in a great complexity to accurately calculate the TCC between two materials. Therefore, most of the experimental techniques to evaluate TCC and its available data are focused on its average value. However, new applications, such as component miniaturization, biotechnology and nanotechnology, have improving the development of methods and techniques to estimate the TCC in a more precise way and its distribution across the interface. Hence, some researches are focused in using more accurate and detailed characteristics of the contacting surfaces between materials. Following this line, mathematical and statistical models have been created to simulate micro irregularities on the interface between two materials by developing roughness patterns. Zhao *et al*., 2005, predicted the TCC based on the contact peaks of irregular surfaces on two-dimensional statistics models of the roughness profile. To develop the model, they measured the roughness profiles of some common machined surfaces. The results showed that is very difficult to establish a complete topography of the surfaces and concluded that the model still needs to be refined to a three-dimensional version in order to be more effective.

More recent works focused not only in the averaged TCC value, but in the determination of its spatial distribution on the whole interface of materials as well as its variation in time. Yang (2007) presented the solution of a transient inverse heat transfer problem to estimate the TCC in single-coated optical fibers. In the work of Shojaeefard *et al*., 2009, a transient inverse heat transfer problem was used to estimate TCC in periodically contacting surfaces. The inverse problem technique used in these two articles was the conjugate gradient method with adjoint problem for function estimation, where temperature measurements were taken intrusively. Although good results to the time variation of the TCC were obtained, it was necessary to use intrusive measurements of temperature in both schemes. Also following this line, Fieberg and Kneer (2008) solved a transient inverse heat transfer problem using a sequential estimation method by optimizing the least square difference between the measured and calculated temperature in each time step in the contacting interface. Temperature measurements were recorded with a long wave infrared camera in this region, resulting in an intrusive approach also.

Gill *et al*. (2009) developed a technique to estimate non-uniform TCC values in a two-dimensional configuration using an inverse problem. To circumvent the sensitivity to noise in the measured temperatures, a boundary element method (BEM) was used to determine the sensitivity coefficients, aiming to find the best location for specific measurement points in the geometry. A regularization scheme was also used, based on a genetic algorithm. Although this approach presented good results related to TCC estimation, intrusive measurements and intensive computational processing were required due to the regularization scheme used.

In 2012, Colaço and Alves presented a non-intrusive approach to solve a two-dimensional steady-state inverse problem for estimation unknown TCC spatial distribution at the interface between two materials, by means of the reciprocity functional combined with the method of fundamental solutions (MFS). This methodology proved to be extremely fast and temperature measurements were taken only on an external surface of the materials.

Further works of these researchers, applying this same approach: reciprocity functional combined with MFS were developed to improve the results of the first work (Colaço and Alves, 2013; Abreu *et al*., 2014). At the same time, also applying reciprocity functional technique, Colaço and Alves developed a transient version of this technique (Colaço *et al*., 2013; Colaço and Alves, 2014; Colaço *et al*., 2014).

As pointed before, the determination of the spatial distribution of TCC is a complex process and requires complicated experiments and/or refined physical models. In this paper, we propose an analytical version of the method developed previously by Colaço and Alves to estimate TCC in double-layered materials by combining the reciprocity functional and the classical integral transform approaches. The aim of this work is to improve this technique, where the estimate can be performed in a fraction of second. Moreover, the resultant method is fully non-intrusive and does not rely on any iterative techniques.

2. PHISICAL PROBLEM

2.1 Physical problem description

The two-dimensional steady-state physical problem considered here is the same proposed by Colaço and Alves in their former work of 2012. This configuration has been used since then by these researchers, as a benchmark, to compare the results of different techniques.

Figure 1 shows the geometry of the physical problem. It can be noticed that this arrangement is consistent with most experimental arrangements used to determine TCC in the interface between two materials.

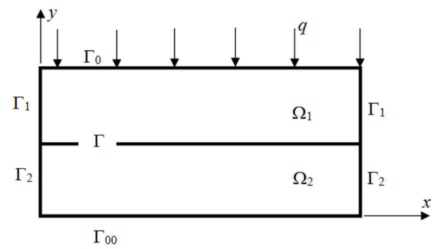

Figure 1. Schematic of the physical problem

Two rectangular isotropic materials (Ω_1 and Ω_2) are placed in contact with each other creating an interface (Γ) between them where a thermal contact conductance $h(x)$ is present. Materials have constant thermal conductivities k_1 and k_2. Lateral boundary surfaces (Γ_1 and Γ_2) of both materials are assumed to be thermally insulated, while the bottom surface (Γ_{00}) of material Ω_2 is subject to a prescribed temperature. The upper surface (Γ_0) of material Ω_1 is subject to a prescribed heat flux (q), where temperature measurements (Y) are also taken, for example, by an infrared camera. The contact surface in the interface Γ is assumed to have a Robin boundary condition: $-k_1\ \partial T_1/\partial \mathbf{n}_1=h(T_1\text{-}T_2)$, where $\mathbf{n}_1$ is the normal derivative outward the interface boundary Γ, related to material Ω_1, and T_1 and T_2 are the temperatures on the contacting surfaces of materials Ω_1 and Ω_2, respectively.

2.2 Mathematical formulation

The mathematical formulation for this two-dimensional steady-state heat transfer problem is formulated as the following direct problem. It should be noticed that the coupling between regions Ω_1 and Ω_2 is done through the interface Γ, the boundary where the thermal contact conductance needs to be estimated.

$$\nabla^2 T_1 = 0 \quad \text{in } \Omega_1 \tag{2.a}$$

$$-k_1 \frac{\partial T_1}{\partial \boldsymbol{n}_1} = q \quad \text{at } \Gamma_0 \tag{2.b}$$

$$\frac{\partial T_1}{\partial \boldsymbol{n}_1} = 0 \quad \text{at } \Gamma_1 \tag{2.c}$$

$$-k_1 \frac{\partial T_1}{\partial \boldsymbol{n}_1} = h(T_1 - T_2) \quad \text{at } \Gamma \tag{2.d}$$

$$\nabla^2 T_2 = 0 \quad \text{in } \Omega_2 \tag{2.e}$$

$$\frac{\partial T_2}{\partial \boldsymbol{n}_2} = 0 \quad \text{at } \Gamma_2 \tag{2.f}$$

$$T_2 = 0 \quad \text{at } \Gamma_{00} \tag{2.g}$$

$$k_2 \frac{\partial T_2}{\partial \boldsymbol{n}_2} = -k_1 \frac{\partial T_1}{\partial \boldsymbol{n}_1} \quad \text{at } \Gamma \tag{2.h}$$

3. INVERSE PROBLEM

The inverse heat transfer problem proposed in this work consists of estimating the thermal contact conductance distribution $h(x)$ at the inaccessible interface Γ between the two materials by using temperature measurements Y at an accessible external boundary surface Γ_0. As it can be verified in Eq. (1), thermal contact conductance is the ratio of the heat flow (q) to the temperature drop (ΔT) across the interface Γ. Therefore, the determination of $h(x)$ is done indirectly, through the estimation of $q(x)$ and $\Delta T(x)$ along the interface Γ. To estimate them two auxiliary problems will be defined, as proposed by Colaço and Alves (2012): the first one to estimate the temperature jump $\Delta T(x) = T_1\text{-}T_2$ and the second one to estimate the heat flux $q(x) = -k_1\, \partial T_1/\partial \mathbf{n}_1$, both at the interface Γ. The results of these two auxiliary problems will be combined to obtain the thermal contact conductance distribution along the interface Γ.

3.1 First auxiliary problem: estimating $\Delta T(x) = T_1\text{-}T_2$ at Γ

Consider the first auxiliary problem for some harmonic test functions $F_1 \in C^2(\Omega_1)$ and $F_2 \in C^2(\Omega_2)$, with the modifications introduced by Abreu *et al.* (2014) in order to avoid the use of Cauchy problems, where ψ is a $L^2(\Gamma_0)$ orthonormal basis system:

$$\nabla^2 F_1 = 0 \qquad \text{in } \Omega_1 \tag{3.a}$$

$$F_1 = \psi \qquad \text{at } \Gamma_0 \tag{3.b}$$

$$\frac{\partial F_1}{\partial \mathbf{n}_1} = 0 \qquad \text{at } \Gamma_1 \tag{3.c}$$

$$F_1 = F_2 \qquad \text{at } \Gamma \tag{3.d}$$

$$\nabla^2 F_2 = 0 \qquad \text{in } \Omega_2 \tag{3.e}$$

$$\frac{\partial F_2}{\partial \mathbf{n}_2} = 0 \qquad \text{at } \Gamma_2 \tag{3.f}$$

$$F_2 = 0 \qquad \text{at } \Gamma_{00} \tag{3.g}$$

$$k_2 \frac{\partial F_2}{\partial \mathbf{n}_2} = -k_1 \frac{\partial F_1}{\partial \mathbf{n}_1} \qquad \text{at } \Gamma \tag{3.h}$$

Considering the same manipulations done by Colaço and Alves (2012) in their early work, combining the above equations of the first auxiliary problem with the mathematical formulation of the physical problem (Section 2.2), using the Green's second identity and substituting temperatures T_1 in the boundary surfaces Γ_0 by its available measurements values Y, the following expression can be obtained

$$k_1 \int_{\Gamma_0} \left[F_1\left(\frac{-q}{k_1}\right) - Y \frac{\partial F_1}{\partial \mathbf{n}_1} \right] d\Gamma_0 = \int_{\Gamma} k_1 \frac{\partial F_1}{\partial \mathbf{n}_1} [T_1 - T_2] d\Gamma \tag{4}$$

Notice that the previous expression links properties of external boundary Γ_0 with properties of boundary interface Γ. Now it is defined $\Re(F_1)$ as the reciprocity functional in terms of test functions F_1, as proposed by Colaço and Alves (2012), as

$$\Re(F_1) = \int_{\Gamma_0} \left[F_1\left(\frac{-q}{k_1}\right) - Y \frac{\partial F_1}{\partial \mathbf{n}_1} \right] d\Gamma_0 \tag{5}$$

It is also observed that, once F_1 is determined, only the thermal conductivity k_1, the imposed heat flux q, and the measured temperatures Y in Γ_0 are needed for the calculation of $\Re(F_1)$. Substituting Eq. (5) in Eq. (4) we obtain

$$k_1 \Re(F_1) = \left\langle [T_1 - T_2], k_1 \frac{\partial F_1}{\partial \mathbf{n}_1} \right\rangle_{L^2(\Gamma)} \tag{6}$$

Now it is possible to define β at the boundary Γ as (Abreu *et al.*, 2014)

$$\beta_j = k_1 \left. \frac{\partial F_{1,j}}{\partial \mathbf{n}_1} \right|_{\Gamma} \tag{7}$$

where each β_j is obtained after solving the auxiliary problem for a corresponding ψ_j. Thus, β can be considered as a set of generic basis function and can be used to represent the temperature difference $[T_1\text{-}T_2]$ at the boundary interface Γ as

$$[T_1 - T_2]_\Gamma = \sum_{i=1}^{N} \alpha_i \beta_i \tag{8}$$

where the unknown coefficients α_i are to be determined. Replacing Eq. (7) and (8) in Eq. (6), the solution of the following linear system of equations gives the values of α_i (Abreu *et al.*, 2014)

$$k_1 \Re(F_{1,j}) = \left\langle \sum_{i=1}^{N} \alpha_i \beta_i , \beta_j \right\rangle_{L^2(\Gamma)} = \sum_{i=1}^{N} \alpha_i \left\langle \beta_i , \beta_j \right\rangle_{L^2(\Gamma)} \tag{9}$$

Once the coefficients are obtained, the temperature jump along the interface Γ, $\Delta T(x)$, can be estimated using Eq. (8).

3.2 Second auxiliary problem: estimating $q(x) = -k_1\ \partial T_1/\partial \mathbf{n}_1$ at Γ

Now consider the second auxiliary problem for some harmonic test functions $G_1 \in C^2(\Omega_1)$, also with the modifications introduced by Abreu *et al*. (2014) in order to avoid the use of Cauchy problems, where ψ is the same $L^2(\Gamma_0)$ orthonormal basis system of the first auxiliary problem:

$$\nabla^2 G_1 = 0 \qquad \text{in } \Omega_1 \tag{10.a}$$

$$G_1 = \psi \qquad \text{at } \Gamma_0 \tag{10.b}$$

$$\frac{\partial G_1}{\partial \mathbf{n}_1} = 0 \qquad \text{at } \Gamma_1 \tag{10.c}$$

$$\frac{\partial G_1}{\partial \mathbf{n}_1} = 0 \qquad \text{at } \Gamma \tag{10.d}$$

Following the same procedure used in section 3.1, it is possible to obtain

$$k_1 \Re(G_1) = -\left\langle G_1 , k_1 \frac{\partial T_1}{\partial \mathbf{n}_1} \right\rangle_{L^2(\Gamma)} \tag{11}$$

where here, $\Re(G_1)$ is the reciprocity functional in terms of test functions G_1. Now define γ at the boundary Γ as (Abreu *et al.*, 2014)

$$\gamma_j = G_j\big|_\Gamma \tag{12}$$

where each γ_j is obtained after solving the second auxiliary problem for a corresponding ψ_j. Thus, γ can also be considered as a set of generic basis function and can be used to represent the heat flux $-k_1\ \partial T_1/\partial \mathbf{n}_1$ at the boundary interface Γ as

$$\left[-k_1 \frac{\partial T_1}{\partial \mathbf{n}_1} \right]_\Gamma = \sum_{i=1}^{N} \delta_i \gamma_i \tag{13}$$

where the unknown coefficients δ_i are to be determined. Replacing Eq. (12) and (13) in Eq. (11), the solution of the following linear system of equations gives the values of δ_i (Abreu *et al.*, 2014)

$$k_1 \Re(G_{1,j}) = \left\langle \sum_{i=1}^{N} \delta_i \gamma_i , \gamma_j \right\rangle_{L^2(\Gamma)} = \sum_{i=1}^{N} \delta_i \left\langle \gamma_i , \gamma_j \right\rangle_{L^2(\Gamma)} \tag{14}$$

Once the coefficients are obtained, the heat flux distribution along the interface Γ, $q(x)$, can be estimated using Eq. (13).

3.3 Combining first and second auxiliary problems: obtaining $h(x)$ at Γ

Considering the thermal contact conductance definition Eq. (1) and the previous estimations of $\Delta T(x)$ and $q(x)$, the value of $h(x)$ in the interface Γ can be obtained as

$$h(x)=\frac{q(x)}{\Delta T(x)}=\frac{-k_1\dfrac{\partial T_1}{\partial \mathbf{n}_1}}{T_1-T_2}=\frac{\sum_{i=1}^{N}\delta_i\gamma_i}{\sum_{i=1}^{N}\alpha_i\beta_i} \tag{15}$$

4. SOLVING AUXILIARY PROBLEMS BY CITT

In the previous sections two auxiliary problems, based on test functions F_1 , F_2 and G_1, were defined to estimate the temperature jump $\Delta T(x)$ and the heat flux $q(x)$ on the interface Γ. The reciprocity functional approach was the key strategy to develop expressions to estimate these functions, relating heat processes that occur inside the materials with known properties and data taken on an external boundary.

However, both auxiliary problems should also be solved. In this work, with the aim to improve the accuracy of this method, both auxiliary problems were solved by the classical integral transform techniques (CITT), an analytical method to solve partial differential systems that govern diffusion-convection phenomena. The technique consists of the following steps (Cotta, 1993):

- Find the related eigenvalue problem;
- Develop the appropriate integral-inverse pair;
- Transform the original partial differential equation and make use of the boundary conditions;
- Solve the resulting system of decoupled ordinary differential equations; and
- Invoke the previously established inversion formula to construct the desired complete potential.

These steps were applied to solve the first auxiliary problem and the result for the potentials F_1 or F_2 is

$$F_j(x,y)=\frac{1}{a}\frac{\int_{x=0}^{a}\psi_j dx}{b}y+\sum_{m=1}^{\infty}\frac{2\cos(\mu_m x)}{a}\int_{x=0}^{a}\cos(\mu_m x)\psi_j dx\frac{\sinh(\mu_m y)}{\sinh(\mu_m b)}, \tag{16}$$

where a and $b/2$ are, respectively, the length and the height of each material Ω_1 and Ω_2, and the eigenvalues μ_m are the positive roots of $\sin(\mu_m a)=0$. In this work we are considering that both materials have the same height $b/2$.

CITT was also applied to solve the second auxiliary problem and the result for the potential G_1 is

$$G_{1,j}(x,y)=\frac{1}{a}\int_{x=0}^{a}\psi_j dx+\sum_{m=1}^{\infty}\frac{2\cos(\mu_m x)}{a}\int_{x=0}^{a}\cos(\mu_m x)\psi_j dx\frac{\cosh(\mu_m y)}{\cosh\left(\mu_m\dfrac{b}{2}\right)}, \tag{17}$$

where a and $b/2$ are, respectively, the length and the height of the body Ω_1, and the eigenvalues μ_m are the positive roots of $\sin(\mu_m a)=0$.

5. RESULTS

The same physical and heat transfer problems considered in the work of Colaço and Alves (2012) were used here. Physical problem was described in section 2 and Tab. 1 summarize the values used in each test case, where a is the length and $b/2$ is the height of each material (Ω_1 and Ω_2).

Table 1. Values used in the test cases

a	0.04 m	k_1	54 W/(m K)
$b/2$	0.01 m	k_2	54 W/(m K)
q	-10 W/m2	h_{max}	1000 W/(m2 K)

As also used by Colaço and Alves (2012), Tab. 2 shows six cases considered in this work corresponding to different thermal contact conductance profiles h, where h_{max}=1000 W/(m^2 K).

Table 2. Thermal contact conductance profiles (adapted from Colaço and Alves, 2012)

Case	h [W/(m^2 K)]	Case	h [W/(m^2 K)]
1	h_{max} for ($x < a/4$) and ($x > 3a/4$) 0 for $a/4 < x < 3a/4$	4	$\left\lvert h_{max} \sin\left(\frac{2\pi x}{a}\right) \right\rvert$
2	h_{max} for ($x < a/4$) and ($a/2 < x < 3a/4$) 0 for ($a/4 < x < a/2$) and ($x > 3a/4$)	5	h_{max}
3	$h_{max} \sin\left(\frac{\pi x}{a}\right)$	6	h_{max} for ($x < a/4$) and ($a/2 < x < 3a/4$) $h_{max}/2$ for ($a/4 < x < a/2$) 0 for ($x > 3a/4$)

Simulated measured temperatures taken in the external boundary surface Γ_0 were obtained by solving the direct problem for each h profile defined in Tab. 2. In order to avoid the "inverse crime", a finite-difference technique was used to solve the direct problem. Experimental errors were introduced in exact temperatures according to the following scheme:

$$Y = T(\Gamma_0) + \varepsilon\sigma \tag{18}$$

where ε is a Gaussian variable with zero mean and unity variance and σ is the standard deviation of the measurements, considered here as 5% of the maximum value of $T(\Gamma_0)$.

The first step of the proposed approach is to solve both auxiliary problems via CITT, as indicated in section 4, using a set of orthonormal functions (ψ). In this study a combination of cosine-wave functions was used. Once the potentials F_1, F_2 and G_1 are obtained, the second step is to estimate the temperature jump $\Delta T(x) = T_1 - T_2$ and the heat flux $q(x) = -k_1\, \partial T_1/\partial \mathbf{n}_1$ at the inaccessible interface Γ, using the reciprocity functional, as indicated in section 3. The last step is to determine the thermal contact conductance distribution $h(x)$ at this interface considering its definition given by Eq. (1).

Finally, Fig 2 shows the results of the thermal contact conductance distribution $h(x)$ for each test case considered in this work. Notice that the general behavior of all functions is well captured, where locations of the maximum and minimum values of $h(x)$ are well determined, even for measurements containing experimental errors. However, for test cases where sudden variations of $h(x)$ are present, the estimated values are not very well predicted by this technique. Other orthonormal functions and regularization schemes should be considered to improve the accuracy of the method for such cases.

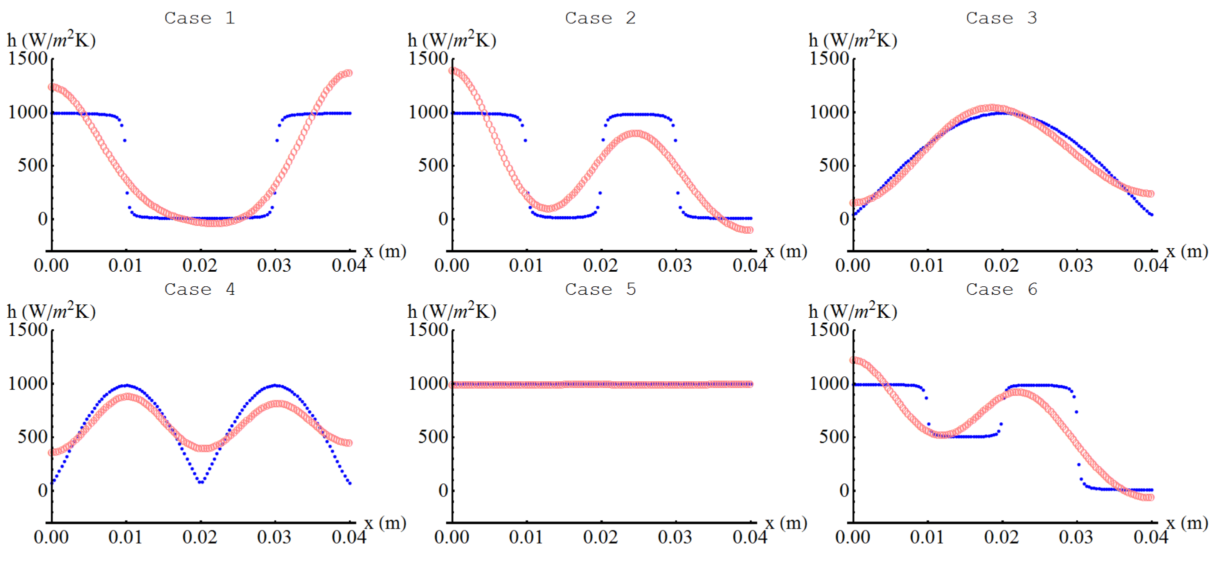

Figure 2. Estimated h (o Reciprocity functional approach; • Exact)

6. CONCLUSIONS

The results obtained in this work showed that reciprocity functional technique is a very promising method for solving inverse heat transfer problems. An analytical approach was used to solve the two auxiliary problems via CITT. Although the estimated functions do not match perfectly the exact solution, the estimates were done in a fraction of second, using only non-intrusive measurements, which are the main advantages of such method. Future works shall investigate proper regularization schemes in order to improve the accuracy of the method.

This method may also be used to detect flaws in composite materials, as the physical nature of the thermal contact conductance at the interface of different materials is related to the discontinuity profile at this interface.

7. ACKNOWLEDGEMENTS

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9. RESPONSIBILITY NOTICE