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HEAD LOSSES IN DUCT FLOWS WITH CONDUCTION-RADIATION HEAT TRANSFER

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Abstract. *Non-isothermal duct flows are among the most important transport phenomena in engineering. In such flows, the viscous frictional head loss parameter is a measure of mechanical energy degradation, but not of the total energy degradation. In non-isothermal flows, both mechanical and thermal energy degradation need be considered. The use of an entropy-based head loss parameter as a measure of energy quality losses in flows with both momentum and conduction-radiation heat transfer is considered in this work. A parametric analysis is also carried out to reveal the role of Reynolds number and Nusselt number in controlling head losses in non-isothermal duct flows. The results show that, under certain flow conditions, selection of a high Reynolds number can significantly reduce the rate of overall head loss. It is also found that Nusselt number is an effective parameter for managing head losses when radiation is a dominant irreversible process.*

Keywords: *non-isothermal flow, head losses, entropy generation, optimization*

1. INTRODUCTION

It can be shown that frictional head loss represents the irreversible degradation of mechanical energy due to entropy generation. Consider for instance steady viscous flow of an incompressible fluid through a duct that communicates thermally with its environment (Fig. 1). The head loss in the duct due to viscous friction is by definition:

$$h_l = \left(P_1/\rho_1 + gz_1 + \frac{1}{2}V_1^2 \right) - \left(P_2/\rho_2 + gz_2 + \frac{1}{2}V_2^2 \right) \tag{1}$$

where, P, ρ, g, z, and V refer to pressure, density, gravitational acceleration, elevation, and velocity, respectively.

Employing the balance of energy, Eq. (2), the frictional head loss can be restated according to Eq. (3):

$$\dot{m}\left(Pv_1 + u_1 + gz_1 + \frac{1}{2}V_1^2 \right) + \sum_{i=1}^{n} Q_i = \dot{m}\left(Pv_2 + u_2 + gz_2 + \frac{1}{2}V_2^2 \right) \tag{2}$$

$$h_l = \left(u_2 - u_1 \right) - \frac{1}{\dot{m}} \sum_{i=1}^{n} Q_i \tag{3}$$

Here, $\dot{m}$ is the mass flow rate, Pv and u are the specific flow and internal energies, respectively, while $\dot{Q}_i$ is the local heat transfer rate.

The balance of entropy can be stated with Gibbs equation as following:

$$\sum_{i=1}^{n} \left(Q_i/T_i \right) + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} \equiv \sum_{i=1}^{n} \left(Q_i/T_i \right) - \dot{m}\int_1^2 \left(du/T \right) + \dot{S}_{gen} = 0 \tag{4}$$

where, T is temperature and $\dot{S}_{gen}$ is the entropy generation rate.

Combining Eq. (3) and Eq. (4) readily leads to an entropy-based formulation of the head loss parameter according to the following expression:

$$h_l = T\dot{S}_{gen}/\dot{m} \tag{5}$$

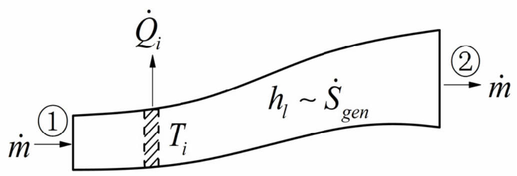

Figure 1. Streamtube for head loss analysis in ducts

Eq. (5) is a thermodynamic second-law metric that may characterize the energy efficiency of any system (Adeyinka and Naterer, 2005). A more exact derivation of Eq. (5) is given by Naterer and Camberos (2008). Based on Eq. (5), the real overall losses in ducts with both momentum and conduction-radiation heat transfer are studied. Only turbulent conditions are considered because engineering flows are mostly of this nature

2. THEORY

2.1 Entropy generation in ducts with momentum and conduction-radiation heat transfer

All momentum and heat transfer processes are irreversible from a thermodynamic point of view. The irreversibility is measurable through variations in the velocity and temperature fields. Entropy generation is the non-measurable effect of thermodynamic irreversibilities. For combined momentum and heat transfer processes, the rate of entropy generation is defined by Bejan (1982) as:

$$\dot{S}_{gen} = \dot{S}_{gen,f} + \dot{S}_{gen,h} \tag{6}$$

where,

$$\dot{S}_{gen,f} = \int_V \mu_{eff} \left(\frac{\Psi}{T} \right) \mathrm{d}V \tag{7}$$

is for the entropy generation associated with viscous friction, and it is dependent on the viscous dissipation function Ψ and effective viscosity μ_{eff}. $\dot{S}_{gen,h}$ is the heat transfer entropy generation rate. In the case of conduction-radiation heat transfer in a confined space, the following definition holds (see for instance Makhanlall and Liu, 2010):

$$\dot{S}_{gen,h} = \dot{S}_{gen,c} + \dot{S}_{gen,r}^{Medium} + \dot{S}_{gen,r}^{Wall} \tag{8}$$

with,

$$\dot{S}_{gen,c} = \int_V k_{eff} \left(\frac{\nabla^2 T}{T^2} \right) \mathrm{d}V \tag{9}$$

$$\begin{aligned} \dot{S}_{gen,r}^{Medium} &= \int_V \int_0^\infty \int_{4\pi} -\left(\kappa_{a\lambda} + \kappa_{s\lambda}\right) \frac{I_\lambda(\mathbf{r},\mathbf{s})}{T_\lambda(\mathbf{r},\mathbf{s})} \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V + \int_V \int_0^\infty \int_{4\pi} \kappa_{a\lambda} \frac{I_{b\lambda}(\mathbf{r},\mathbf{s})}{T_\lambda(\mathbf{r},\mathbf{s})} \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V \\ &+ \int_V \int_0^\infty \int_{4\pi} \frac{\kappa_{s\lambda}}{4\lambda} \left(\int_{4\pi} \frac{I_{b\lambda}(\mathbf{r},\mathbf{s})}{T_\lambda(\mathbf{r},\mathbf{s})} \Phi(\mathbf{s},\mathbf{s}') \mathrm{d}\Omega' \right) \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V + \int_V \int_0^\infty \int_{4\pi} \frac{\kappa_{a\lambda}}{T(\mathbf{r})} \left[I_\lambda(\mathbf{r},\mathbf{s}) - I_{b\lambda}(\mathbf{r}) \right] \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V \end{aligned} \tag{10}$$

$$\dot{S}_{gen,r}^{Wall} = \int_V \int_0^\infty \int_{4\pi} \left[\frac{I_\lambda(\mathbf{r}_\mathbf{w},\mathbf{s})}{T(\mathbf{r}_\mathbf{w})} - L_\lambda(\mathbf{r}_\mathbf{w},\mathbf{s}) \right] \left| \mathbf{n}_\mathbf{w} \cdot \mathbf{s} \right| \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V \tag{11}$$

Here, $\dot{S}_{gen,c}$, $\dot{S}_{gen,r}^{Medium}$, and $\dot{S}_{gen,r}^{Wall}$ are for the entropy generation due to heat conduction and convection, radiation transfer in the transported medium, and radiation transfer at solid walls, respectively. In these expressions, k_{eff} is the effective thermal conductivity, κ_a and κ_s are the radiative absorption and scattering coefficient, respectively, λ is wavelength, I_λ is radiation spectral intensity, $I_{b\lambda}$ is blackbody spectral intensity, Φ is the scattering phase function, Ω is the solid angle, $\mathbf{n}$ is a unit surface normal vector, and $\mathbf{s}$ is a unit position vector. For a detailed derivation of Eq. (10) and Eq. (11) see the works of Caldas and Semiao (2005) and Liu and Chu (2007).

2.2 Entropy-based formulation of head losses in ducts with momentum and conduction-radiation heat transfer

Losses in a flow are conventionally determined using a frictional head loss parameter. This parameter measures mechanical energy degradation, i.e. mechanical energy conversion into internal energy. When this conversion occurs on the temperature level $T = T_0$, with T_0 being the temperature of the reference environment, then the frictional head loss is equal to the total thermodynamic head loss. However, for flows at temperature levels $T > T_0$, the conventional head loss parameter measures the mechanical energy degradation, but not the total energy degradation, because now there is also thermal energy degradation to be considered.

For flows with combined momentum and heat transfer, it is logical to determine the overall losses by expanding Eq. (5) as following:

$$h_l = h_{l,f} + h_{l,h} = T\dot{S}_{gen} / \dot{m} \tag{12}$$

where,

$$h_{l,f} = T_0 \dot{S}_{gen,f} / \dot{m} = \frac{1}{\dot{m}} \int_V \frac{T_0}{T} \mu_{eff} \Psi \mathrm{d}V \tag{13}$$

and

$$h_{l,h} = h_{l,h}^{c} + h_{l,h}^{r,Medium} + h_{l,h}^{r,Wall} \tag{14}$$

with

$$h_{l,h}^{c} = T_0 \dot{S}_{gen,c} \big/ \dot{m} = \frac{1}{\dot{m}} \int_V k_{eff} \left(\frac{T_0 \nabla^2 T}{T^2} \right) \mathrm{d}V \tag{15}$$

$$\begin{aligned} h_{l,h}^{r,Medium} = T_0 \dot{S}_{gen,r}^{Medium} \big/ \dot{m} &= \frac{1}{\dot{m}} \int_V \int_0^\infty \int_{4\pi} -\left(\kappa_{a\lambda} + \kappa_{s\lambda}\right) \frac{T_0}{T_\lambda(\mathbf{r},\mathbf{s})} I_\lambda(\mathbf{r},\mathbf{s}) \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V \\ &+ \frac{1}{\dot{m}} \int_V \int_0^\infty \int_{4\pi} \kappa_{a\lambda} \frac{T_0}{T_\lambda(\mathbf{r},\mathbf{s})} I_{b\lambda}(\mathbf{r},\mathbf{s}) \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V \\ &+ \frac{1}{\dot{m}} \int_V \int_0^\infty \int_{4\pi} \frac{\kappa_{s\lambda}}{4\lambda} \left(\int_{4\pi} \frac{T_0}{T_\lambda(\mathbf{r},\mathbf{s})} I_{b\lambda}(\mathbf{r},\mathbf{s}) \Phi(\mathbf{s},\mathbf{s}') \mathrm{d}\Omega' \right) \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V \\ &+ \frac{1}{\dot{m}} \int_V \int_0^\infty \int_{4\pi} \kappa_{a\lambda} \frac{T_0}{T(\mathbf{r})} \left[I_\lambda(\mathbf{r},\mathbf{s}) - I_{b\lambda}(\mathbf{r}) \right] \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V \end{aligned} \tag{16}$$

$$h_{l,h}^{r,Wall} = T_0 \dot{S}_{gen,r}^{Wall} \big/ \dot{m} = \frac{1}{\dot{m}} \int_V \int_0^\infty \int_{4\pi} \left[\frac{T_0}{T(\mathbf{r_w})} I_\lambda(\mathbf{r_w},\mathbf{s}) - L_\lambda(\mathbf{r_w},\mathbf{s}) \right] \left| \mathbf{n_w} \cdot \mathbf{s} \right| \mathrm{d}\Omega \mathrm{d}\lambda \mathrm{d}V \tag{17}$$

In above expressions, $h_{l,f}$ represents mechanical energy degradation due to viscous dissipation, $h_{l,h}^{c}$ represents the energy degradation associated with heat conduction, $h_{l,h}^{r,Medium}$ is the energy degradation due to radiation heat transfer in the medium, and $h_{l,h}^{r,Wall}$ is the energy degradation due to wall radiation heat transfer processes.

3. RESULTS AND DISCUSSION

3.1 Head losses distribution and optimization of duct geometry

The procedure outlined in the previous section for determining the head losses in non-isothermal duct flows can in principle be applied to ducts of any shape. The following results are however limited to rectangular ducts through which a participating gas flows (Table 1). The cross section optimization of long rectangular ducts with both momentum and conduction-radiation heat transfer is shown in Fig. 2. Square rectangular ducts are predicted to have the lowest overall head loss rate. This may be because of their relatively small perimeter (discussed next).

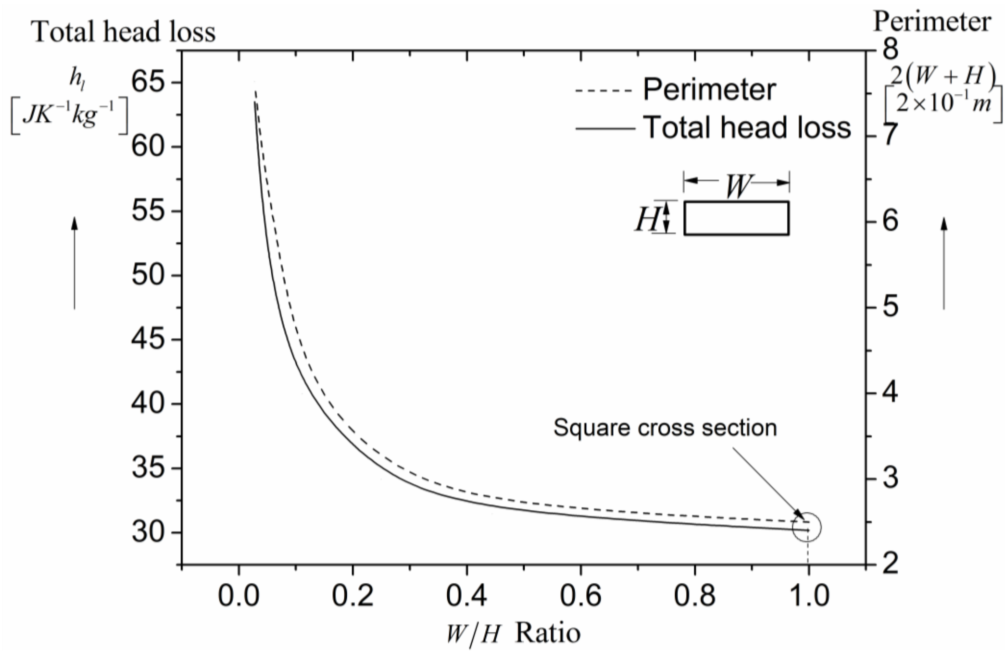

Figure 2. Influence of duct geometry on overall head loss

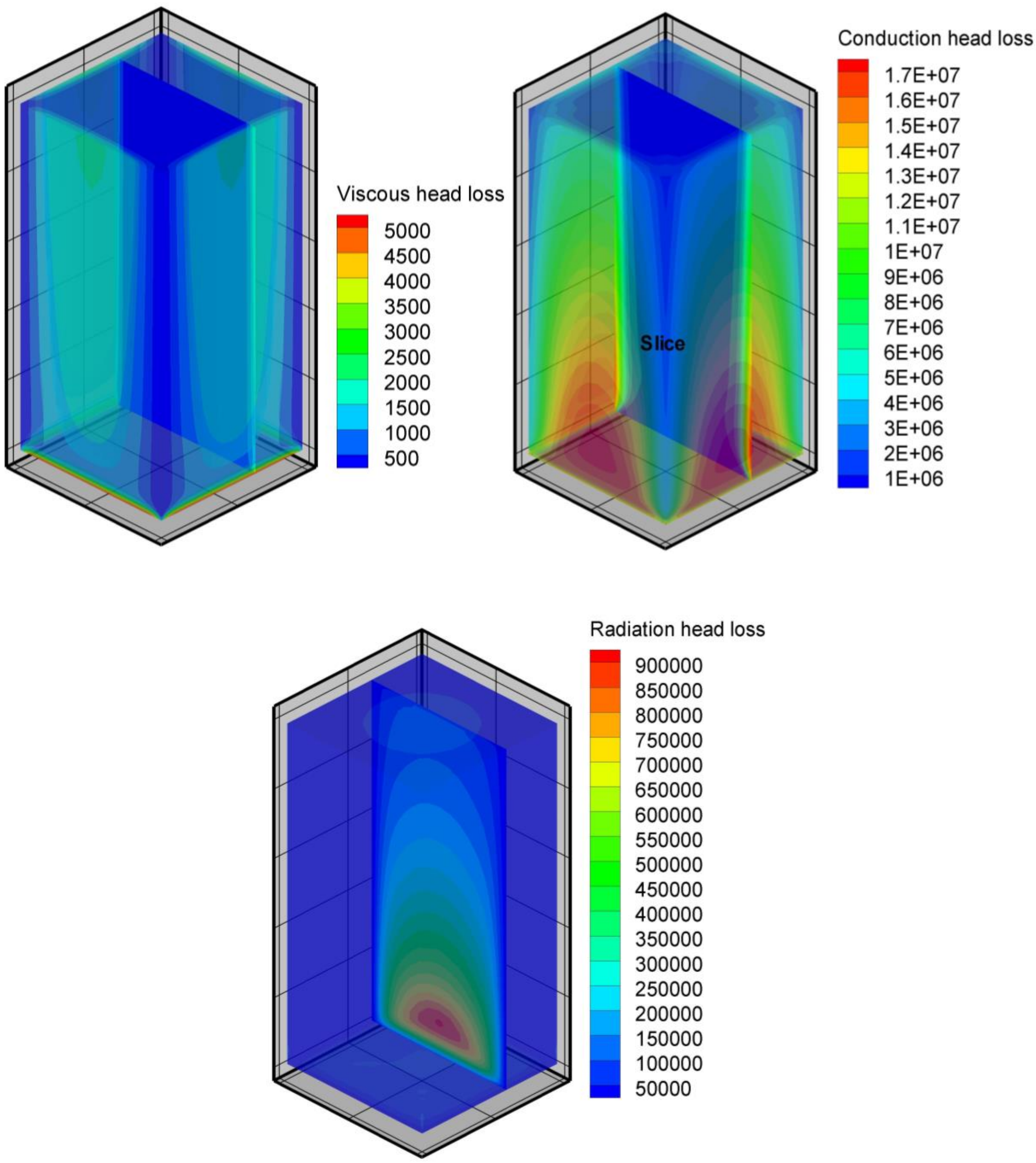

Figure 3. Head losses distribution in square ducts, $W/\left(kgm^3\right)$

Figure 3 depicts the head losses distribution in square ducts. The simulation model details are provided in Table 1. It is clear that head loss in non-isothermal duct flows is almost entirely due to the heat transfer processes. Generally, thermodynamic effects associated with viscous dissipation are only important in flows for which heat transfer effects can be neglected. The results also reveal that the duct near wall region acts as a strong concentrator of head loss due to heat conduction and convection. This is not unexpected, because the near wall zones in non-isothermal duct flows are commonly marked by relatively high local temperature gradients. Such large temperature gradients are the driving forces for heat conduction and its associated entropy generation.

Unlike heat conduction, thermal radiation transfer processes are usually not driven by local temperature gradients. Thermal radiation is a long range phenomenon, and therefore its thermodynamic effects are governed by the entire temperature field of the system under consideration. In semitransparent participating gasses (this study), radiation heat absorption and emission is often most significant in high temperature regions (Modest, 2003). This also explains why (in Fig. 3) the head losses associated with radiation transfer processes in the medium is most significant in the duct core region.

3.2 Role of Reynolds number

A parametric study has been carried out on the duct with square cross section to determine the effect of Reynolds number on the head losses. The results are shown in Fig. 4. It is apparent that the overall head loss reduces as Reynolds number becomes larger (especially in the region $\mathrm{Re} < 1.5\times10^4$). This can be explained by considering the effect of Reynolds number on the temperature field. The increase of Reynolds number is accompanied by a more uniform temperature field within the duct (not shown). As the overall temperature difference and local temperature gradients reduces with increasing Reynolds number, the head losses both due to radiation and heat conduction become less significant.

It is important to note that for low Reynolds number, i.e. when large temperature differences exist over the entire duct, thermal radiation is the dominant mechanism of head loss. Attempts to effectively reduce the head losses in non-isothermal flows with conduction-radiation heat transfer at low Reynolds number should thus be focused on how to reduce the radiation problem. One possible option is discussed in the next section.

3.3 Role of Nusselt number

Nusselt number is the dimensionless heat transfer coefficient that determines the extent to which the wall cooling effect influences the temperature distribution in the fluid. The total transverse average Nusselt number for duct flows with conduction-radiation heat transfer can be defined as following (see for instance Krishnaprakas, *et al*., 1999):

$$Nu_t = Nu_c + Nu_r = \frac{1}{N}\sum_{Z=0}^{Z=L} \frac{q'' D_h}{k_{\mathrm{av,eff}}\left(T_w - T_b\right)} \tag{18}$$

with,

$$T_b = \frac{1}{\dot{m}} \int \rho u\left(x,y,z\right) T\left(x,y,z\right) dA \tag{19}$$

Here, Nu_c and Nu_r are the conduction and radiation Nusselt numbers, respectively, q'' is the total wall heat flux, $\dot{m}$ is the mass flow rate, N is the node number, T_w is the local wall temperature, T_b is the bulk mean fluid temperature at duct cross section, and $k_{\mathrm{av,eff}}$ is the average duct cross section effective thermal conductivity.

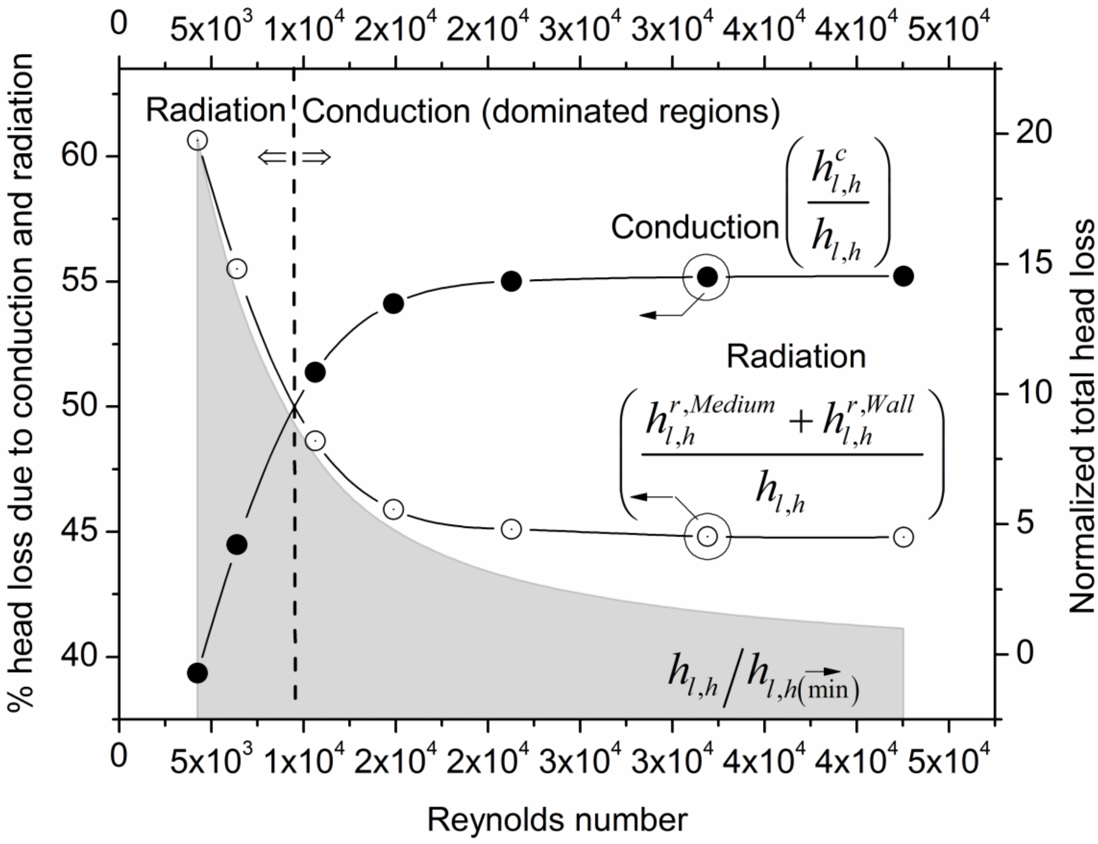

Figure 4. Influence of Reynolds number

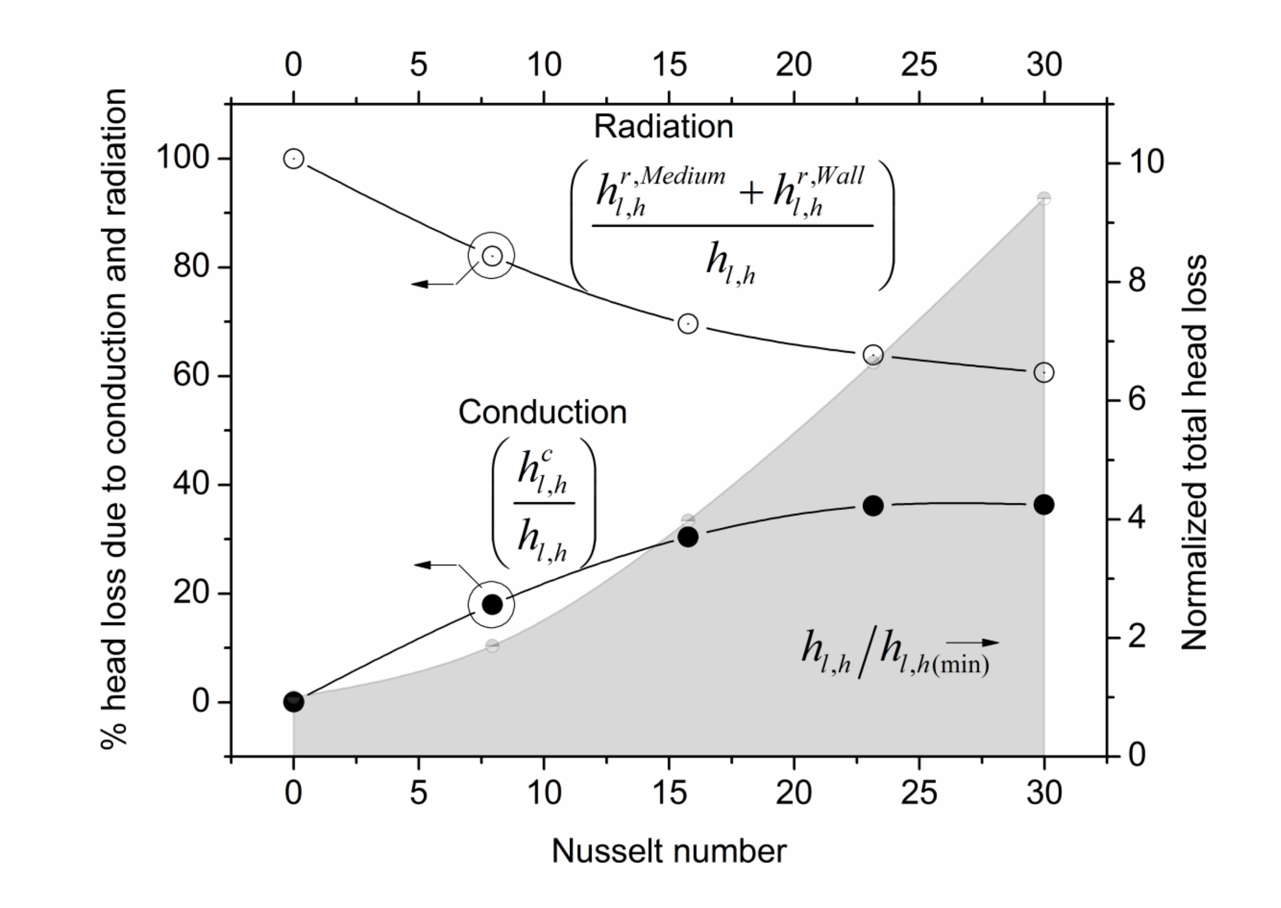

Figure 5. Influence of Nusselt number

As shown in Fig. 5, Nusselt number is an effective parameter for managing head loss when thermal radiation is the dominant irreversible process. Selection of a low Nusselt number increases the influence of radiation, but at the same time reduces the influences of heat conduction and convection. Since the rates of head loss due to conduction is usually very strong in duct flows (see Fig. 3), reducing the heat conduction irreversibilities will often result in better performance.

Table 1. Model parameters for duct flow with conduction-radiation heat transfer.

Properties	**Values**
Duct length (m)	10
Duct width (m)	0.2
Duct height (m)	0.2
Density (kg/m^3)	0.082
Specific heat (J/kgK)	14275-0.4T
Thermal conductivity (W/mK)	0.08-0.004T
Viscosity (kg/ms)	$3.773\times10^{-6}+1.932\times10^{-8}T$
Absorption coefficient (1/m)	0.62
Scattering Coefficient (1/m)	0.01
Fluid inlet temperature (K)	1000
Reference wall Nusselt number	30
Reference temperature (K)	300

4. CONCLUSIONS

An entropy-based model for evaluation of head losses in non-isothermal duct flows with conduction-radiation heat transfer is discussed in this work. The approach is applied to long rectangular ducts. The optimum duct cross sectional shape is found and the role of Reynolds number and Nusselt number is studied. The results show that:

1) Rectangular ducts with square cross section has the lowest overall rate of head loss
2) Reynolds number is an effective parameter for managing head losses in low turbulent Reynolds number flows
3) Nussult number is an effective parameter for managing head losses when radiation is the dominant irreversible process.

5. ACKNOWLEDGEMENTS

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