23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

THERMAL ANALYSIS OF HEATED PITOT PROBES IN ATMOSPHERIC CONDITIONS OF ICE ACCRETION

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Abstract. *The purpose of this work is to propose a simple combined model for the thermal analysis of Pitot probes that includes conjugated heat transfer and a modified Messinger ice formation formulation, in order to describe ice accretion in electrically heated aeronautical Pitot tubes. The proposed lumped-differential model is solved by making use of the Method of Lines as implemented in the routine NDSolve, from the numerical-symbolic computational system Mathematica v.9.0. It is then illustrated the importance of considering the heat transfer within the probe structure, that accounts for heat conduction along the probe and the thermal capacitance of its internal elements, in adequately predicting the ice formation on the stagnation and tip regions. Solutions are provided for actual adverse atmospheric conditions, extracted from a recently proposed certification envelope for Pitot probes*.

Keywords: *Ice accretion; Aeronautical Pitot tube; Anti-icing system; Conjugated problem; Lumped analysis; Hybrid methods*

1. INTRODUCTION

Aircraft icing has been found to be a considerably negative effect on the aircraft flight performance and can threaten the safety of the flight. Ice may occur on wings, control surfaces, horizontal and vertical stabilizers, fuselage nose, landing gear doors, engine intakes, Pitot tubes, and other sensors and drain system outputs (Caliskan et al., 2008; Özgen et al., 2009). Among these, the case of the Pitot tube is emphasized in this work since it is still an essential sensor for guidance and navigation of modern aeronautical systems. The catastrophic effects of this kind of failure can be observed when analyzing a few accidents that occurred in recent years, such as flights X-31A (1995) in USA, Birgen Air Flight 301 (1996), Aero Perú 603 (1996), Northwest Orient Airlines Flight 6231 (1997) and the more recent tragedy with the Air France AF447 flight, in which ice clogged Pitot probes, in a high altitude flight with loss of lift, triggered the sequence of events that led to the disaster, according to the investigations conducted by the Bureau d'Enquêtes et d'Analyses pour la Sécurité de l'Aviation Civile (BEA), France.

The first efforts to understand the effects of icing on performance and flight mechanics began in the 1940´s (Langmuir et al., 1946), mainly based on experiences in test flights. Among the pioneering works, the modeling research of Messinger (1953) is still today a fundamental and applicable model. While the literature is quite rich in terms of icing and de-icing of wings and engine intakes, reporting a few well-known computer codes, such as LEWICE, ONERA2D and TRAJICE2 for icing of airfoils (Stefanini et al., 2008), it is still fairly poor on Pitot probes ice accretion studies. Some of the most relevant contributions to the simulation of ice formation on aircraft components were provided by the NASA Lewis Research Center (USA). A NASA report (Abbott et al., 1945) summarizes the results of a survey along ten years of collaboration in the simulation of ice formation, involving the National Aeronautics and Space Administration (NASA), The Defense Research Agency (DRA) and the Office National d'Études et de Recherches Aérospatiales (ONERA, France), in a large number of theoretical and experimental studies of ice formation processes, especially in wings. Aircraft icing is a very complex phenomenon that occurs during flights through certain clouds at temperatures below freezing, when supercooled water droplets impinge and freeze on the unprotected areas on which they impact (Sheriff et al., 1997). The rate and amount of ice accretion on an unheated surface depends on several parameters such as the shape of the component, including surface finishing, the size, the speed at which the body is travelling, the temperature, liquid water content (LWC), the size of the droplets in the cloud and local collection efficiency (β) (Gent et al., 2000; Myers et al., 2002; Fortin et al., 2006).

Thus, the present paper focuses on the thermal analysis of anti-ice systems for Pitot tubes, based on electric heating of the sensor body, in view of the serious consequences that can result from freezing tolerance of these probes. Recently, in (Souza et al., 2011; Souza et al., 2012; Lisboa et al., 2013; Souza et al., 2015), a mixed differential-lumped transient thermal analysis of aeronautical Pitot probes was proposed, and the model was properly validated through experimental results in wind tunnel and flight tests. This paper is aimed at incorporating into these recent studies a simplified but robust model of ice formation, for prediction of the transient thermal behavior of heating systems for aeronautical speed sensors under severe icing conditions. The main objective of this work is thus to theoretically study the conjugated heat transfer problem associated with the transient thermal behavior of an aeronautical Pitot tube undergoing icing, through the implementation of a modified Messinger-type ice formation formulation. The purpose is to demonstrate the importance of considering conjugated heat transfer in the implementation of more complex models for predicting ice formation on aircraft structures. Due to the lack of specific experimental results for the icing on Pitot probes, the model was first applied to wing sections, allowing for the model validation from results for airfoils in icing wind tunnels and then it is applied to the Pitot tube situation. Finally, the proposed theoretical model is applied in critical flight situations.

2. PROBLEM FORMULATION

The proposed conduction model consists of a partially lumped formulation, with lumping in the radial direction and local differential formulation along the probe length. The curvature at the handle of the tube is neglected, due to the lack of interest on an accurate thermal analysis of the Pitot probe support. In the actual probe configuration here considered, the tip of the Pitot tube has a conical section, which together with most of the cylindrical portion is made of copper, until mechanical forming joint switches the material to brass, then followed by the wing shaped base. In addition, the heater and the porcelain insulator can be found only at the central portion of the cylindrical copper tube. The external convection problem is simplified using empirical correlations to describe the heat transfer coefficient in each region of the probe. Thus, appropriate correlations were obtained from the literature to estimate the heat transfer coefficients at the stagnation region, cone and cylindrical bodies, and wing shaped base, (Souza et al. (2011), Sparrow et al. (2004) and Tong & Hu (2009)).

The detailed knowledge of the flow and heat transfer processes over and within the probe itself are essential for its thermal management, especially near the stagnation region due to its functional importance, while the supporting base region holds little importance to the thermal behavior of the sensor as a whole (Souza et al., 2011). The probe more closely considered in the development of this study belongs to the A4 Skyhawk aircraft of the Brazilian Navy, the Pitot sensor PH-510, from Aero Instruments Co., USA (Figures 1.a, b).

Figure 1.a. Pitot probe PH-510 used for illustration of the proposed thermal model

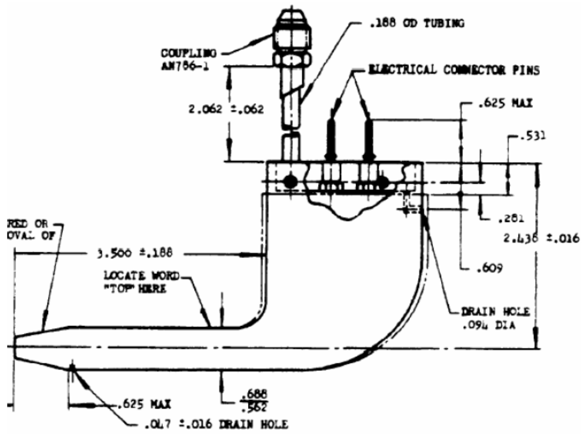

Figure 1.b. Technical drawing of the Pitot tube (source: Aero Instruments Co., USA)

The mathematical formulation of the present problem starts with the heat conduction equation for transient heat transfer along the structure of the Pitot tube, coupled with correlations for the convection heat transfer coefficients along its surface. The model of ice formation is employed in the sensing region of the Pitot, both as a boundary condition in the stagnation region and modifying the energy equation at the sensor tip. The problem formulation for the radially averaged temperature distribution, $T_{av}(x,t)$, with ice accretion, is given by:

$$\rho c_{p,ef}(x)\frac{\partial T_{av}(x,t)}{\partial t}=\frac{1}{A(x)}\frac{\partial}{\partial x}\left[k_{ef}(x)A(x)\frac{\partial T_{av}(x,t)}{\partial x}\right]-h(x)\frac{p(x)}{A(x)}\left[T_{av}(x,t)-T_{\infty}\right]+g_{av}(x,t),\; 0<x<L,\; t>0 \tag{1}$$

with initial and boundary conditions given by:

$$T_{av}(x,0)=T_0(x),\quad 0<x<L \tag{2}$$

$$-k_{ef}(0)\left.\frac{\partial T_{av}(x,t)}{\partial x}\right|_{x=0}=h_e^*[T_{\infty}-T_{av}(0,t)]+q_L+q_G,\quad t>0 \qquad \left.\frac{\partial T_{av}(x,t)}{\partial x}\right|_{x=L}=0,\quad t>0 \tag{3,4}$$

$$h_e^*=\left(\frac{1}{h_0}+\frac{L_{Rime\ Ice}(0,t)}{K_{Rime\ Ice}}+\frac{L_{Glaze\ Ice}(0,t)}{K_{Glaze\ Ice}}+\frac{L_{Water}(0,t)}{K_{Water}}\right)^{-1} \tag{5}$$

where the variables shown above correspond to ρ, density [kg/m^3], c_p, specific heat [J/(kg ºC)], *ef* stands for effective property, T_{av}, radially averaged temperature [ºC], x, longitudinal coordinate [m], k_{ef}, thermal conductivity [W/(m ºC)], A, cross-sectional area [m^2], p, cross-section perimeter [m], h, heat transfer coefficient [W/(m^2 ºC)], t, time [s], T_{∞}, air temperature outside boundary layer [ºC], $g_{av}(x,t)$, transversally averaged energy generation density [W/m^3], $T_0(x)$, initial temperature distribution [ºC], h_e^* is the effective heat transfer coefficient at stagnation region [W/(m^2 °C)], h_0 is the coefficient of heat transfer due to the flow in the stagnation region [W/(m^2 °C)], q_G is the gained heat flux [W/m^2], q_L is lost heat flux, K's are the thermal conductivities of each layer [W/(m °C)], and L's are the thicknesses of each layer [m].

The governing equations for determining the ice accretion rates were implemented using the extended Messinger model. At lower temperatures and low liquid water contents, rime ice occurs, for which the ice shape is determined by a simple mass balance. At higher temperatures and high liquid water contents, glaze ice forms, for which the energy and mass conservation equations are combined to yield a single first order ordinary differential equation, to be solved numerically. Once the ice thickness is obtained, the water height and the temperatures in the different layers may be calculated (Myers et al., 2002; Özgen et al., 2009). The mass and energy balances employed in the analysis of ice accretion over wings can be directly applied to the study of ice formation on Pitot probes. However, the heat fluxes by convection (q_c) and the latent heat (q_L) are the most important terms in the energy balance. The terms here considered, are listed below (Mingione et al., 1998; Özgen et al., 2009):

Convective heat flux at the water surface (q_c):

$$q_c=h_c(T_{sur}-T_a) \tag{6}$$

where h_c is local heat transfer coefficient [W/(m^2 °C)], T_{sur} is the temperature at the ice surface [ºC], T_a is the ambient temperature [ºC].

Cooling heat flux by incoming droplets (q_{drop}):

$$q_{drop}=LWC\,\beta V_{\infty}c_{pw}(T_{sur}-T_a) \tag{7}$$

where LWC is liquid water content [kg/m^3], β , the local collection efficiency, V_{∞} , freestream velocity [m/s], c_{pw} is specific heat of water [J/kg K].

Evaporative heat flux loss (q_e):

$$q_e=\lambda_e e_0(T_{sur}-T_a) \tag{8}$$

where λ_e is the evaporation coefficient, e_0 = 27.03, saturation vapor pressure constant [Pa/K].

The evaporation coefficient is expressed as:

$$\lambda_e=\frac{0.622h_cL_E}{c_pP_tLe^{2/3}} \tag{9}$$

where h_c is local heat transfer coefficient [W/(m^2 °C)], L_E is latent heat of evaporation [J/kg], P_t is the total pressure of the airflow [Pa], c_p is the specific heat of air [J/kg K], and Le is Lewis number.

Sublimation heat flux loss (q_s):

$$q_s = \lambda_s e_0 (T_{sur} - T_a) \tag{10}$$

where the sublimation coefficient λ_s is expressed as:

$$\lambda_s = \frac{0.622 h_c L_s}{c_p P_t Le^{2/3}} \tag{11}$$

where Le is Lewis number and L_s is the latent heat of sublimation [J/kg].

Aerodynamic heating term (q_{aero}):

$$q_{aero} = \frac{r\, h_c V_\infty^2}{2 c_p} \tag{12}$$

where r is the adiabatic recovery factor ($r=Pr^{1/2}$ for laminar flow, $r=Pr^{1/3}$ for turbulent flow).

Heat flux due to kinetic energy of incoming droplets (q_k):

$$q_k = LWC\, \beta V_\infty \frac{V_\infty^2}{2} \tag{13}$$

Heat flux due to incoming runback water (q_{in}):

$$q_{in} = \dot{m}_{in} c_{pw} (T_f - T_{sur}) \tag{14}$$

where $\dot{m}_{in}$ is the mass flux of the incoming runback water [kg/m^2.s], T_f is the freezing temperature [ºC] and T_{sur} is the temperature at the ice surface (for rime ice) or the water surface (glaze ice) [ºC].

Heat flux due to solidification (Q_l):

$$q_l = \rho_r L_F \frac{\partial B}{\partial t} \tag{15}$$

where ρ_r is the density of rime ice [kg/m^3] and L_F is latent heat of solidification [J/kg].

The rime ice thickness and the expression for the water height (B) can be obtained directly from the mass conservation, as water droplets freeze immediately on impact:

$$B(t) = \frac{LWC\, \beta V_\infty + \dot{m}_{in} - \dot{m}_{e,s}}{\rho_r} t \tag{16}$$

$$h(t) = \frac{(LWC\, \beta\, V_\infty + \dot{m}_{in} - \dot{m}_e)}{\rho_w}(t - t_g) - \frac{\rho_g}{\rho_w}(B(t) - B_g) \tag{17}$$

where $\dot{m}_s$ is the evaporating (or sublimating) water mass flux [kg/m^2.s], ρ_w is the density of water [kg/m^3], t is time [s], $\dot{m}_e$ is the evaporation mass fllux [kg/m^2.s], and ρ_g is density of glaze ice [kg/m^3]. Also, B_g and t_g are the ice thickness [m] and the corresponding time [s] at which glaze ice first appears, respectively:

$$B_g = \frac{k_i (T_f - T_s)}{(LWC\, \beta\, V_\infty + \dot{m}_{in} - \dot{m}_{sub}) L_F + (q_{aero} + q_k + q_{in}) - (q_c + q_{drop} + q_e)} \tag{18}$$

$$t_g = \frac{\rho_r}{LWC\beta V_\infty + \dot{m}_{in} - \dot{m}_{sub}} B_g \tag{19}$$

where k_i is thermal conductivity of ice [W/(m °C)], T_s is the temperature at the Pitot tube surface [ºC], $\dot{m}_{sub}$ is the ice sublimation mass flux [kg/m^2.s].

In the present paper, the equations of ice growth and ice shape for airfoil, as presented by Anderson & Tsao (2005), Tsao & Lee (2012) and Sherif et al. (1997), are employed for the Pitot PH-510 in atmospheric conditions of ice accretion. The dimensionless ice thickness, Δ, is given by:

$$\frac{\Delta}{d} = nA_c\beta_0 \tag{20}$$

Here d is the cylinder diameter, n is freezing fraction, A_c is the accumulation parameter and β_0 is the collection efficiency of water droplets at the stagnation line.

The accumulation parameter is expressed as:

$$A_c = \frac{LWC\, V_\infty\ \tau}{d\,\rho_i} \tag{21}$$

where τ is the accretion time and ρ_i is the ice density, and the freezing fraction is given by

$$n = \frac{c_{p,ws}}{\Lambda_f}(\phi + \frac{\theta}{b}) \tag{22}$$

The terms in this expression are $c_{p,ws}$, specific heat of water at the surface temperature, ϕ , the water energy transfer parameter, b , the relative heat factor, θ, the air energy transfer parameter, and Λ_f latent heat of solidification for water, where:

$$b = \frac{LWC\ V_\infty\ \beta_0\ c_{p,ws}}{h_c} \tag{23}$$

3. SOLUTION METHODOLOGY

The proposed combined model, together with the parameters and properties described herein, have been implemented in the symbolic-numerical platform *Mathematica* 9.0 (Wolfram, 2013) and solved numerically by the method of lines available in its subroutine for solving differential equations, NDSolve. The NDSolve function handles both partial and ordinary nonlinear differential equations, with automatic absolute and relative errors control. This function provides the solution as an object code that approximates the behavior of the dependent variables as a continuous function in terms of the independent variables. The numerical results were also verified against the error controlled solution through the UNIT code (Unified Integral Transforms) (Sphaier et al., 2009; Cotta et al., 2010), which implements an automatic hybrid numerical-analytical solution based on the Generalized Integral Transform Technique (GITT) (Cotta, 1993; Cotta & Mikhailov, 1997; Cotta & Mikhailov, 2006) .

4. RESULTS AND DISCUSSION

The Pitot tube fitted to the aircraft A-4 Skyhawk of the Brazilian Navy, PH-510, from Aero Instruments Co., Figs.1.a,b, was used as a basis for evaluation of the model. The heater is located within the central portion of the cylindrical body of the probe and has a nominal value of 41 Ohms of electrical resistance, coiled over a porcelain electrical insulation.

4.1 A-4 Skyhawk Flight Conditions

Theoretical predictions for the transient thermal behavior of the heated Pitot tube were first obtained for typical flight conditions of the A4 Skyhawk (Lisboa et al., 2013; Souza et al., 2015). Figure 2.a presents the temperature distribution at steady state along the surface of the Pitot tube, once only the external temperature T_∞ is abruptly changed to -40 °C while going through a cloud formation, with Mach 0.51 and 15000ft of altitude. In this situation, for which the transient evolution of surface temperatures is shown in Fig.2.b, the anti-icing system still manages to keep the temperature at the tip of the probe above the freezing temperature, while the area of the wing-shaped support, less important in the thermal control, is already at temperatures below the freezing point.

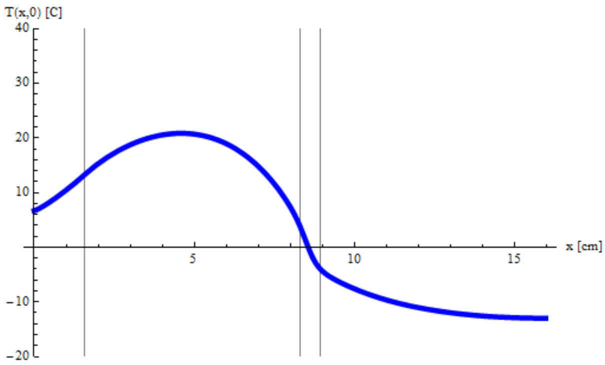

Figure 2.a. Prediction of steady-state temperatures along the probe PH510 with ice formation (T_∞=-40 °C, Ma=0.51, 15000 ft altitude)

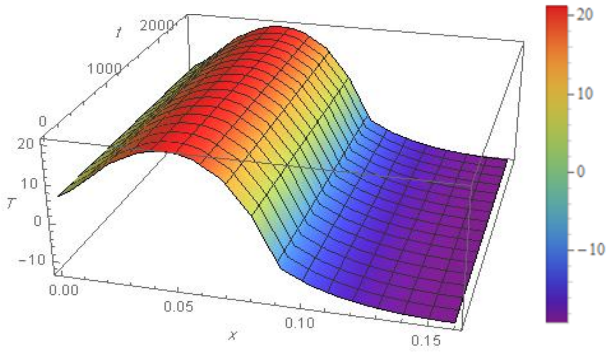

Figure 2.b. Transient distribution of the surface temperature of the Pitot tube along its length (T_∞=-40 °C, Ma=0.51, 15000 ft altitude)

Figures 3.a, b show the transient behavior of the temperature distribution along the probe surface under much more favorable conditions to ice formation, again with T_∞ = -40 °C and with particularly high values of LWC (1.115 g/m^3). Also, the MVD (water droplet median volume diameter), which has effects on the rime ice shape, is taken as 45 µm. Here, it is clearly shown the reduction in temperatures at the probe tip below 0 °C, with ice formation.

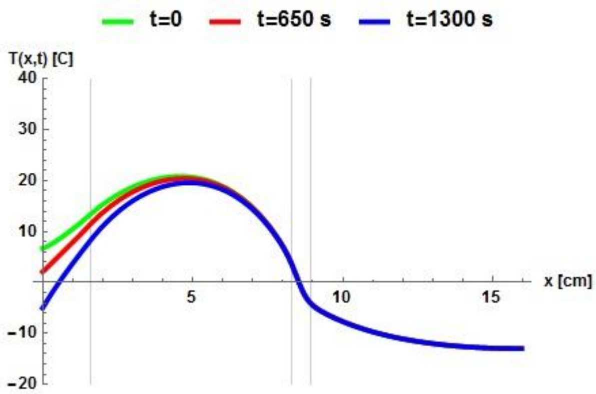

Figure 3.a. Comparison of the temperature prediction along the PH510 probe before (green line) and after starting the icing conditions (red and blue line) (T_∞=-40 °C, LWC=1.115 g/m^3, MVD=45 µm, V_∞=800 km/h)

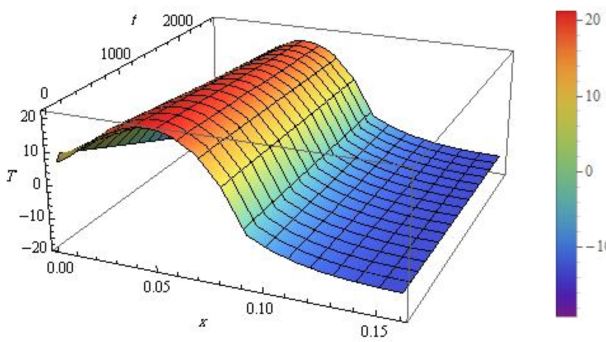

Figure 3.b. Transient distribution of the surface temperature along the A4 Pitot with icing at-flight (T_∞=-40 °C, LWC=1.115 g/m^3, MVD=45 µm, V_∞=800 km/h)

This specific behavior of the stagnation point temperature is best observed in Figure 3.c, which shows the transient behavior of the temperature at the probe tip, where ice formation in the stagnation region is triggered at about 840 s after starting the transient process.

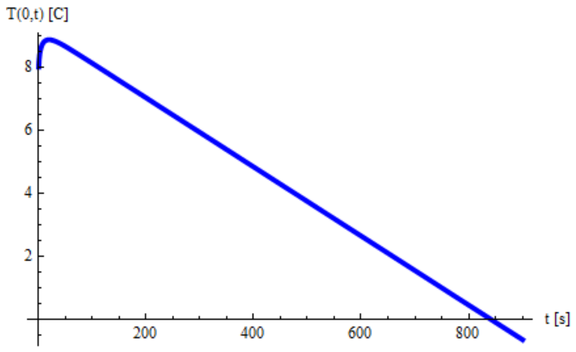

Figure 3.c. Transient behavior of the temperature at the stagnation region of the Pitot probe.
A4 Skyhawk (T_∞=-40 °C, LWC=1.115 g/m^3, MVD=45 µm, V_∞=800 km/h)

4.2 AF 447 Flight Conditions

The Airbus A330 has three Pitot probes (Fig.4) and six static pressure sensors. These probes are fitted with drains enabling removal of water, and an electric heating system in principle for preventing ice formation.

As we have no data on the Thales Pitot tube, which was installed on the AF447 at the time of the accident, in the present simulation we have used data relative to the PH510 but with the conditions of the AF 447 reported in the final report of the BEA, French agency responsible for the accident investigation. The conditions adopted were altitude of 11 km and Mach number between 0.82-0.86 (871-913 km/h).

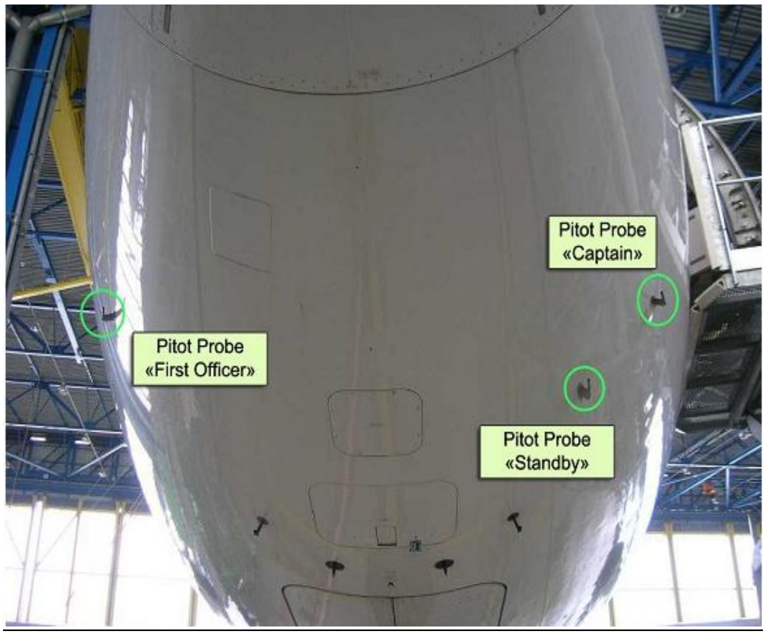

Figure 4. Position of the Pitot tubes of the Airbus A330
(source: BEA Final Report-Air France-flight AF 447 Rio de Janeiro - Paris, 2012)

Figure 5.a illustrates the transient behavior of the temperature distribution along the surface of the Pitot probe, where it is clearly observed that the temperature at the stagnation region achieves values below freezing temperature during the time interval investigated. Figure 5.b shows the time evolution of the temperature at the stagnation region, which allows one to evaluate that after about 3080 sec (about 50 minutes), ice formation starts. The estimated time for icing of the Pitot tubes in the actual accident was about 30 minutes, as discussed in the BEA report (BEA Final Report, 2012) essentially the time elapsed along the flight within the cloud formation until the beginning of the probes freezing. The results here obtained considered only the liquid water content (LWC), and ice crystals were not accounted for.

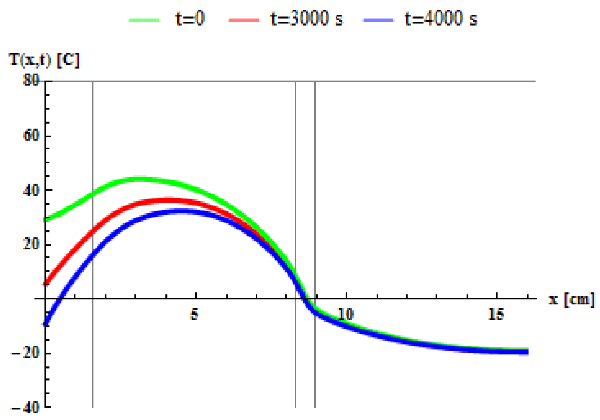

Figure 5.a. Comparison of the predicted temperatures along the probe PH510 before and after icing formation onset for the flight conditions of the AF447 (T_∞ = -54 °C, LWC = 1.115 g/m^3, MVD = 45 µm, V_∞ = 800 km/h)

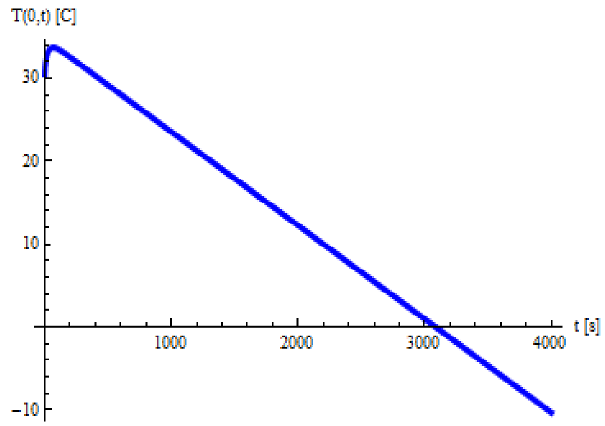

Figure 5.b. Transient behavior of the temperature in the stagnation region of the Pitot probe for the flight conditions of the AF447, with ice formation (T_∞ = -54 °C, LWC = 1.115 g/m^3, MVD = 45 µm, V_∞ = 800 km/h)

4.3 Ice Formation on the Pitot Probe

Figure 6.a shows a prediction of ice formation on the Pitot tube PH-510, for the conditions used by (Silva, 2010). From the ice shape it can be observed that the accretion occurs mainly at the stagnation region, with a distinct reduction along the conical region and no occurrence in the cylindrical region. The simplified ice formation model is obtained from airfoil equations and then geometry factors and collection efficiency relations are adopted for Pitot tube in stagnation line and conical region of Pitot tube. The ice accretion on the conical region and beyond is negligible due to the very small value of the collection efficiency (β <0.2).

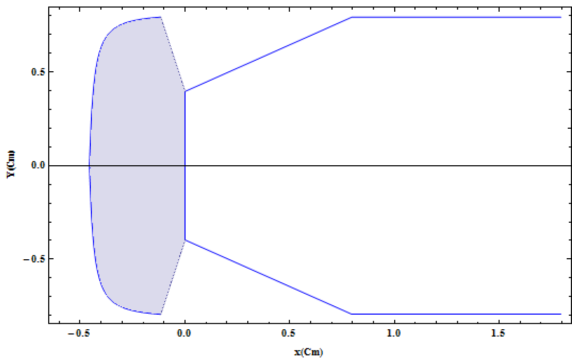

Figure 6.a. Ice shape in the Pitot probe PH-510 under conditions of (Silva, 2010). (V_∞= 56 m/s, LWC=0.59 g/m^3, T_{sur}= -10 ^{0}C, MVD=26.2µm)

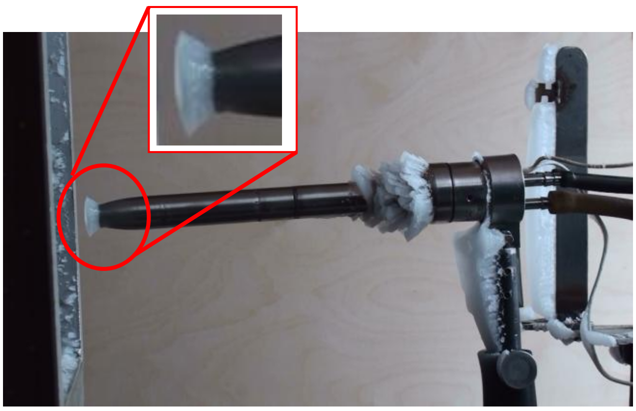

Figure 6.b. Ice shape over Pitot probe as obtained in the TsAIG Climatic Icing Wind Tunnel, Russia (Prof. Ivan Egorov, Head of TsAGI research department, private communication)

Although experimental and theoretical analysis of ice formation on Pitot tubes were not yet available in the open literature, we have obtained from the TsAIG Climatic Icing Wind Tunnel, Russia, some sample images from their Pitot tube icing experiments, as displayed in Figure 6.b. TsAGI has a climatic wind tunnel for investigations of aircraft parts and aviation materials at icing conditions. From a qualitative point of view, one can conclude that the ice shape in these images indicate that the present simulation is consistent with the expected physical behavior on ice accretion.

This work proposed a simplified model for ice formation in the framework of a conjugated heat transfer problem formulation, towards the thermal analysis of a Pitot tube under atmospheric conditions favorable to icing. The thermal analysis is based on the heat conduction equation for the structure of the probe, employing a lumped formulation in the radial direction and a local differential formulation along the longitudinal coordinate starting from the probe tip. The geometric and material variations along the length of the Pitot then become effective coefficients and variable properties as a function of the longitudinal coordinate. Appropriate coefficients were used for heat transfer in the stagnation, conical, cylindrical and elliptical wing shaped support regions, which form the structure of the Pitot.

This model was validated in previous studies using experimental results in transient conditions for incompressible flow obtained in wind tunnel, and for compressible flow on flight tests with the A4 Skyhawk aircraft of the Brazilian Navy. The coupling between the heat conduction problem and the icing accretion model is accomplished through modified effective heat transfer coefficients at both the tip and along the conical region of the Pitot probe.

5. ACKNOWLEDGEMENTS

The authors are indebted to the Brazilian Navy for providing the Pitot tubes here employed and sponsoring the A4 flight tests. The financial support of CNPq and FAPERJ is also acknowledged. This work is dedicated to the 228 victims of the AF447 flight and their families.

6. REFERENCES

Abbott, I.H., V. Doenhoff, A.E., Stivers, L.S. JR., 1945, *Summary of airfoil data*. NACA ACR, L5C05.

Al-khalil, K. M., Horvath, C., Miller, D. R., Wright, W., 2001, *Validation of NASA thermal ice protection computer codes. Part 3 – Validation of ANTICE. Cleveland*, OH: National Aeronautics and Space Agency, P: 18. (Contractor Report, 2001-210907).

Anderson, D. N., Tsao, J-C., 2005, *Additional Results of Ice-Accretion Scaling at SLD Conditions*, Ohio Aerospace Institute, Brook Park, Ohio, NASA/CR—2005-213850.

Anderson, D. N., Tsao, J. C., 2005a , *Evaluation and Validation of the Messinger Freezing Fraction*, Ohio Aerospace Institute, Brook Park, Ohio, NASA/CR—2005-213852.

Anderson, D. N., Tsao, J. C., 2005b , *Overview of Icing Physics Relevant to Scaling*, Ohio Aerospace Institute, Brook, Park, Ohio, NASA/CR—2005-213851.

Atmospheric Properties Calculator, Based on US Standard Atmosphere 1976, Version 2.1.4, released August 2005: <http://www.aerospaceweb.org/design/scripts/atmosphere/>.

Bragg, M.B., 1982, *Rime ice accretion and its effect on airfoil performance.* NASA Contractor Report 165599, Lewis Research Center, Grant NAG 3-28, March 1982.

Caliskan, F., Aykan, R., and Hajiyev, C., 2008, *Aircraft Icing Detection, Identification, and Reconfigurable Control Based on Kalman Filtering and Neural Networks*, J. of Aerospace Engineering, v. 21, no. 2, pp.51-60.

Cotta, R.M., 1993, *Integral Transforms in Computational Heat and Fluid Flow*, CRC Press, Boca Raton.

Cotta, R.M. and Mikhailov, M.D., 1997, *Heat Conduction: Lumped Analysis, Integral Transforms, Symbolic Computation*, Wiley-Interscience, Chichester, UK.

Cotta, R.M. and Mikhailov, M.D., 2006, *Hybrid methods and symbolic computations*, in: Handbook of Numerical Heat Transfer, 2nd edition, Chapter 16, Eds. W.J. Minkowycz, E.M. Sparrow and J.Y. Murthy, John Wiley, New York.

Donatti, C. N., Silveira, R. A., Bridi, G., Maliska, C.R., Silva, A.F.C., 2007, *Ice Accretion Simulation in Presence of a Hot Air Anti-Icing System*, Proc. of the 19th International Congress of Mechanical Engineering - COBEM 2007, Brasilia, DF, Brazil, November 5-9.

Fortin, G., Laforte, J-L., Ilinca, A., 2006, *Heat and mass transfer during ice accretion on aircraft wings with an improved roughness model*. Int. J. Thermal Sciences,V. 45, P:595–606.

Gent, R.W., Dart, N.P., Cansdale, J.T., 2000, *Aircraft icing*, Phil. Trans R Soc Lond A, V. 358. P: 2873–2911.

Hardy, J.K., 1946, *Protection of aircraft against ice*. Report S.M.E. 3380, British Royal Aircraft Establishment, July.

Langmuir, I., Schaefer, V., Vonnegut, B., 1946, *Thinking outside the Cold Box: How a Nobel Prize Winner and Kurt Vonnegut's Brother Made White Christmas on Demand*, <http://www.gereports.com/post/78010633546/thinking-outside-the-cold-box-how-a-nobel-prize>

Lisboa, K.M., Souza, J.R.B., and Cotta, R.M., 2013, *Thermal Analysis of an Aeronautical Pitot Probe with Anti-Icing System: Modeling, Simulation and Parametric Studies*, Proc. of the 22nd International Congress of Mechanical Engineering - COBEM 2013, Ribeirão Preto, SP, Brazil, November 3-7.

Messinger, B.L., 1953, *Equilibrium temperature of an unheated icing surface as a function of airspeed*. J. of Aeronautical Sciences, V. 20, n. 1, P: 29–42.

Mingione, G., Brandi, V., 1998, *Ice accretion prediction on multielement airfoils.* J. Aircraft, V. 35, n. 4. P : 240–246.

Myers, T.G., 2001, *Extension to the Messinger model for aircraft icing*. AIAA J. The American Institute of Aeronautics and Astronautics, V. 39, n. 2, P: 211–218.

Myers, T.G., Charpin, J.P.F., Thompson, C.P., 2002, *Slowly accreting ice due to supercooled water impacting on a cold surface*. Phys Fluids, V. 14, n. 1, P: 240–256.

Özgen, S., Canibek, M., 2009, *Ice accretion simulation on multi-element airfoils using extended Messinger model*, Heat and Mass Transfer, V. 45, Issue 3 , pp 305-322.

Ruff, G. A., Berkowitz, B. M., 1990, *User Manual for the NASA Lewis Ice Accretion Prediction Code (LEWICE)*. Cleveland: National Aeronautics and Space Administration: 240 (Contractor Report, 185129).

Sherif, .S.A., Pasumarthi, N., Bartlett, C.S., 1997, *A semi-empirical model for heat transfer and ice accretion on aircraft wings in super cooled clouds.* Cold Regions Science and Technology, P: 165–179.

Silva, G. A. L., 2009, *Transferência de calor e massa no escoamento bifásico em torno de aerofólios equipados com sistemas de antigelo aeronáuticos,* Tese de Doutorado, Escola Politécnica, Universidade de São Paulo, 2009.

Silva, G. A. L., 2010, *Similarity Studies for Air Data Probes New Standard – AS5562, Coupled Heat and Mass Transfer Effects*, SAE AC-9C Aircraft Icing Technology Committee Meeting October 8th-10th 2010 – Portland.

Silva, G. A. L., 2009, *Transferência de Calor e Massa no Escoamento Bifásico em Torno de Aerofólios Equipados com Sistemas de Antigelo Aeronáticos,*Tese (Doutorado)- Universidade de São Paulo, São Paulo, Brasil.

Silveira, R. A., 2001, *Simulação Numérica da Formação de Gelo na Borda de Ataque de Perfis Aerodinâmicos*. Dissertação (Mestrado) — Universidade Federal de Santa Catarina, Florianópolis.

Silveira, R. A., Clovis R. M., D. A., Estivam, R. M., 2003, *Evaluation of Collection Efficiency Methods for Icing Analysis*, 17th International Congress of Mechanical Engineering, COBEM, November 10-14, 2003, São Paulo, SP.

Souza, J.R.B., J.L. Zotin, J.B.R. Loureiro, C.P. Naveira-Cotta, A.P. Silva Freire, and Cotta, R.M., 2011, *Conjugated Heat Transfer Analysis of Heated Pitot Tubes: Wind Tunnel Experiments, Infrared Thermography and Lumped-Differential Modeling*, 21st International Congress of Mechanical Engineering, COBEM-2011, ABCM, Natal, RN, Brazil.

Souza, J.R.B., Lisboa, K.M., Cerqueira, I.G., Naveira-Cotta, C.P., Cotta, R.M., and Zotin, J.L.Z., 2012, *Conjugated: Heat Transfer Models for Heated Aeronautical Pitot Tubes Experimental Validation with A4 Skyhawk Flight Tests*, 14th Brazilian Congress of Thermal Sciences and Engineering, ENCIT 2012, Rio de Janeiro, RJ.

Souza, J.R.B., Lisboa, K.M., Cerqueira, I.G., Naveira-Cotta, C.P., Cotta, R.M., and Zotin, J.L.Z., 2015, *Conjugated Heat Transfer Analysis of Heated Aeronautical Pitot Probes with Flight Tests Experimental Validation*, Heat Transfer Eng, V.36, pp.991-1000.

Sparrow, E.M., Abraham, J.P., Tong, J.C.K., 2004, *Archival Correlations for Average Heat Transfer Coefficients for Non-circular and Circular Cylinders and for Spheres in Cross-flow*, Int. J. Heat & Mass Transfer, V.47, P: 5285-5296.

Sphaier, L. A., Naveira Cotta, C. P., Cotta, R. M., Quaresma, J. N. N., 2009, *The UNIT (Unified Integral Transforms) Symbolic- Numerical Computational Platform for Benchmarks in Convection-Diffusion Problems*, Proceedings of the 30th CILAMCE, Congresso Ibero Latino Americano de Métodos Computacionais em Engenharia (CILAMCE), 2009, Buzios, RJ, Brazil.

Stefanini, L. M., 2009, *Efeito Da Camada Limite Térmica Na Formação de Gelo Aerofólios de Uso Aeronáutico.* Dissertação de Mestrado - Universidade de São Paulo, São Paulo, Brasil.

Tribus, M., 1949, *Intermittent Heating for Protection in Aircraft Icing*. Ph.D. Dissertation, University of California, Los Angeles, CA.

Tsao, J. C., Lee, S., 2012, *Evaluation of Icing Scaling on Swept NACA 0012 Airfoil Models*, Ohio Aerospace Institute, Brook Park, Ohio and ASRC Aerospace Corporation, Cleveland, Ohio NASA/CR—2012-217419.

Tong, Z.M., and Hu, Y.H., 2009, *Convective Heat Transfer and Flow Resistance Characteristics of Various Types of Elliptical Tubes*, Proc. of the Int. Conf. on Energy and Environment Technology, IEEE.

WAS, *2nd Workshop on Aviation Safety*, 2011, Núcleo Interdisciplinar de Dinâmica dos Fluidos (NIDF,COPPE/UFRJ), CT2, <http://www.segurancaaerea.coppe.ufrj.br/2ndwas.html.>

White F. M., 1992, *Viscous Fluid Flow*, McGraw-Hill.

Wolfram, S., 2005, The *Mathematica* Book, Cambridge-Wolfram Media.

Wright W.B., Gent R.W., Guffond, D., 1997, DRA/NASA/ ONERA *Collaboration on icing research, Part II. Prediction of airfoil ice accretion*, NASA CR-202349.

Wright, W.B., (1995), *User Manual for the Improved NASA Lewis Ice Accretion Code, LEWICE 1.6*, NASA CR 198355, June 1995

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