23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

NON LINEAR EDDY VISCOSITY MODELLING IN STEADY TURBULENT FLOWS DEVELOPED FROM LES ANALYSIS

Fernando Soares Alves
Luiz Eduardo Bittencourt Sampaio
Roney Leon Thompson
Laboratory of Theoretical and Applied Mechanics,
Graduate Program of Mechanical Engineering, Universidade Federal Fluminense - Rua Passo da Pátria 156, São Domingos, Niterói-RJ, Brasil
fernandosoaresalves@id.uff.br, luizebs@vm.uff.br, rthompson@mec.uff.br

Abstract. *Concerning incompressible flows of newtonian fluids the Reynolds Average is applied to the balance equations in order to obtain governing equations for the mean variables. The result is a set of equations which need further closure since a new term rises from the averaging procedure, namely the so called Reynolds stress tensor. Although many different models can be found in the literature for non linear eddy viscosity models, mainly linear models are still applied, specially in engineering, for the closure of the problem. The main objective of the work here presented is improving the linear model mostly adopted using tensor decomposition and analysis by orthogonal projection from LES results for a better modelling of the deviatoric part of the Reynolds stress tensor.*

Keywords: *RANS, non Linear Eddy Viscosity Model, Reynolds Stress Tensor*

1. INTRODUCTION

Turbulent fluid flow still remains as a major unsolved problem in classical physics and for this reason one the most active areas of research in fluid mechanics concerns turbulence modelling. As for newtonian fluids and, more specifically, incompressible flows, the balance of mass and momentum yields the set of equations known as the Navier-Stokes equation. As stated by Kolmogorov (1941a,b) the fine structure of turbulence in such flows is better known to be captured by what is called Direct Numerical Simulation (DNS) method. However since such scales are extremely difficult to represent numerically, DNS is still too prohibitive from the computational point of view. Most results concern simple geometries at low Reynolds number flows.

On the opposite side stands the method known as Reynolds Average Navier-Stokes equations. Not aiming in capturing all the scales within a turbulent flow, time averaging is applied to the governing equations in other to obtain transport equations for the mean variables. Such average is defined as a Reynolds operator (see Frisch (1995); Pope (2000)) and when applied to the momentum equation a new term results, namely the Reynolds stress tensor $\mathcal{R} = -\varrho\overline{\mathbf{u}' \otimes \mathbf{u}'}$ [1] . Such entity represents how the fluctuation of the velocity field interacts with the mean flow and RANS models main concern deals with properly representing such tensor field. Eddy viscosity models, linear and nonlinear, (LEVM NLEVM), and Reynolds Stress Transport Models (RSTM) are the main branches within RANS modelling and they simply differ on modelling the explicit relation or the transport equation for the Reynolds stresses with objective kinematic tensors of the mean flow. The present analysis focus on eddy viscosity models concerning the Boussinesq hypothesis.

Lying in between RANS and DNS stands what is called Large Eddy Simulation (LES). In such method only the large structures of the turbulent flow, which contain most of its energy, are numerically solved, while the fine ones are left to be modelled. Although more costly than RANS, large eddy simulation is definitely more versatile than DNS since it does not aim in represent all the fine structures within the flow. Therefore LES stands as an alternative to RANS simulations when accuracy is mostly desired.

Nevertheless as for industrial applications RANS is still mainly adopted when comes to simulating turbulent flows. Certainly large eddy simulation predict better results, but is way more costly and it does not fulfill the need for rapid decision making in such scenarios. This is why RANS modelling probably will still be an useful tool for many years to come and therefore the need to better describe turbulence in such approach. Most of the inaccuracy of RANS models comes from not properly capturing the structure of the Reynolds stresses and its transport for complex flows.

In the present paper LES results are used in order to capture the Reynolds stress tensor for the flow over a backward facing step. The disadvantage of bridging LES-RANS models is that there are two stages of modeling, which means

[1] The over line stands for the time average, the prime represents the fluctuation with respect to the mean of velocity field and ϱ is the (constant) mass density of the fluid.

that it is a model of a model. The advantage, however, is the possibility of simulating flows in complex geometries with high Reynolds numbers with a high level of accuracy. Therefore, the corresponding RANS model can be generated from complex flows. This advantage cannot be overemphasized, since, in general the results used to feed RANS models come from very simple geometries where the Reynolds stress has a simple structure.

As noted by Thompson *et al.* (2010) the methodology of building RANS models from LES results is basically divided in two parts. The first one would be, specifically concerning EVM, to investigate how the Reynolds stresses relate to other mean objective kinematical tensors according to the theory of tensor decomposition present in Thompson *et al.* (2010). The second one would be investigating how such coefficients describing the linear combination which better describes the Reynolds stresses relates to the mean flow. However in this paper only the Boussinesq hypothesis is studied and therefore its only coefficient is investigated.

2. DEFINITIONS AND GOVERNING EQUATIONS

2.1 Time average and fluctuations

Within the context of this paper the Reynolds operator adopted is

$$\overline{(\cdot)} = \lim_{T\to\infty} \frac{1}{T}\int_0^T (\cdot)\mathrm{d}t \tag{1}$$

and the fluctuations of any field are given by

$$(\cdot)' = (\cdot) - \overline{(\cdot)}. \tag{2}$$

2.2 RANS equations

After applying the Reynolds operator defined in (1) to the Navier-Stokes equations one gets

$$\nabla \cdot (\overline{\mathbf{u}}) = 0 \tag{3}$$

$$\nabla \cdot (\overline{\mathbf{u}} \otimes \overline{\mathbf{u}}) = -\nabla \overline{p} + \nu \nabla^2 \overline{\mathbf{u}} + \nabla \cdot (\mathbf{R}) \tag{4}$$

where the Reynolds stress tensor is defined as the dyadic product of the fluctuations of the velocity field $\mathbf{R} \equiv -\overline{\mathbf{u'} \otimes \mathbf{u'}}$. As treated in Pope (2000) and Wilcox (2006) in incompressible turbulent flows, equation (4) may be expressed in terms of a modified pressure $\mathfrak{p}$ noticing that only the deviatoric part of the Reynolds stresses

$$\mathbf{B} = \mathbf{R}_{\text{dev}} = \mathbf{R} + \frac{2}{3}\kappa \mathbf{1} \tag{5}$$

contribute to the momentum equation (4), where $\kappa \equiv -\frac{1}{2}\mathrm{tr}\,(\mathbf{R}) = \frac{1}{2}\overline{\mathbf{u'} \cdot \mathbf{u'}}$ is the turbulent kinetic energy (per unit mass) and $\mathbf{1}$ is the identity tensor in tridimensional euclidean vector space. The modified turbulent pressure is therefore defined as $\mathfrak{p} = \overline{p} + \frac{2}{3}\kappa$ and for incompressible turbulent flows the momentum equation (4) may be written as

$$\nabla \cdot (\overline{\mathbf{u}} \otimes \overline{\mathbf{u}}) = -\nabla \mathfrak{p} + \nu \nabla^2 \overline{\mathbf{u}} + \nabla \cdot (\mathbf{B})\,. \tag{6}$$

In incompressible turbulent flows RANS is only concerned with modelling the deviatoric part of the Reynolds stress tensor. In this matter EVM's deal generally with representing $\mathbf{B}$ in terms of linear combinations of objective kinematical tensors of the mean flow.

3. THE BOUSSINESQ HYPOTHESIS AND THE EDDY VISCOSITY

Boussinesq (1877, 1897) suggested that the Reynolds stress should be linearly proportional to the mean rate of deformation tensor. The analogy with the constitutive equation for the viscous stresses in newtonian fluids is clear and since the flow is said to be incompressible

$$\mathbf{B} = \alpha\overline{\mathbf{D}} \tag{7}$$

is also deviatoric. It should be noted that in general that such hypothesis does not always hold (Pope (2000); Frisch (1995); Wilcox (2006)). As a matter of fact Thompson *et al.* (2010) have shown that $\mathbf{B}$ can be decomposed in two orthogonal parts, one which is proportional and another which is orthogonal to $\overline{\mathbf{D}}$. So actually

$$\mathbf{B} = \alpha\overline{\mathbf{D}} + \mathbf{B}_{\perp\overline{\mathbf{D}}}\,. \tag{8}$$

Simply taking the double dot product of the above equation with $\overline{\mathbf{D}}$ gives

$$\alpha = \frac{\mathbf{B} : \overline{\mathbf{D}}}{\|\overline{\mathbf{D}}\|^2}\;, \tag{9}$$

since orthogonality between the parts must hold in such a decomposition. Linear eddy viscosity simply relates to the proportionality coefficient by

$$\nu_T = \frac{\alpha}{2}\,. \tag{10}$$

Eddy viscosity modelling mostly deals with the Boussinesq hypothesis and how to relate such proportionality coefficient α, or the eddy viscosity, to objective kinematical tensors of the mean flow. Equation (9) expresses how α relates the mean rate of deformation $\overline{\mathbf{D}}$ however implicitly with the (deviatoric) Reynolds stress itself. Since α is believed to be an isotropic function of its explaining tensors, according to Korsgaard (1990); Smith (1971); Itskov (2013) this coefficient should be only a function of the algebraic invariants of such tensors.

3.1 Scaling α

Since α, or ν_T, has dimensions of kinematical viscosity, or diffusivity, usual scaling involving characteristic lengths and velocity of the flow, or the turbulent kinetic κ and some other turbulent field, e.g. the rate of dissipation ϵ or the frequency ω, are usually applied (see Wilcox (2006)). For the present paper the scaling adopted is

$$\alpha = \tilde{\alpha}\frac{\kappa}{\|\overline{\mathbf{D}}\|} \tag{11}$$

where the new field $\tilde{\alpha}$ is dimensionless. The proposed model is therefore stated as

$$\mathbf{B} = \tilde{\alpha}\frac{\kappa}{\|\overline{\mathbf{D}}\|}\overline{\mathbf{D}}. \tag{12}$$

3.2 Motivation for the proposed scaling

The present paper investigates how the dimensionless coefficient $\tilde{\alpha}$ field may relate with invariants of objective kinematical tensors of the mean flow. As defined in (9) and (11) the proportionality coefficient, and therefore its dimensionless version, is still an unknown variable concerning the closure of turbulence. For bidimensional incompressible flow the first and third invariants of $\overline{\mathbf{D}}$ are not as relevant as its second one $II_{\overline{\mathbf{D}}} = \frac{1}{2}\left((\mathrm{tr}(\overline{\mathbf{D}}))^2 - \mathrm{tr}(\overline{\mathbf{D}}^2)\right)$ and therefore not taken in account when trying to describe $\tilde{\alpha}$. Other fields of interest are the turbulent kinetic energy κ and also the second invariant of the mean non-persistence-of-strainning tensor $\overline{\mathbf{P}} = \overline{\mathbf{D}}\,\overline{\mathbf{W}}^* - \overline{\mathbf{W}}^*\,\overline{\mathbf{D}}$, where $\overline{\mathbf{W}}^*$ is the relative vorticity tensor, defined as the vorticity measured with respect to the rate of rotation of the eigenvectors of $\overline{\mathbf{D}}$. Numerical issues in computing $\overline{\mathbf{P}}$ and how to circumvent them are extensively discussed in Thompson *et al.* (2010). The motivation behind choosing the tensor $\overline{\mathbf{P}}$ as an important entity to explain the Reynolds stresses lies in Thompson *et al.* (2010) previous results in extending the basis of tensors used to describe $\mathbf{B}$, which yield better results when considering the non-persistence-of-strainning tensor. For this reason here it was also adopted its second invariant as an important field that might could explain the coefficient $\tilde{\alpha}$ even though the basis consisted only of the rate of deformation tensor $\overline{\mathbf{D}}$, namely the Boussinesq hypothesis.

The proposed model as stated in (12) is similar to others such as the $\kappa-\epsilon$ and $\kappa-\omega$ models when trying to scale the eddy viscosity with the turbulent kinetic energy κ. In such models the scalings proposed are $\frac{\kappa^2}{\epsilon}$ and $\frac{\kappa}{\omega}$, respectively. The model here proposed would still lie in the class of two-equation-models since the rate of dissipation play its role in the transport of turbulent kinetic energy. However when only utilising κ as part of the scaling factor it is believed to smooth better the dimensionless coefficient than rather also using ϵ, which transport equation has so been modelled that it has very little to do with its true physics.

Alves (2014) has shown for turbulent plane Poiseuille flows that such a scaling as proposed in (11) translates better the dimensionless coefficient behaviour than the ones in $\kappa-\epsilon$ and $\kappa-\omega$ models. Familiar results in y_+ units involving steep slopes and uniformity only far away from the wall were shown to be diminished when dealing with the proposed scaling present in Alves (2014). As a matter of fact Alves (2014) shows that for such flows the new proposed scaling returns an almost uniform dimensionless coefficient for a wide range of Reynolds number in opposition to the usual C_μ field found in standard $\kappa-\epsilon$ model (see Launder and Spalding (1974)).

Sampaio (2012); Nave *et al.* (2010); Yabe *et al.* (1991) proposed that in order to increase accuracy in numerically describing $\|\overline{\mathbf{D}}\|$ a transport equation for the velocity gradient should be derived and solved along with the remaining equations of the problem. Although increasing the computational cost the accuracy of the discrete field would also be augmented. Therefore for the proposed model one way of increasing accuracy would be solving the transport equation for the mean velocity gradient or for the norm of the mean rate of deformation tensor $\|\overline{\mathbf{D}}\|$.

4. METHODOLOGY

4.1 Normalization

Before carrying on with the analysis a normalization is proposed in order to describe the field of interest. Since $\tilde{\alpha}$ is dimensionless it would be interesting to try to explain it as function of other dimensionless fields. However the turbulent kinetic energy and the invariants all have dimensions which differs from one another.

The normalization adopted is defined as

$$(\cdot)^\star = \frac{(\cdot)-\min(\cdot)}{\max(\cdot)-\min(\cdot)} \tag{13}$$

which not only returns a dimensionless field but also limits its image in $[0,1] \subset \mathbb{R}$.

4.2 Orthogonal Projection of $\tilde{\alpha}^\star$

Orthogonal projection is used for describing the behaviour of the dimensionless field $\tilde{\alpha}$ on the spanned linear sub spaced given by the non-orthogonal basis of functions $\mathfrak{B} = \{\kappa^\star, II^\star_{\overline{\mathbf{D}}}, II^\star_{\overline{\mathbf{P}}}, \dots\}$, where the basis may be extended by taking into account higher powers of the turbulent kinetic energy and the invariants in question. Therefore the coefficient may be expressed as

$$\tilde{\alpha}^\star = \sum_i^N c_i \Gamma_i + \tilde{\alpha}^\star_{\perp\mathfrak{B}} \tag{14}$$

where $\Gamma_i \in \mathfrak{B}$ and N is the dimension of the linear subspace spanned by the linearly independent functions in $\mathfrak{B}$. The further coefficients c_i may be obtained considering that the functions above belong to an infinite dimension Hilbert space with the following defined inner product

$$\langle f, g\rangle \equiv \int_\Omega fg \mathrm{d}V \ . \tag{15}$$

Since for all i from 1 to N the following orthogonality must hold

$$\langle \Gamma_i, \tilde{\alpha}^\star_{\perp\mathfrak{B}}\rangle = 0 \ , \tag{16}$$

the coefficients for the orthogonally projection of $\tilde{\alpha}$ are given as the solution of the following linear system of equations

$$\langle \tilde{\alpha}^\star, \Gamma_k \rangle = \sum_i^N c_i \langle \Gamma_i, \Gamma_k \rangle \ , \tag{17}$$

where k ranges from 1 to N, simply yielding N linearly independent equations.

The orthogonal counterpart $\tilde{\alpha}^\star_{\perp\mathfrak{B}} \equiv \tilde{\alpha}^\star - \sum_i^N c_i \Gamma_i$ remains as the part of $\tilde{\alpha}$ not explainable by the chosen fields which constitute the basis $\mathfrak{B}$. As closer $\tilde{\alpha}$ gets to the space spanned by the functions in $\mathfrak{B}$ the smaller such orthogonal part $\tilde{\alpha}_{\perp\mathfrak{B}}$ becomes and therefore the better the behaviour of $\tilde{\alpha}$ is captured by the basis functions.

For the present paper three basis were chosen: $\mathfrak{B}_I = \{\kappa^\star, II^\star_{\mathbf{D}}, II^\star_{\mathbf{P}}\}$, $\mathfrak{B}_{II} = \{\kappa^\star, II^\star_{\mathbf{D}}, II^\star_{\mathbf{P}}, {\kappa^\star}^2, {II^\star_{\mathbf{D}}}^2, {II^\star_{\mathbf{P}}}^2\}$ and $\mathfrak{B}_{III} = \{\kappa^\star, II^\star_{\mathbf{D}}, II^\star_{\mathbf{P}}, {\kappa^\star}^2, {II^\star_{\mathbf{D}}}^2, {II^\star_{\mathbf{P}}}^2, {\kappa^\star}^3, {II^\star_{\mathbf{D}}}^3, {II^\star_{\mathbf{P}}}^3\}$. Further basis may be chosen by only taking higher powers of $\kappa^\star$, $II^\star_{\mathbf{D}}$ and $II^\star_{\mathbf{P}}$.

5. NUMERICAL SIMULATION

5.1 Geometry: Backward facing step

The backward facing step is a classical benchmark test case and has been extensively used in the literature to validate turbulence models. Here we use the same geometry as Moin *et al.* (1997), with a 1.2 expansion ratio ($ER = \frac{L_y - h}{L_y}$, with $L_y = 6h$), shown in Fig. 1. The other dimensions are $L_x = 23h$, $L_z = 4h$, and $L_i = 3h$, where h is the step height, L_i is the inlet length, before the step, and L_x , L_y , L_z are the total length, height and span, respectively.

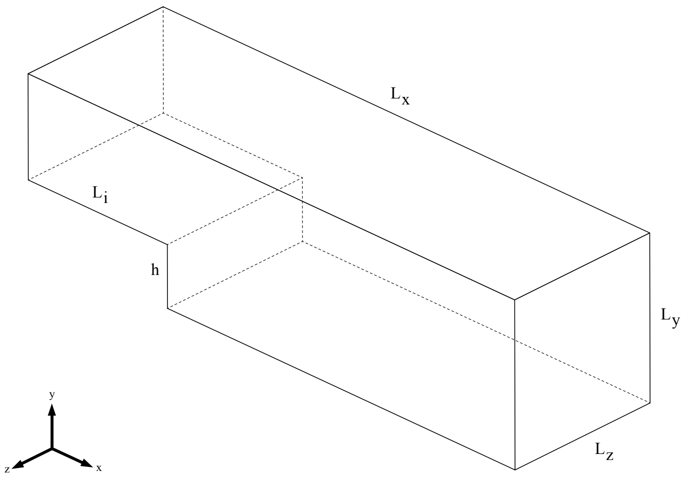

Figure 1: Backward facing step geometry.

The inlet boundary conditions are: imposed velocity, according to a profile obtained with DNS (Moin *et al.* (1997)) at $x = 3h$ from the step. The step height based Reynolds number was $Re = U_0 h/\nu = 5100$, where U_0 is the maximum velocity, and ν is the kinematic viscosity. Zero gradient in normal direction is also imposed on the pressure field. For simplicity, turbulence is not set there, in a hope that at least some part of the spectrum would have developed before reaching the step, and that the abrupt geometry will provide most of the turbulence downstream. The outlet is set as an outflow, with fixed pressure (zero) and null normal gradient for the velocity and turbulence variables. The bottom surfaces are set as walls, while the top, as an impenetrable, no-stress plane. The lateral planes are set as periodic.

5.2 Discretization

To solve the governing equations in a computer, a discretization process is needed to project the complete and continuous function space and operators into a vectorial space, resulting in a linear system to be solved.

The mass, momentum, and turbulence equations were discretized using the Finite Volume Method (FVM) implemented in OpenFOAM CFD (2014b,a). The field values are stored at the control volume centroids. Whenever fluxes or other face quantities are needed, the centroid values are interpolated to the faces centroids, using the most appropriate scheme. Here, a linear interpolation is used for the velocity field, corresponding to the classical central differences scheme of Finite Difference Method (FDM). This avoids extra dissipation which would otherwise affect the turbulence spectrum, and it is the recommended scheme when a physical modelling approach is chosen over an implicit LES. On the other hand, for variables like the subgrid turbulent kinetic energy, which cannot assume negative values and should be bounded, a Total Variation Diminishing (TVD) scheme based on Minmod limiter is employed. This guarantees a bounded behavior when high frequency oscillations are eminent, yet an accurate discretization for smoother modes.

The discretization of these equations generates three sets of linear systems, corresponding to the mass, momentum, and subgrid turbulence equations, which are solved separately, in a segregated approach. During a time step of the transient evolution, the momentum is solved first, then the pressure is found to correct the mass conservation and lastly an inner loop coupling is performed only for the velocity and pressure fields, using the PISO methodology Issa (1986).

An hexahedral mesh consisting of $300 \times 160 \times 32$ subdivisions in the streamwise (x), vertical (y) and spanwise (z) directions respectively was employed. In the x-direction, 40 subdivisions out of the total (300) were placed before the step, with a small concentration towards the step. Also, in the y-direction a non-uniform distribution of subdivisions was employed, concentrating control volumes close to the wall and in the step level. After simulation was ran and the mean wall shear stress was obtained, it was possible to evaluate the near wall normal spacing in wall units $\Delta y_+ = \Delta y/\delta$. The wall characteristic length is given by $\delta = \nu/u_* = \nu/\sqrt{\tau_w/\varrho}$ is the local mean wall shear stress. The resulting near wall mesh spacing in wall units were $y_+ = 0.56$ in average, and $y_+ = 7.57$ in the worst case, indicating that a suitable resolution was obtained to tackle turbulence with LES.

5.3 LES model

The Navier-Stokes equation and continuity are implicitly filtered in space, taking advantage of the Finite Volume Methods integral over each control volume. The resulting equation presents an additional term representing the subgrid (filtered) scales and its interaction with the resolved scales, and needs modeling as it cannot be closed. This term is modeled according to the Boussinesq hypothesis, in the framework of a functional model, where the dissipative role it plays in the energy cascade is more important than the exact capturing of the subgrid stress tensor structure.

The subgrid stress tensor, or more precisely, its deviatoric part, is thus assumed tied to the symmetric part of the resolved velocity gradient, by means of a "subgrid" viscosity, which is the very essence of all the modeling involved.

To provide the local value of this viscosity, two characteristics quantities are generally needed. Among the various modeling options, we choose the one equation subgrid model of Yoshizawa and Horiuti (1985), in which the two characteristic values are the subgrid kinetic energy (κ_{SGS}), given by its own additional transport equation, and a length scale, given by the cubic root of the control volume volume. The constant parameters of the model were dynamically adjusted, locally and for each time step, following the Germano methodology (Germano *et al.* (1991)).

6. RESULTS

In this section we must emphasise that all images are plotted using the same range, namely the dimensionless interval $[0, 1] \subset \mathbb{R}$, and with the same colour pallet, in which black stands for 0 and white for 1.

The first field shown in Figure 2 is the normalised dimensionless coefficient $\tilde{\alpha}^\star$. The ones that follow in pictures 3, 4 and 5 are the normalised fields of the turbulent kinetic energy $\kappa^\star$, and the second invariants $II^\star_{\mathbf{D}}$ and $II^\star_{\mathbf{P}}$, respectively.

As seen in Figure 2 the structure of de scaled (normalised) coefficient is quite complex, specially when compared to simple shear flows such as in Alves (2014). High spatial gradients are observed right after the step and in the middle of the geometry, particularly closer to the outlet. As mentioned before the explaining chosen fields are the ones pictured in Figures 3, 4 and 5 and their respective powers to the expanded basis of functions.

The following Figures from 6 to 11 show how the orthogonal projection explained in 4.2with the basis $\mathfrak{B}_I$, $\mathfrak{B}_{II}$ and $\mathfrak{B}_{III}$ looks like for the backward facing step flow in question. Figures 7, 9 and 11 shows the orthogonal counterpart $\tilde{\alpha}^\star_{\perp\mathfrak{B}}$ regarding each one of projections. As it can be seen by the pictures the extended basis which contain higher powers of the chosen fields give better approximations to the target field $\tilde{\alpha}^\star$, as expected. The analysis could be carried further with even wider basis of functions than $\mathfrak{B}_I$, $\mathfrak{B}_{II}$ and $\mathfrak{B}_{III}$, however it would yield qualitatively the same result. The authors believe that the orthogonal projection method is well depicted here as confirmed by the results.

Figure 2: $\tilde{\alpha}^\star$.

Figure 3: $\kappa^\star$.

Figure 4: $II^{\star}_{\mathbf{D}}$.

Figure 5: $II^{\star}_{\mathbf{P}}$.

Figure 6: Orthogonal projection of $\tilde{\alpha}^{\star}$ onto the basis of functions $\mathfrak{B}_I = \{\kappa^{\star}, II^{\star}_{\mathbf{D}}, II^{\star}_{\mathbf{P}}\}$.

Figure 7: Orthogonal counterpart $\tilde{\alpha}^{\star}_{\perp\mathfrak{B}}$ relative to $\mathfrak{B}_I$.

Figure 8: Orthogonal projection of $\tilde{\alpha}^{\star}$ onto the basis of functions $\mathfrak{B}_{II} = \{\kappa^{\star}, II^{\star}_{\mathbf{D}}, II^{\star}_{\mathbf{P}}, \kappa^{\star^2}, II^{\star^2}_{\mathbf{D}}, II^{\star^2}_{\mathbf{P}}\}$.

Figure 9: Orthogonal counterpart $\tilde{\alpha}^{\star}_{\perp\mathfrak{B}}$ relative to $\mathfrak{B}_{II}$.

Figure 10: Orthogonal projection of $\tilde{\alpha}^{\star}$ onto the basis of functions
$\mathfrak{B}_{III} = \{\kappa^{\star}, II^{\star}_{\mathbf{D}}, II^{\star}_{\mathbf{P}}, \kappa^{\star^2}, II^{\star^2}_{\mathbf{D}}, II^{\star^2}_{\mathbf{P}}, \kappa^{\star^3}, II^{\star^3}_{\mathbf{D}}, II^{\star^3}_{\mathbf{P}}\}$.

Figure 11: Orthogonal counterpart $\tilde{\alpha}^{\star}_{\perp\mathfrak{B}}$ relative to $\mathfrak{B}_{III}$.

7. FINAL REMARKS AND FURTHER DEVELOPMENT

The presented work dealt with scaling the proportionality field, or the eddy viscosity, in the Boussinesq hypothesis and aimed in describing it by orthogonal projection onto chosen basis of functions consisting of meaningful fields of the mean flow. The major problem associated with it lies first in the eddy viscosity approximation that for the backward facing step case performs poorly. The second one is related with describing such a viscosity as a finite linear combination of fields and therefore approximating a not quite so good approximation. The results agreed with what was expected from the orthogonal projection method, however as for a model perhaps does not deserve time to implement it having in mind that the exact description of α would still be a not so good approximation for the (deviatoric) Reynolds stresses $\mathbf{B}$.

On the other hand, as shown by Thompson *et al.* (2010), regarding EVM the further extension of the basis of tensor used in describing $\mathbf{B}$ yields significant better results. The further developments would deal with linear combinations of mean objective kinematical tensor of the mean flow such as $\overline{\mathbf{D}}$, $\overline{\mathbf{D}}^2$, $\overline{\mathbf{P}}$, $\overline{\mathbf{P}}^2$ and even combinations as $\overline{\mathbf{P}}\,\overline{\mathbf{D}} + \overline{\mathbf{D}}\,\overline{\mathbf{P}}$ to better capture the complex structure of $\mathbf{B}$. The resulting coefficients would be projected onto a even wider linear subspace spanned by new invariants of the new basis tensors, as well as the turbulent kinetic energy κ. Analysis tend to continue in studying also implicit relations involving also the second invariant $II_{\mathbf{B}}$ regarding the scaling and also the projections always aiming in describing $\mathbf{B}$ more accurately.

8. REFERENCES

Alves, F.A.V.B., 2014. *O uso de base de dados DNS para a construção de modelos RANS utilizando decomposições tensoriais e coeficientes de bases normailizadas*. Master's thesis, Universidade Federal Fluminense.

Boussinesq, J., 1877. "Essai sur la theorie des eaux courantes". *Mem. Pres. Acad. Sci.*, Vol. XXIII, p. 46.

Boussinesq, J., 1897. "Théorie de l'ecoulement tourbillonnant et tumultueux des liquides". *Gauthier-Villars et fils*.

CFD, O., 2014a. *Programmer's guide*.

CFD, O., 2014b. *User's guide*.

Frisch, U., 1995. *Turbulence. The legacy of A. N. Kolmogorov*. Cambridge University Press.

Germano, M., Piomelli, U., Moin, P. and Cabot, W., 1991. "A dynamic subgrid-scale eddy viscosity model". *Physics of Fluids*.

Issa, R.I., 1986. "Solution of the implicilty discretized fluid flow equations by operator-splitting". *Journal of Computational Physics*.

Itskov, M., 2013. *Tensor Algebra and Tensor Analysis for Engineers*. Springer Verlag, terceira edition.

Kolmogorov, A.N., 1941a. "The local structure of turbulence in incompressible viscous fluids at very large reynolds number". *Dokl. Akad. Nauk. SSSR*.

Kolmogorov, A.N., 1941b. "On the degeneration of isotropic turbulence in incompressible viscous fluids". *Dokl. Akad. Nauk. SSSR*.

Korsgaard, J., 1990. "On the representation of symmetric tensor-valued isotropic functions". *International Journal of Engineering Science*.

Launder, B.E. and Spalding, D.B., 1974. "The numerical computation of turbulent flows". *Computer Methods in Applied Mechanics and Engineering*.

Moin, P., Le, H. and Kim, J., 1997. "Direct numerical simulation of a turbulent flow over a backward-facing step". *Journal of Fluid Mechanics*, Vol. 330, pp. 349 – 374.

Nave, J.C., Rosales, R. and Seibold, B., 2010. "A gradient-augmented level set method with an optimally local, coherent advection scheme". *Journal of Computational Physics*, Vol. 229, No. 10, pp. 3802–3827.

Pope, S.B., 2000. *Turbulent Flows*. Cambridge University Press.

Sampaio, L.E.B., 2012. "A transport-physics-accurate convection discretization based on first order upwind and the need for a new error assessment strategy". *Computers and Fluids*, Vol. 67, No. 30, pp. 87 – 103.

Smith, G.F., 1971. "On isotropic functions os symmetric tensors, skew-symmetric tensors and vectors". *International Journal of Engineering Science*, Vol. 9, pp. 899 – 916.

Thompson, R.L., Mompean, G. and Thais, L., 2010. "A methodology to quantify the nonlinearity of the reynolds stress tensor". *Journal of Turbulence*, Vol. 11, No. 33, p. 27.

Wilcox, D.C., 2006. *Turbulence modelling for CFD*. DCW Industries.

Yabe, T., Aoki, T., Sakaguchi, G., Wang, P. and Ishikawa, T., 1991. "The compact cip (cubic-interpolated pseudo-particle) method as a general hyperbolic solver". *Computers and Fluids*, Vol. 19, No. 3-4, pp. 421–431.

Yoshizawa, A. and Horiuti, K., 1985. "A statisticlly-derived subgrid-scale kinetic energy model for the large eddy simulation of turbulent flows". *Journal of the Physical Society of Japan*, Vol. 54, No. 8, pp. 2834–2839.

9. RESPONSIBILITY NOTICE