

ON AN OVERVIEW OF NONLINEAR DYNAMICS OF ELECTRO-MECHANICAL ENGINEERING SYSTEMS, EXCITED BY SMALL MOTORS

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Abstract. *The aim of this work is to describe a large number of phenomena caused by the action of vibrations on nonlinear electro-mechanical systems. The main goal of this paper is to give a general description of this class of phenomena which has been discussed in recent years. We remarked that a large number of publications were based on the assumption that the external excitations are produced by an ideal source of power with prescribed time history: prescribed magnitude, phase and frequency, or in random problems with prescribed characteristics. In reality, the excitations sources are non-ideal, they have always limited power, limited inertia and their frequencies varies according to the instantaneous state of the vibrating system. We remarked that small motors with limited power are used in laboratory test and therefore the investigations of mutual interaction of driven and driving subsystems are very important. We will discuss several contributions on the past, nowadays and perspective futures in engineering that may be modeled by non-ideal systems with many degrees of freedom. In this lecture, we will present examples of this kind of engineering devices.*

Keywords: *non-ideal sources, small motors, limited power supply, energy transfer, Sommerfeld effect.*

1. INTRODUCTION

It is well known that the analysis of the motion of real electro-mechanical systems were carried out by means of mathematical models, which have been always simplified and therefore describe the “exact” behavior with the same degree of approximations. The study of problems involving the coupling of several systems was widely explored, in the last years, essentially in function of the change of constructive characteristics of the machines and structures. Accordingly, oscillatory processes can be divided into the following types: free, forced, parametric and self-excited oscillations and we remarked that two or more oscillations can interact in the same oscillatory system. This fact is of important scientific and practical interest. In this way, some phenomena were observed in a composed dynamic systems supporting structures and rotating machines, where were verified that the unbalancing of the rotating parts, was the greatest causer of the vibrations. We noted that a lot of oscillatory (vibrating) phenomena of real systems cannot be explained by and solved on the basis of linear theory and it is important to introducing nonlinear characteristics into the mathematical models of vibrating systems and in particular to electro-mechanical systems. The main difficulty in comparisons of linear systems mainly because of absence of validity of superposition principle. Every nonlinear vibrating system must be solved individually and special methodology must be developed for each class of problems.

Here, we will deal with a special class of nonlinear systems called non-ideal systems (NIS). In this paper, an overview on non-ideal vibrations (NIS) is presented. We analyzed the physical phenomena involved; the adequate methodologies to deal with them and presented a short report of some recent progress, in the period from 2003 to 2015,

published over the current literature. We verify that the study of the vibrating systems when the external excitation was influenced by the response of the system had been considered a major challenge in theoretical and practical engineering research. It is also well known that structures supporting unbalanced machines capable of a limited power output are considered, as in the case of real motors. The motion of a vibrating structure under the action of such energy source, it is accompanied by the interaction between these non-ideal motors and their supports. In usual approaches, the excitation is considered as ideal (IS); that is, the influence of the motion of the structure on the motor is disregarded. Here, the reciprocal influence of the system on the energy source is considered (non-ideal systems-(NIS)). As a direct consequence, in the region of resonance, unstable conditions of motion occur and the form of resonance curve depends on which direction the frequency of the excitation is being altered. We know that the problem of passage through resonance of unbalanced equipment, with an operational speed higher than the lower frequencies of vibration of the supporting structure, has been studied for a long time. However, the machine may not be able to supply the power necessary for this technique, as a large part of its energy is used in shaking the structure and not in accelerating the rotation of the shaft. This is well known as **Sommerfeld effect**.

Note that as long as the rotation of the motor is assumed to be uncoupled from the vibrations excited upon the supporting structure; one has a known forcing function of time corresponding to an unlimited or ideal power supply. Note that if the structure has linear behavior, the problem will be a linear one. The introduction of real torque-speed curves for non-ideal motors renders the system nonlinear and capable of multiple steady-state periodic motions whose stability must be assessed. Further complexities may be introduced if the structure itself exhibits nonlinear behavior. We also found that the right branch of the amplitude frequency response curve may be unstable if the magnitude of its negative slope is larger than that of the torque-speed characteristic of the driving motor. Further comments on the subject are found in the book by (Kononenko 1969), without undeserved of others. A complete overview on different theories of (NIS) was done by (Balthazar *et al.*, 2003). We announced that (Bolla *et al.*, 2007), used the method of multiple scales, instead the usual averaging procedure; in this kind of problem, by the first time, with success and that (Dantas and Balthazar, 2007) presented the so called **Sommerfeld effect**, as a bifurcation of periodic orbits, by using the regular perturbation theory (Blekman *et al.*, 2008) studied the problem of the motion of an imbalanced rotor when passing through a resonance zone solved by the iteration method combined with the method of the direct separation of motions (this methodology can find in the book of (Blekman, 2000)). (El-Badawy, 2007) demonstrated that a simple, reduced-order, nonlinear model provides an effective analytical tool for the treatment of source-structure interactions by applying the theory to an actual structure and developing the appropriate model. The equations of motion were analyzed using perturbation analysis to determine the frequency response of the system in the vicinity of the primary resonance and the theoretical and experimental frequency response curves were compared to deduce the effective nonlinearity coefficient, and it was shown that, at certain critical speeds, the excitation source was severely limited by the structural vibrations generated. (Gonçalves *et al.*, 2014) found that the *Sommerfeld effect* depends on some system parameters and the motor operational procedures. These parameters were explored to avoid the resonance capture in the *Sommerfeld effect*. Numerical simulations and experimental tests were used to help gather insight of this dynamic behavior. We also remarked that we may be unable to operate a rotating vibrational system beyond the critical speed due to a limited power supply. Special points were determined in the presence of Sommerfeld effect and jump phenomenon, in the passage through resonance region for a NIS of Duffing and Rayleigh type, by using an averaging method by (Palacios and Balthazar, 2009a). Recently (Cveticanin and Zukovic, 2015) studied the influence of the order of nonlinearity on the dynamic properties of the (NIS). The authors considered the motor, with the torque which is the cubic function of the angular velocity, and the nonlinear oscillator, with certain order, the numerical calculation and analytical solutions were also done.

The main purpose of this work is to present results obtained for some non-ideal models. New dynamic phenomena and differences between regular and chaotic motion, in each level of simplifications, will be emphasized. Here, we will overview them in a systematic way, including of new results. By another hand, new approaches to the classical non-ideal researches were done, with success, through a number of new publications, in last years.

2. MODELLING AND GOVERNING EQUATIONS OF MOTION OF (NIS)

The general (NIS) mathematical model of a vibrating system, may be characterized by its governing equations of motion (Felix *et al.*, 2009a):

$$m_1 \ddot{x} + f(x, \dot{x}) + \frac{\partial U(x)}{\partial x} = F(\dot{\phi}, \ddot{\phi}, r, m_0) \quad (1)$$

$$I \ddot{\phi} + H(\dot{\phi}) = L(\dot{\phi}) + R(\phi, \dot{\phi}, \ddot{x}, r, m_0) \quad (2)$$

where m_1 is the mass, x is displacement of the (NIS), ϕ is angular displacement of the rotor, $F(\phi, \dot{\phi}, \ddot{x}, r)$ expresses the action of the source of energy on the oscillating system (angular velocity of motor, that is not constant), parameters r and m_0 are the eccentricity and mass of unbalanced shaft of the electric motor, I is the moment of inertia of the rotor, the function $R(\phi, \dot{\phi}, \ddot{x}, r)$ expresses the action of the oscillating system on the source of energy, the function $H(\dot{\phi})$ is the resistive torque applied to the motor, the function $L(\dot{\phi})$ is the driving torque of the source of energy (motor). Note that, usually, the inductance is much smaller than the mechanical constant time of the system and, then in stationary regime, we can take $L(\dot{\phi})$ as (linear) $L(\dot{\phi}) = u_1 - u_2 \dot{\phi}$, where u_1 is related to voltage applied across to the armature of the DC motor, that is, a possible control parameter of the problem and u_2 is a constant for each model of DC motor considered. $f(x, \dot{x})$ is the nonlinear and non-conservative part of the restoring force, while $\frac{\partial U(x)}{\partial x}$ is its conservative part (U is the potential, or strain energy). $f(x, \dot{x}) = c\dot{x}$, $U(x) = \frac{1}{2}k_1x^2 + \frac{1}{4}k_2x^4$

2.1 (NES)-nonlinear energy sink absorbers to (NIS)-Energy transfer(“pumping”)between oscillators

Based in an extension of the (IS) investigated by (Costa *et al.*, 2009) and (Dantas and Balthazar, 2008) and taking into account that in a vibrating system, with small low viscous dissipation, the energy initially imparted to the primary subsystem, can be transferred to the nonlinear energy sink, named as (NES)-(Vakakis *et al.*, 2008) reducing the amplitudes of vibrations of the (NIS) and eliminated (or reduced) the occurrence of the *Sommerfeld effect*, both inside and outside resonance region, respectively.

In this particular case, we will need to consider more one equation in Eq (1) (Felix *et al.*, 2009b):

$$m_2 \ddot{y} + c_2 \dot{y} + g(y) + \frac{\partial V(x, y)}{\partial y} = 0 \quad (3)$$

where m_2 is the mass, c_2 is the coefficient of the viscous damping and y is the displacement of the (NES), $g(y)$ is the stiffness term not necessarily linear of the (NES), V is the potential energy not necessarily linear associated with the coupling spring.

We also mention that, using a combination of the optimal active control for nonlinear systems (Rafikov and Balthazar, 2008a), (Rafikov *et al.*, 2008b). We also announced that the vibrations absorption (in resonance, nonlinear and chaotic) were analyzed in a mathematical model of a simple portal frame, excited by a non-ideal power source and connected to an NES device, which renders descriptions that are close to engineering situations encountered in practice. In this model, we introduced the kinetic energy of the motor in the formulation.

Furthermore, the proposed NES was quite effective in reducing of the amplitude of oscillations of the structure, the *Sommerfeld effect* and the jump phenomenon. For the suppression of chaos, the NES device was an effective tool (Felix *et al.*, 2013a).

(Felix *et al.*, 2013b), developed a dynamic vibration absorber, using a viscoelastic material, with nonlinear essential stiffness and time-dependent damping properties for a non-ideal vibrating system with *Sommerfeld effect*, resonance capture, and jump phenomenon. The used absorber was a mass-bar subsystem, which consisted of a viscoelastic bar with memory, attached to mass, in which the internal dissipative forces depended on current, deformations and its operational frequency varies with limited temperature. The non-ideal vibrating system consists of a linear (nonlinear) oscillator (plane frame structure) under excitation, via spring connector of a DC-motor with limited power supply. A viscoelastic dynamic absorber, modeled with elastic stiffness essentially nonlinearities, was itself developed to further reduce the *Sommerfeld effect* and the response of the structure. The numerical results showed the performance of the absorber on the non-ideal system response through the resonance curves, time histories, and Poincare sections. Furthermore, the structure responses using the viscoelastic damper with and without memory were studied.

And in the work done by (de Godoy *et al.*, 2013), The authors considered a vibrating system that consisted of a snap-through truss absorber, coupled to an oscillator under excitation of an electric motor, with an eccentricity and limited power, characterizing a non-ideal oscillator. It was aimed to use the non-linearity and quasi-zero stiffness of absorber (snap-through truss absorber) in order to obtain a significantly attenuation the jump phenomenon. There was also an interest to exhibit the reduction of *Sommerfeld effect*, to confirm the saturation phenomenon occurrence and

show the power transfer in a non-linear structure, evidencing of the pumping energy. As shown by simulations in this referenced work, this absorber allowed the energy pumping before and during the jump phenomenon, decreasing of the higher amplitudes of considered system. Additionally, the occurrence of saturation phenomenon due to the use of snap-through truss absorber was verified. The analysis of parameter uncertainties was introduced and then the sensitivity of system with parametric errors demonstrated a trustable system.

2.2 (NEVA) –Electro-mechanical absorbers to (NIS)

(Felix and Balthazar, 2009c) extended the study of (Yamapi, 2006) and studied the attenuation of the *Sommerfeld effect*, through of a nonlinear electromechanical vibration absorber, called (NEVA). They have dealt with the study of a (NIS), coupled to a nonlinear electromechanical vibration absorber. This electromechanical damping device consisted of an electrical system coupled magnetically to a mechanical structure under a (NIS). They also considered a nonlinear friction of type cubic-quantic Duffing oscillator and by considering the voltage of the resistor as being a nonlinear function of type Rayleigh oscillator. We observed that the passage of the resonance region, a dramatic decreasing of the vibration amplitude of the (NIS) and to with the possibility of the minimization of the Sommerfeld effect of (NIS). They taking into account that the electric part of the controller was consisted of a linear inductor L , a nonlinear capacitor C , and nonlinear resistor R and the expression of the voltage over the resistor and the condenser were a nonlinear function of the instantaneous electrical charge q .

In the above, mentioned case, we need to consider more one equation in Eq (1):

$$L\ddot{q} - R\left(1 - \frac{1}{i_0^2}\dot{q}^2\right)\dot{q} + \frac{1}{C_p}q + i_0^2\alpha_a q^3 + i_0^4\alpha_b q^5 + T\dot{x} = 0 \quad (4)$$

Where i_0 is an initial current, in the electrical part, C_0 is the linear value of the capacitive characteristic and the parameters α_3 and α_5 , are a nonlinear coefficients, those depending on the type (or kind) of the capacitor. The quantity T is the transducer (constant), which relates the current in the coil to the magnetic force on the considered coil. The transducer constant is given by $T = 2\pi n l B$, where n is the number of turns in the coil, l is the radius of the coil, and B is the uniform radial magnetic field strength in the annular gap. The transducer constant T also relates the electrical potential e , across to the terminals of the coil to the velocity of the coil, with respect to the permanent magnet. Note that the Eq (1), must be replaced by

$$m_1\ddot{x} + f(x, \dot{x}) + \frac{\partial U(x)}{\partial x} + T\dot{q} = F \quad (5)$$

In the paper by (Felix *et al.*, 2011) the attenuation of a nonideal vibrating system, using (NEVA) and (MR) damper has been presented. The novelty of the (MR) damper application, in this study, was the performance of attenuating the interaction between (NEVA) and (NIS)-nonideal system, in the post-resonance region. The following phenomenon was eliminated: Sommerfeld effect and transient motion of long time. The most important results of this paper may be expressed as: in the nonideal system (NIS), without (NEVA) and (MRD), the Sommerfeld effect (“Energy Transfer” the strong interaction between the foundation and the DC motor); the influence of the viscous damping; the jump and transient motion, during a long period of time were found. The authors analyzed the influence of (NEVA) on (NIS), in the post-resonant region, which causes, on a large band in the resonance curve, a set of jumps, two maxima’s resulting in a strong interaction, between the (NIS) and (NEVA), and a big jump due to a transient motion, during a long period of time. We also analyzed the influence of the parameters: the magnetic coupling and the nonlinear capacitor for the effective action of (NEVA). We analyzed the effectiveness of the (MR) damper: it increases the damping force amplitude.

By other hand, (Alişverişç *et al.*, 2014) analyzed the chaotic dynamics of an electromechanical damped Duffing oscillator coupled to a rotor. The electromechanical damped device or electromechanical vibration absorber consists of an electrical system coupled magnetically to a mechanical structure (represented by the Duffing oscillator) and that works by transferring the vibration energy of the mechanical system to the electrical system. A Duffing oscillator with double-well potential is considered. Numerical simulations results are presented to demonstrate the effectiveness of the electromechanical vibration absorber. Lyapunov exponents are numerically calculated to prove the occurrence of a chaotic vibration in the non-ideal system and the suppressing of chaotic vibration in the system using the electromechanical damped device.

We also mention that (Picirillo et al, 2014) dealt with the Sommerfeld effect (Jump phenomena) attenuation in a non-ideal mechanical oscillator connected with an unbalanced motor excitation with a limited power supply (non-ideal system) using a magneto rheological damper (MRD). The dynamical response of systems with MRD presented different behavior due to their nonlinear characteristic. MRD nonlinear response was associated with adaptive dissipation related to their hysteretic behavior. The Bouc-Wen mathematical model was used to represent the MRD behavior. Numerical simulations show different aspects about the Sommerfeld effect, illustrating the influence of the different electric current applied in the MRD to control the force developed by this damper.

(Castão *et al.*, 2011) obtained the attenuation of the jump phenomena associated with the Sommerfeld Effect introduced by the nonlinearities of a magnetic rheological damper (MRD) in a non-ideal vibrational system, excited by a DC motor modeled as limited power source. Numerical simulations of the nonlinear vibrations of the system are carried out for different values of the MRD control parameter in order to show the amplitude reduction of the vibrations close to the system resonance introduced by the nonlinear damping effect of the MR system.

2.3 Controls Approaches to (NIS)

(De Souza *et al.*, 2006) studied Tuned liquid column dampers are U-tubes filled with some liquid, acting as an active vibration damper in structures of engineering interest like buildings and bridges. The authors studied the effect of a tuned liquid column damper in a vibrating system consisting of a cart which vibrates under driving by a source with limited power supply (non-ideal excitation). The effect of a liquid damper was studied in some dynamical regimes characterized by coexistence of both periodic and chaotic motion. Also (Felix *et al.*, 2005) presented a modeling and numerical analysis of a vibration control liquid damper for a structural frame under non-ideal excitation. The authors considered a portal frame with concentrated mass on the horizontal beam and two identical columns assumed as linear elastic and with negligible mass, one non-ideal source (DC motor of limited power supply and unbalanced) and a TLCD mounted on the structural frame.

A work interested on control of chaos is to be found in (Souza *et al.* 2005). In this mentioned work the authors investigated numerically the dynamical behavior of a non-ideal mechanical system consisting of a vibrating cart containing a particle which can oscillate back and forth colliding with walls carved in the cart. This system represented an impact damper for controlling high-amplitude vibrations and chaotic motion. The motion of the cart was induced by an in-board non-ideal motor driving an unbalanced rotor. We studied the phase space of the cart and the bouncing particle, in particular the intertwined smooth and fractal basin boundary structure. The control of the chaotic motion of the cart due to the particle impacts was also investigated. Our numerical results suggest that impact dampers of small masses are effective to suppress chaos, but they also increase the final-state sensitivity of the system in its phase space.

(Zukovic and Cveticanin, 2009) studied the existence of clearance between the connecting force between motor and the fixed part of the system is discontinuous but linear. The mathematical model of the system was represented by two coupled second-order differential equations. The transient and steady-state motion and also the stability of the system were analyzed and the Sommerfeld effect was detected. For certain values of the system parameters the motion is chaotic. This is caused by the period doubling bifurcation. The existence of chaos was proved with maximal Lyapunov exponent. A new chaos control method based on the known energy analysis was introduced and the chaotic motion was transformed into a periodic. By one hand, we announced that an adopted active control strategy consists of two controls (Tusset et al, 2013) the nonlinear (feedforward) in order to keep the controlled system in a desirable orbit, and the feedback control, which may be obtained by considering state-dependent Riccati equation control to bringing the system into the desired orbit using a magneto rheological (MR) damper. To control the electric current applied in control of the MR damper the Bouc-Wen mathematical model was used to the MR damper. The passive control was obtained by means of a nonlinear sub-structure with properties of nonlinear energy sink. Simulations showed the efficiency of both the passive control (energy pumping) and active control strategies in the suppression of the chaotic behavior. In the work done by (Tusset and Balthazar, 2013) the dynamics of the ideal and non-ideal Duffing oscillator with chaotic behavior is considered. In order to suppress the chaotic behavior and to control the system, a control signal is introduced in the system dynamics. The control strategy involves the application of two control signals, a nonlinear feedforward control to maintain the controlled system in a periodic orbit, obtained by the harmonic balance method, and a state feedback control, obtained by the state dependent Riccati equation, to bring the system trajectory into the desired periodic orbit. Additionally, the control strategy includes an active magneto rheological damper to actuate on the system. The control force of the damper is a function of the electric current applied in the coil of the damper, which is based on the force given by the controller and on the velocity of the damper piston displacement. Numerical simulations demonstrate the effectiveness of the control strategy in leading the system from any initial condition to a desired orbit, and considering the mathematical model of the damper (MR), it was possible to control the force of the shock absorber (MR), by controlling the applied electric current in the coils of the damper.

We remarked that several studies have also carried out theoretical and experimental analysis of portal frames excited by non-ideal excitation devices. The nonlinear control technique based on saturation phenomenon and of systems coupled with quadratic nonlinear terms is utilized to suppress the high amplitude of the first-mode vibration of a reduced non-ideal system of type shear building (plant). A two-to-one internal resonance condition is maintained between the plant and the controller. In this case, energy is transferred completely from one part of the combined system to the other. When the plant is forced at resonance (a selection of the control parameter), this energy transfer mechanism limits the response of the plant. The method of averaging was used by (Felix *et al.*, 2005) to determine an approximation solution (by using averaging process) to this mentioned controlled non-ideal system. We announced that the obtained results showed that the motion of the portal frame saturated and most of the energy imparted to the portal frame by the non-ideal source was transferred to the controller and the effectiveness of the saturation controller was adjusting the gains.

2.4 Self Synchronization on shaft (NIS)

We announced that the nonlinear phenomena of shaft's self-synchronization have been studied extensively since the late 1940s, mostly with the aim of their application in designs of various shakers and exciters for purposeful vibration of objects (Blekhman, 1988). The phenomenon had been studied extensively by asymptotic methods to predict possible (multiple) steady-state rotational motions and to evaluate their stability—mostly with application to the design of vibrators with a reduced number of driving motors. Certain cases of undesirable shaft self-synchronization in engineering had also been studied, but only steady-state motions were analyzed. Results of numerical simulation of transient self-synchronization of rotating shafts with one potential application being gas turbine engines with multiple shafts was studied by (Dimentberg *et al.*, 2001).

In the paper (Balthazar *et al.*, 2004) a practical problem of synchronization of a (NIS) was posed and investigated by means of numerical simulations. Two rotating unbalanced motors compose the mathematical model were considered with limited power supply mounted on the horizontal beam of a simple portal frame. As a starting point, the problem was reduced to a four-degrees-of-freedom model and its equations of motion and the numerical results showed the expected phenomena associated with the passage through resonance with limited power. Further, for a two-to-one relationship between the frequencies associated with the first symmetric mode and the sway mode, by using the variation of torque constants, the control of the self-synchronization and synchronization (in the system) were observed at certain levels of excitations. The paper of (Djavan *et al.*, 2015) showed that two motors on the same plate can enter into synchronization with the phase difference equal to zero, π or 2π depending on the physical characteristics of the motors and the plate. Both motors were considered as NIS oscillators and act as external excitation on a specific area of the plate. The analysis of the vibration of the plate indicated through numerical simulation that one can obtain a reduction of vibration when the motors phase difference is equal to π .

2.5 Pseudoelastic Behavior of (NIS)

It is known that intelligent and adaptive material systems and structures have become very important in engineering applications. The fundamental characteristic of these systems is the ability to adapt to environmental conditions. A new class of materials with promising applications in structural and mechanical systems is the shape memory alloy (SMA). The mechanical behavior of shape memory alloys, in particular, shows strong dependence on temperature. Shape Memory Alloys (SMAs) consist of a group of metallic materials that demonstrate the ability to return to some previously defined shape or size when subjected to the appropriate thermal procedure. The SMAs appear in a low (usually martensite) and a high temperature phase (austenite). In literature, the shape memory effects (SMEs) are classified into the following three types: two-way effect, one-way effect and pseudoelasticity. The effects can appear in this sequence with increasing temperature. In the pseudoelastic effect, a SMA material achieves a very large strain upon loading that is fully recovered in a hysteresis loop upon unloading.

(Picirillo *et al.*, 2008) presented a nonlinear dynamic behavior of a (NIS), which consists of a mass connected to a rigid support by a shape memory alloy (SMA) element and a damper and in order to disturb the system, a DC motor with limited power supply was connected to the mass, causing an interaction between the vibrating structure and the energy source. The used SMA element was characterized using a one-dimensional phenomenological constitutive model, based on the classical Devonshire theory. The mentioned authors analyzed the (NIS) in form of two coupled nonlinear differential equations and some interesting nonlinear phenomena as the Sommerfeld effect and nonlinear resonance including periodic, chaotic and hyperchaotic regime were presented and discussed.

We also announced that in (Picirillo *et al.*, 2015) the authors studied the nonlinear dynamics behavior of a shape memory oscillator (SMO) subjected to an ideal or nonideal excitation: The restoring force of the oscillator was provided by a shape memory device (SMD), described by a thermomechanical model capable of reproducing the

hysteretic behavior via the evolution of a suitable internal variable. Due to nonlinearities in The model, the SMO exhibited periodic or non-periodic behaviors. The effects of the external sources on the response of SMO were studied through the scalogram analysis of continuous wavelet transform by using a new measure, called the Scale index, with success.

2.6 Vibro-Impact of (NIS)

We have successfully discussed and connected two interesting issues in nonlinear dynamics: a) non-ideal power sources, represented, in this case, by the in-board non-ideal motor driving an unbalanced rotor that induces the motion of the structure; b) impact damper for controlling high-amplitude vibrations. An unbalanced non-ideal DC motor shear-building type foundation structure that suffers the Sommerfeld Effect of getting stuck at resonance (energy imparted to the motor being used to excite large amplitude motions of the supporting structure). We have shown that the impact damping may mitigate or even suppress the undesired phenomena without dissipation of energy.

As an example, we mention that (De Souza *et al.*, 2008) discussed the dynamics of two kinds of mechanical systems: considering vibro-impact systems which have many implementations in applied mechanics, ranging from drilling machinery and metal cutting processes to gear boxes. Moreover, from the point of view of dynamical systems, vibro-impact systems exhibited a rich variety of phenomena, particularly chaotic motion. In this paper, we review recent works on the dynamics of vibro-impact systems, focusing on chaotic motion and its control. The considered systems are a gear-rattling model and a smart damper to suppress chaotic motion. Furthermore, we investigated systems with non-ideal energy source, represented by a limited power supply. As an example of a non-ideal system, we analyzed chaotic dynamics of the damped Duffing oscillator coupled to a rotor. Then, we showed how to use a tuned liquid damper to control the attractors of this non-ideal oscillator.

(Brasil *et al.*, 2006) presented a simple mathematical model of a wind turbine tower. Here, the wind excitation is considered to be a non-ideal power source. In such a consideration, there is interaction between the energy supply and the motion of the supporting structure. If power is not enough, the rotation of the generator may get stuck at a resonance frequency of the structure. This is a manifestation of the so-called Sommerfeld Effect. In this model, at first, only two degrees of freedom are considered, the horizontal motion of the upper tip of the tower, in the transverse direction to the wind, and the generator rotation. Next, we add another degree of freedom, the motion of a free rolling small mass inside a chamber. Its impact with the walls of the chamber provides control of both the amplitude of the tower vibration and the width of the band of frequencies in which the Sommerfeld effect occur. Some numerical simulations are performed using the equations of motion of the models obtained via a Lagrangian approach.

And (Moraes *et al.*, 2013) analyzed the dynamics of a vibro-impact system consisting of two blocks of different mass coupled by a spring the analysis included time histories of displacements and velocities, phase portraits and diagrams of the displacement and frequency, used to show the Sommerfeld effect. For certain values of the parameters of the system, the motion is chaotic. The mathematical model of the system was used to obtain an insight to the global dynamics of the vibro-impact system.

2.7 Nonideal Energy Harvester (NIEH)

In the last years, we have been seeing a necessity of an energy source smaller than the usual and more efficient, for design of vibrating systems based in new technologies, with that the research on Energy Harvesting, has increasing substantially. To build energy harvesting devices, many researchers have concentrated their efforts on finding the best configuration for these systems and to optimize its power output. In the process of Energy harvesting, the electrical energy was obtained through of conversion of mechanical energy, created by an ambient vibration source by a type of transduction, for example as a piezo-ceramic thin film. A number of different electromechanical coupling mechanisms have been developed for harvesting devices, and because of the constitutive laws of piezoelectric materials, the role of nonlinearities, in the electromechanical coupling of the design of energy harvesting system, must be taken into account. We mention that some nonlinearities of the piezoelectric material was experimentally found by (Crawley and Anderson, 1990), and an analytical approximated was proposed by (Tripplet and Quinn, 2009). Recently, the nonlinearities of vibratory energy harvesting was widely exploited by (Daqaq, *et al.*, 2014).

The mathematical model (NIEH), we need to substitute the Eq (1), by the following governing equations of motion:

$$M\ddot{x} + f(x, \dot{x}) + \frac{\partial U(x)}{\partial x} - \frac{d(x)}{C} q = F(\phi, \ddot{\phi}, r, m_0) \quad (6)$$

where the quantity $M = m_1 + m_0$ is the total mass of the (NIEH), x is displacement of the (NIEH), $P1$ and $P2$ are thin film piezoelectric applied layers and the electrical charge developed in the coupled circuit given by q , the term $\frac{d(x)}{C}q$ represents the piezoelectric coupling to the mechanical component, with a strain-dependent coupling coefficient $d(x)$. The voltage V across the piezoelectric material has the form:

$$V = -\frac{d(x)}{C}x + \frac{q}{C} \quad (6)$$

Here the C represents the piezoelectric capacitance and with $V = -R\dot{q}$ the (NIEH) coupled governing equations of motion are:

$$M\ddot{x} + c\dot{x} + k_1x + k_2x^3 = m_0r(\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi) + \frac{d(x)}{C}q \quad (7)$$

$$I\ddot{\phi} = \Gamma(\dot{\phi}) + m_0r\ddot{x}_1\cos\phi \quad (8)$$

$$R\dot{q} - \frac{d(x)}{C} + \frac{q}{C} = 0 \quad (9)$$

In the paper of (Iluk *et al.*, 2013) a new model of energy harvester based on a simple portal frame structure was presented. The system was considered to be (NIS). The nonlinearities present in the piezoelectric material were considered in the piezoelectric coupling mathematical model. The system was a bi-stable Duffing oscillator presenting a chaotic behavior. Analyzing the average power variation, and bifurcation diagrams, the value of the control variable that optimizes power or average value that stabilizes the chaotic system in the periodic orbit was determined. The control sensitivity was also determined.

In another paper (Iluk *et al.*, 2014a) an analysis of a new energy harvester model was presented, based on a simple portal frame structure, considered a NIS due to the kind of excitation influenced by the response of the system, such as a direct current motor with limited power supply. The horizontal motion of the portal frame is considered under a non-ideal excitation, and the approximated mathematical model of the system is obtained, considering the coupled oscillators. To model the piezoelectric coupling, the nonlinearities of the piezoelectric material were considered. A constantly sustained energy harvesting is essential for using these devices in real applications; for this, a control strategy was required. Passive control was obtained by means of a nonlinear substructure with properties of nonlinear energy sink. Numerical simulations were performed in order to find best values of control parameters. To checked the robustness of the control strategy, an analysis considering uncertainties in the parameters of the model was performed, showing the efficiency of the passive control (energy pumping) in the suppression of the chaotic behavior, as well as the sensitivity of the control system to parametric errors. Passive control leads the system to a stable periodic orbit, allowing a more efficient energy harvest, due to the higher peak-to-peak amplitude of oscillation mean value. The passive control strategy eliminated the need for an active microcontroller to stabilize the system in a periodic orbit, improving the energy budget (harvested versus expended). The results showed the displacement of the structure and the maximum power harvested by the device with and without passive nonlinear energy sink. It can be concluded that the application of passive control was successful. The control was robust and improved the energy harvested through the suppression of the chaotic motion, leading the system to a periodic orbit with stable amplitude of vibration, without damaging the structure. We also mention that (Iluk *et al.*, 2014b) presented a Finite Element Analysis (FEA) of modal energy exchange and harvesting in a simple portal frame structure. The authors considered both horizontal and vertical support excitations resonant first with the first mode (sway mode) and latter with the second mode (the first symmetrical mode). As 2:1 internal resonance was present between these two modes, the phenomena of mode saturation and energy exchange (modal coupling) may occur. Thus, energy pumped into the system through one of the modes, was partially transferred to the other mode, not directly excited. An evaluation of the energy available for harvesting in each of the considered mode was computed.

3. CONCLUSIONS

In this paper, we have analyzed a possible practical application concerning than unbalanced non-ideal DC motor type foundation structure that suffers the *Sommerfeld Effect* of getting stuck at resonance (energy imparted to the motor being used to excite large amplitude motions of the supporting structure). We overviewed recent studies, new phenomena were addressed, concerning structures supporting unbalanced machines capable of a limited power output

were considered (NIS) and the motion of an oscillating structure under the action of such energy source was accompanied by a full interaction between these non-ideal motors and their supports, amplitude vibrations.

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