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# AN EFFICIENT TIME-DOMAIN FINITE ELEMENT MODEL REDUCTION METHOD FOR NONLINEAR SYSTEMS

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***Abstract.*** *More recently a number of works have demonstrated that the mathematic models can represent with a reasonable accuracy the dynamic behavior of mechanical structures incorporating non-linearities in the time-domain. However, there is a great interest in the search for more efficient numerical procedures to solve the resulted equations of motion of non-linear systems composed by many thousands of degrees of freedom requires. Such difficulties motivate the study reported herein in which a strategy of reanalysis called Combined Approximation Method (CA) is employed in order to reduce the non-linear equations of motion of the system and its computational cost. After the discussion of the theoretical foundations, the full and reduced dynamic responses of a plate incorporating geometrical non-linearities are presented and the main aspects of the proposed methodology are highlighted.*

***Keywords:*** *Geometrical nonlinearities, thin plates, Combined Approximation Method*

## 1. INTRODUCTION

In recent years, there has been a growing demand for durability, reliability and comfort in structures, machines and equipment due to modernization of processes and products (Kocer, 2010). These goals become more complex with the tendency to build increasingly slender structures, so that they also promote (or at least not hinder) the increase in operating speed of machines and processes.

Another important aspect to consider in the current scenario of technological development, especially when using thin or moderately thin structures in engineering projects, is the nonlinear character, the linear phenomena, in reality, are an exception. So, do not consider non-linear effects in practical projects may result in performance degradation, reduced lifetime, increase operating costs and lead to catastrophic failure.

In this context, the structure analysis becomes a very important issue for security and reliability of projects however, there has to consider that this issue is complex and broad. Basically, this analysis involves the prior establishment of a mathematical model that try to relate as closely as possible to the relations of cause and effect throughout the entire analyzed structure, or at least the part of interest. According Gerges (2013) for structural dynamic analysis, the sources of nonlinearities may be global or local, these being dependent material behavior, the nature of the deformation and the imposed boundary conditions.

In this article, specifically, we analyze the behavior of a thin plate from the point of view of geometric nonlinearity associated. In this sense, we accept the hypotheses that the structure will be subject to large displacements and moderate rotations, combined with the infinitesimal deformation and considering that the structure is always under the elastic range when applied harmonic point loads.

In order to establish a *Finite Element Model* for the equation of motion of plate takes into account, in addition to the assumptions described in the preceding paragraph, the hypothesis of *Love-Kirchhoff*, amalgamated the *HSDT theory* (*Higher-Order Shear Deformation Theory*), once this provides better prediction of transverse shear stresses. However, models generated by this theory have high computational costs (Bajoria and Kulkarni, 2003).

In this context, it is important to search for numerical and computational techniques for modeling nonlinear dynamic systems subject to mechanical perturbations that lead to the appearance of large displacements and rotations moderate.

However, the resolution of such problems involving finite element models of complex structures of industrial interest involves a high computational cost. In this case, the use of models reduction methods is needed to reduce the cost of calculation of dynamic responses.

Therefore, in order to surpass the associated computational cost will be used a models reduction method, the method known as Combined Approaches (CA) (Kirsch, 2002), which aims to establish a significant savings of computational effort, in addition to accuracy and efficiency also available in others model reduction methods. Thus, using this reduction method is possible to establish a dynamic analysis of model associated with the system.

## 2. REVIEW OF FINITE ELEMENT MODELING OF NON-LINEAR SYSTEMS

The plates are structural elements bounded by two plane surfaces spaced out from each other by a magnitude called thickness h. In the case of the thickness dimension is much less than the dimensions of the plane surfaces (length and width), these plates are called thin plates. The plane equidistant of external planar surfaces is called midplane of the plate and, denoted by $\Gamma_m$. Consider a thin plate, Fig. 1.

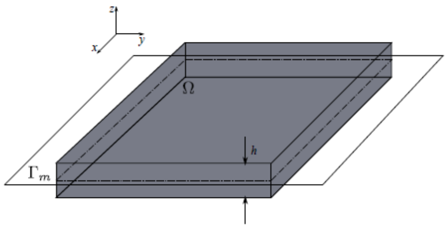



Figure 1: Geometry of a plate and it midplane (extracted from Gerges, 2013).

Figure 1 illustrates a plate limited by volume Ω, and h the thickness in global coordinates. Suppose a load is applied, which produces, on the plate, deformations and displacements, in this case the large displacement is considered, so that the plate thickness remains unchanged. Accordingly various types of nonlinearities may occur from these considerations, however in this work will be analyzed only the geometric nonlinearity, which arise when external forces produce small deformations in structure, but having large variations in displacements fields.

In this case, it is valid the following assumptions: (i) the validity domain is elastic; (ii) the deformations are infinitesimal; (iii) the rotations are moderates. In practice, this behavior is observed from the moment that the transverse displacement field of the structure exceeds the value of thickness.

Thus, assuming that the plate will suffer moderate rotations, however, still small enough to be considered, without loss of generality, the displacement field expressed by Eq. (1).

$$U(x,y,z,t) = \begin{Bmatrix} u(x,y,t) \\ v(x,y,t) \\ w(x,y,t) \end{Bmatrix} - z \begin{Bmatrix} w_{,x} \\ w_{,y} \\ 0 \end{Bmatrix} \tag{1}$$

where $w_{,x} = \frac{\partial w}{\partial x}$ $e$ $w_{,y} = \frac{\partial w}{\partial y}$.

Therefore, from Eq. (1) and the fact that the imposed loads will cause large displacements, is established, in accordance with the *Green-Lagrange* tensor expression that relates the deformations suffered in relation to displacement, as explicit in Eq. (2).

$$\boldsymbol{E} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix} - z \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} \left(w_{,x}\right)^2 \\ \left(w_{,y}\right)^2 \\ 2w_{,x}w_{,y} \end{Bmatrix} \tag{2}$$

where The first term of Eq. (2) is the vector deformations due to the membrane effect , the second, the vector of deformation due to the bending effect, the last term, the nonlinear distortion vector reflecting the coupling between the effects on membrane and bending on the midplane, which is the nonlinear contribution to the structure.

On the other hand, Eq. (2) shows this problem has three variational variables (u, v, w); in this model, it is considered small quantities, thus, it is assumed that their rotations can be expressed by derivatives of w, therefore, are dependent variables. Thus, the finite element model should include the degrees of freedom for translations {U, V, W}, and the rotations $\{\theta_x; \theta_y\}$, therefore, just five degrees of freedom per node. Consequently, the number of unknown variables is less than the number of degrees of freedom per node. Therefore, it is possible to establish the Eq. (3) that relates the variational variable according to the degrees of freedom of translations and rotations, where $N$, $N^{w}$, $N^{\theta_x}$ $e$ $N^{\theta_y}$ are the interpolation functions.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} N & 0 & 0 & 0 & 0 \\ 0 & N & 0 & 0 & 0 \\ 0 & 0 & N^{w} & N^{\theta_x} & N^{\theta_y} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \\ \theta_x \\ \theta_y \end{bmatrix} \tag{3}$$

From Eqs. (2 and 3) you can get Eq. (4) that establishing the tensor of *Green-Lagrange* in its discretized form *by Finite Element*:

$$\boldsymbol{E} = \begin{bmatrix} N_{,x} & 0 \\ 0 & N_{,y} \\ N_{,y} & N_{,x} \end{bmatrix} \begin{Bmatrix} U \\ V \end{Bmatrix} - z \begin{bmatrix} N^{w}_{,xx} & N^{\theta_x}_{,xx} & N^{\theta_y}_{,xx} \\ N^{w}_{,yy} & N^{\theta_x}_{,yy} & N^{\theta_y}_{,yy} \\ 2N^{w}_{,xy} & 2N^{\theta_x}_{,xy} & 2N^{\theta_y}_{,xy} \end{bmatrix} \begin{Bmatrix} W \\ \theta_x \\ \theta_y \end{Bmatrix} + \frac{1}{2} \begin{bmatrix} w_{,x} & 0 \\ 0 & w_{,y} \\ w_{,y} & w_{,x} \end{bmatrix} \begin{bmatrix} N^{w}_{,x} & N^{\theta_x}_{,x} & N^{\theta_y}_{,x} \\ N^{w}_{,y} & N^{\theta_x}_{,y} & N^{\theta_y}_{,y} \end{bmatrix} \begin{Bmatrix} W \\ \theta_x \\ \theta_y \end{Bmatrix} \tag{4}$$

or yet:

$$\boldsymbol{E} = \boldsymbol{B^m}\widetilde{\boldsymbol{U}} + \frac{1}{2}\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}})\widetilde{\boldsymbol{W}} - z\boldsymbol{B^f}\widetilde{\boldsymbol{W}} \tag{5}$$

where

$$\boldsymbol{B^m} = \begin{bmatrix} N_{,x} & 0 \\ 0 & N_{,y} \\ N_{,y} & N_{,x} \end{bmatrix}, \quad \boldsymbol{B^f} = \begin{bmatrix} N^{w}_{,xx} & N^{\theta_x}_{,xx} & N^{\theta_y}_{,xx} \\ N^{w}_{,yy} & N^{\theta_x}_{,yy} & N^{\theta_y}_{,yy} \\ 2N^{w}_{,xy} & 2N^{\theta_x}_{,xy} & 2N^{\theta_y}_{,xy} \end{bmatrix} \quad e \quad \boldsymbol{B^{nl}}(\widetilde{W}) = \begin{bmatrix} w_{,x} & 0 \\ 0 & w_{,y} \\ w_{,y} & w_{,x} \end{bmatrix} \begin{bmatrix} N^{w}_{,x} & N^{\theta_x}_{,x} & N^{\theta_y}_{,x} \\ N^{w}_{,y} & N^{\theta_x}_{,y} & N^{\theta_y}_{,y} \end{bmatrix} \tag{6}$$

On the other hand, considering Eq.(5) and the principle of virtual work between any two instants, and considering that Young's modulus, E, and Poisson coefficient $\nu$ characterize the elastic and isotropic material; establish:

$$\delta\Pi_{def} = \int_{\Gamma_m} \left(\boldsymbol{B^m}\delta\widetilde{\boldsymbol{U}} + \boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}})\delta\widetilde{\boldsymbol{W}}\right)^{T} \boldsymbol{D^m} \left(\boldsymbol{B^m}\widetilde{\boldsymbol{U}} + \frac{1}{2}\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}})\widetilde{\boldsymbol{W}}\right) dS + \int_{\Gamma_m} \left(\boldsymbol{B^f}\delta\widetilde{\boldsymbol{W}}\right)^{T} \boldsymbol{D^f}\boldsymbol{B^f}\widetilde{\boldsymbol{W}} dS \tag{7}$$

where $\begin{cases} \boldsymbol{D^m} = \dfrac{Eh}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}. \\ \boldsymbol{D^f} = \dfrac{h^2}{12}\boldsymbol{D^m} \end{cases}$

Based on Eq. (7) and through matrix properties and integrals obtain:

$$\delta E = \delta\boldsymbol{X^T} \int_{\Gamma_m} \begin{bmatrix} (\boldsymbol{B^m})^{T}\boldsymbol{D^m}\boldsymbol{B^m} & \frac{1}{2}(\boldsymbol{B^m})^{T}\boldsymbol{D^m}\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}}) \\ \left(\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}})\right)^{T}\boldsymbol{D^m}\boldsymbol{B^m} & \frac{1}{2}\left(\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}})\right)^{T}\boldsymbol{D^m}\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}}) + \left(\boldsymbol{B^f}\right)^{T}\boldsymbol{D^f}\boldsymbol{B^f} \end{bmatrix} dS\ \boldsymbol{X} \tag{8}$$

where $\boldsymbol{X} = \begin{Bmatrix} \widetilde{\boldsymbol{U}} \\ \widetilde{\boldsymbol{W}} \end{Bmatrix}$.

Eq. (8) gives an expression that relates the deformation plate with its displacement in time. This equation has nonlinear parcels due to the influence of displacements due to the imposed loads. On the other hand, the stiffness matrix as expressed by Eq. (9).

$$\boldsymbol{K} = \int_{\Gamma_m} \begin{bmatrix} (\boldsymbol{B^m})^{T}\boldsymbol{D^m}\boldsymbol{B^m} & 0 \\ 0 & \left(\boldsymbol{B^f}\right)^{T}\boldsymbol{D^f}\boldsymbol{B^f} \end{bmatrix} dS + \int_{\Gamma_m} \begin{bmatrix} 0 & \frac{1}{2}(\boldsymbol{B^m})^{T}\boldsymbol{D^m}\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}}) \\ \left(\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}})\right)^{T}\boldsymbol{D^m}\boldsymbol{B^m} & \frac{1}{2}\left(\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}})\right)^{T}\boldsymbol{D^m}\boldsymbol{B^{nl}}(\widetilde{\boldsymbol{W}}) \end{bmatrix} dS \tag{9}$$

The first integral represents the linear term, $\boldsymbol{K^l}$, in the model, reflecting the effects of bending and membrane uncoupled and the second integral representing the nonlinear term, $\boldsymbol{K^{nl}}$, highlighting the coupling between these effects. On the other hand, the equation of motion of the plate can be expressed by:

$$\boldsymbol{M}\begin{Bmatrix}\ddot{\widetilde{\boldsymbol{U}}}\\ \ddot{\widetilde{\boldsymbol{W}}}\end{Bmatrix}+\left(\boldsymbol{K}^{l}+\boldsymbol{K}^{nl}\right)\begin{Bmatrix}\widetilde{\boldsymbol{U}}\\ \widetilde{\boldsymbol{W}}\end{Bmatrix}=\begin{Bmatrix}\boldsymbol{F}^{\widetilde{U}}\\ \boldsymbol{F}^{\widetilde{W}}\end{Bmatrix} \tag{10}$$

where the term $\left(\boldsymbol{K}^{l}+\boldsymbol{K}^{nl}\right)\begin{Bmatrix}\widetilde{\boldsymbol{U}}\\ \widetilde{\boldsymbol{W}}\end{Bmatrix}$ show the linear and nonlinear force including linear and nonlinear effects of the system. The solution of Eq. (10) depends on the nature of the load imposed on the structure. Thus, it is necessary to determine X in Eq. (8) such that minimizes Eq. (10). Thus:

$$\boldsymbol{K}^{t}=\int_{\Gamma_m}\begin{bmatrix}(\boldsymbol{B}^{m})^{T}\boldsymbol{D}^{m}\boldsymbol{B}^{m} & 0\\ 0 & \left(\boldsymbol{B}^{f}\right)^{T}\boldsymbol{D}^{f}\boldsymbol{B}^{f}\end{bmatrix}\begin{Bmatrix}\boldsymbol{d}\widetilde{\boldsymbol{U}}\\ \boldsymbol{d}\widetilde{\boldsymbol{W}}\end{Bmatrix}dS+\int_{\Gamma_m}\begin{bmatrix}0 & \frac{1}{2}(\boldsymbol{B}^{m})^{T}\boldsymbol{D}^{m}\boldsymbol{B}^{nl}(\widetilde{\boldsymbol{W}})\\ \boldsymbol{B}^{nl}(\widetilde{\boldsymbol{W}})^{T}\boldsymbol{D}^{m}\boldsymbol{B}^{m} & \frac{1}{4}\boldsymbol{B}^{nl}(\widetilde{\boldsymbol{W}})^{T}\boldsymbol{D}^{m}\boldsymbol{B}^{nl}(\widetilde{\boldsymbol{W}})\end{bmatrix}\begin{Bmatrix}\boldsymbol{d}\widetilde{\boldsymbol{U}}\\ \boldsymbol{d}\widetilde{\boldsymbol{W}}\end{Bmatrix}dS+$$
$$+\int_{\Gamma_m}\begin{bmatrix}0 & \frac{1}{2}(\boldsymbol{B}^{m})^{T}\boldsymbol{D}^{m}\boldsymbol{B}^{nl}(\widetilde{\boldsymbol{W}})\\ 0 & \frac{1}{4}\boldsymbol{B}^{nl}(\widetilde{\boldsymbol{W}})^{T}\boldsymbol{D}^{m}\boldsymbol{B}^{nl}(\widetilde{\boldsymbol{W}})+\boldsymbol{G}^{T}\boldsymbol{T}^{m}\boldsymbol{G}\end{bmatrix}\begin{Bmatrix}\boldsymbol{d}\widetilde{\boldsymbol{U}}\\ \boldsymbol{d}\widetilde{\boldsymbol{W}}\end{Bmatrix}dS \tag{11}$$

Finally, Eq. (11) is the formulation of plate stiffness matrix in finite element subjected to loading perpendicular to the midplane of the plate and it is still necessary impose other boundary conditions to the system. However, regardless of the boundary conditions to be imposed, the Eq. (11) makes clear that the resolution of the system is too time consuming, with the computational cost directly related to the magnitude of the matrices involved. Because it is a significant size system and by the fact that it is necessary evaluate the nonlinear force at each step, since the nonlinear term of the plate stiffness directly depends on the displacement of the structure at each time point.

## 3. SYSTEMS MODEL REDUCTION METHOD OF NON-LINEAR

### 3.1 APPROXIMATIONS REANALYSIS

Reanalysis methods are designed to assess the structural response, but without solving the complete set of simultaneous equations. Solution procedures generally uses the original response of the structure.

Several studies as Kirsch (2002), Abu Kasim (1987) and Barthelemy (1993), present considerations of reanalysis methods, which are generally based on approaches. However, the nonlinear reanalysis problems can be solved by some methods that provide sufficiently accurate solutions of the linear equations updated in each iteration cycle. Many studies on the theme are based on reanalysis of eigenvalues (eigenvalues and eigenvector). A comparison of several reanalysis methods of eigenvalues can be seen in Chen et al. (2000).

The common methods of reduction may be divided into global, local and combined approaches (Kirsch, 2002). Local approaches are effective only for small changes in design variables. Global Approaches method are more accurate for large changes in the structure, but they require a lot of computational effort.

According to Lima et al (2015) in complex structures of industrial interest, EF models generally consist of a large number of DOFs. In such cases, it becomes impossible to calculate the response directly in the time domain from Eq. (10) due to the computational cost high (memory and processing time). This fact motivates the use of model reduction procedures, aimed at reducing the model dimensions (and the associated computational cost) while maintaining a reasonable predictive power of the numerical models.

### 3.2 USING THE METHOD OF COMBINED APPROXIMATIONS

From the considerations presented in the previous section, in this work we used the theoretical developments of the method of the Combined approaches (CA), available in Kirch, Bogomolni & Sheinman (2005) as an approximate reanalysis method, aiming its essential properties: precision and accuracy. Thus, based on the method of superposition and CA method, the aim is to build a reduced model that represents as closely as possible the system expressed by Eq. (10), from the original system data.

In this sense, based on the development of Lima et al (2015), is generated a reduction base of modified model due to the reduction base of the original model without, however, pass through the eigenvalue problem of the modified system, since this increases the computational cost.. In this context, Eq. (10) is interpreted as a reanalysis problem where the nonlinear force is decomposed into a linear term, $K_l$, and a perturbation, $\Delta K_{nl}$. However, since this latter term evolves with time, reducing base must also evolve with time. Thus, based on the assumption that the exact answer, given by resolution of Eq. (10), can be solved by a projection in a reduced base of vectors:

$$\widehat{\boldsymbol{u}}(\boldsymbol{t})=\boldsymbol{T}^{-1}\boldsymbol{u}(\boldsymbol{t}) \tag{12}$$

where $\boldsymbol{T} \in C^{NxNR}$ is the transformation matrix, each column of this matrix, $\boldsymbol{T}$ contains the information of plate vibrate mode, $\hat{u} \in C^{NR}$ is the vector of generalized coordinates and, $NR << N$ is the number of basis vectors.

Once associated with the transformation expressed in Eq. (12), the equations of motion Eq. (10) can be rewritten as follows:

$$\widehat{\boldsymbol{M}}\ddot{\widehat{\boldsymbol{u}}} + \widehat{\boldsymbol{K}^l}\widehat{\boldsymbol{u}} = \widehat{\boldsymbol{F}} - \widehat{\boldsymbol{f}}_{\boldsymbol{nl}} \tag{13}$$

where $\widehat{\boldsymbol{M}} = \boldsymbol{T^T M T}$, $\widehat{\boldsymbol{K}^l} = \boldsymbol{T^T K^l T}$, $\widehat{\boldsymbol{F}} = \boldsymbol{T^T F}$, $\widehat{\boldsymbol{f}}_{\boldsymbol{nl}} = \boldsymbol{T^T f_{nl}}$, with $\boldsymbol{f_{nl}} = \boldsymbol{K^{nl} u}$, are matrices and vectors reduced. So that the transformation base is obtained from the resolution of the eigenvalue and eigenvector problem associated:

$$\begin{cases} (\boldsymbol{K^l} - \boldsymbol{\lambda_i M})\phi_i = 0 & i = 1, \dots, N \\ \boldsymbol{\phi_0} = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_{NR}] & \boldsymbol{\Lambda_0} = diag([\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_{NR}]) \end{cases} \tag{14}$$

The base, $\phi_0$, it is more enriched by introducing the residue formed by the external excitation force:

$$\boldsymbol{R} = \left(\boldsymbol{K}^l\right)^{-1}\boldsymbol{b} \tag{15}$$

where $\boldsymbol{b} \in R^{N\times f}$ is a Boolean matrix, which allows to select among the DOFs, in which the forces of external excitation are applied. Thus, it is possible to obtain the transformation matrix $\boldsymbol{T_0}$, which will be taken as the initial basis for the reanalysis method to be used in this work:

$$\boldsymbol{T} = [\boldsymbol{\phi_0} \quad \boldsymbol{R}] \tag{16}$$

Thus, from Eq. (16) it is possible to obtain an initial reduction base for the problem to be solved by Eq. (10) so that it is able to reduce the dimensions of the expressions measures and information provided by the *EF method*. On the other hand, geometric nonlinearity, as already mentioned, evolves in time and is expected, by conception, that this base is not able of represent all measures in the entire time domain. Thus, it is presumed with the evolution of the level of nonlinearities that this fact will directly influence the system, with respect to non-linear stiffness matrix, as explicit in Eq. (11). Suggesting that the reduction base needs evolve so that it is still able to represent the same information as given by Eq. (10) without losing the property of condensing system.

### 3.3 COMBINED APPROXIMATIONS METHOD

Based on information from the previous section and to obtain the evolution of the form of reduced basis that it remains with the properties of efficiency and accuracy, is used the Method of Combined Approaches illustrated in Fig. 2.

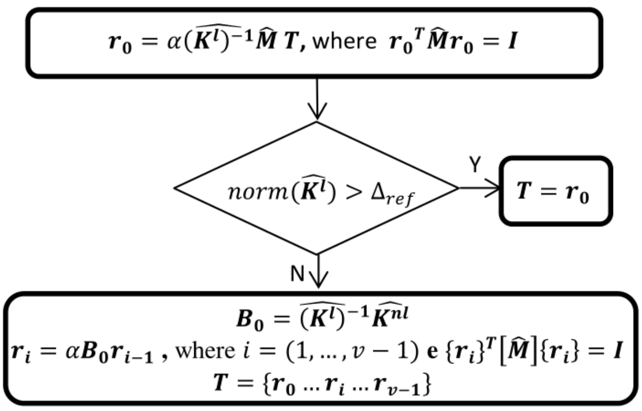



Figure 1: Flowchart of Combined Approximations Method.

Fig. 4 schematically illustrates the main steps of the numerical procedure used to calculate the system response represented by Eq. (10), the structural damping, $\boldsymbol{D}$, is equal to 0.5% of the linear stiffness of the plate. Thus, the steps can be summarized as follows:

Step (*i*): the beginning of the process, $t_0$, is defined material properties $E, \nu, \rho$, with the passage of time and the time interval to be analyzed, $\Delta t$ and $t$, the thickness of the plate, $h$, the parameters of the *Newmark Method*, $\beta$ and $\gamma$, mass matrices, $\boldsymbol{M}$, stiffness $(\boldsymbol{K^l}, \boldsymbol{Knl}, \boldsymbol{Kt})$ determined by Eq. (13) and damping, $\boldsymbol{D}$, and the fields of displacement, velocity and acceleration $(\boldsymbol{u}(t_0), \dot{\boldsymbol{u}}(t_0), \ddot{\boldsymbol{u}}(t_0))$, respectively, having as parameter the conditions of mechanical boundary.

Step (*ii*): next, an eigenvector and eigenvalue problem is solved according to Eq. (14), in order to obtain the vibration modes of the plate. Extracting some of these modes associated with the membrane and bending effects, so that the reduction base of the plate vibration includes more information, worth pointing out that the modes of vibration chosen depend on the type of structure to be analyzed. In this work, the reduction of base is composed of a percentage of 80% of vibration modes relating to bending and the rest of the modes associated to membrane effect.

So, based on that, and in the eigenvector matrix, it's possible establish a matrix, $\boldsymbol{\phi_0}$, containing the shape information from vibrating of the plate and joining this matrix the residue of external excitation force given by Eq. (15), obtains the first reduction base, $\boldsymbol{T}$, as described in Eq(16). With this first reduction base, can apply a first reduction of the matrices and vectors associated with the system of Eq. (10), thus forming matrices $(\widehat{\boldsymbol{M}}, \widehat{\boldsymbol{K^l}}, \widehat{\boldsymbol{Knl}}, \widehat{\boldsymbol{Kt}})$, and the vector $(\hat{\boldsymbol{u}}, \dot{\hat{\boldsymbol{u}}}, \ddot{\hat{\boldsymbol{u}}}, \widehat{\boldsymbol{F}}, \widehat{\boldsymbol{fnl}})$, as defined in Eq. (13);

Step (iii): next, iterative process is started by exciting the system in the force application point, according to the vector of external loads at each time step to evaluate the fields displacement, velocity and acceleration is used it Newmark method, testing the accuracy of the approach via calculation of residue. So if they do need to correct the fields $\hat{\boldsymbol{u}}(t_i), \dot{\hat{\boldsymbol{u}}}(t_i), \ddot{\hat{\boldsymbol{u}}}(t_i)$, It is used for this method of evaluating whether Newton-Raphson this instant, new values for the non-linear stiffness matrix, Eq. (11), this process continues until convergence to the residue. It is noteworthy that the CA method has the characteristic change the size of the database depending on the energy dispensed to calculate the displacement field. Thus, it is appropriate, at each time step, the determination of the fields, not reduced, $\boldsymbol{u}, \dot{\boldsymbol{u}}\ e\ \ddot{\boldsymbol{u}}$, using the expressions, $\boldsymbol{u} = \boldsymbol{T}\hat{\boldsymbol{u}}$, $\dot{\boldsymbol{u}} = \boldsymbol{T}\dot{\hat{\boldsymbol{u}}}$ e $\ddot{\boldsymbol{u}} = \boldsymbol{T}\ddot{\hat{\boldsymbol{u}}}$, respectively. So based on the estimated energy waived in the convergence process proceeds to the reorganization of a further reduction of base, T, (if necessary), as described in CA method, Fig. 2, in this case proceeds to reduce vectors and system matrices according to this new base, finally the time step is incremented and the iterative process (iii) is repeated to go all the pre-set time domain.

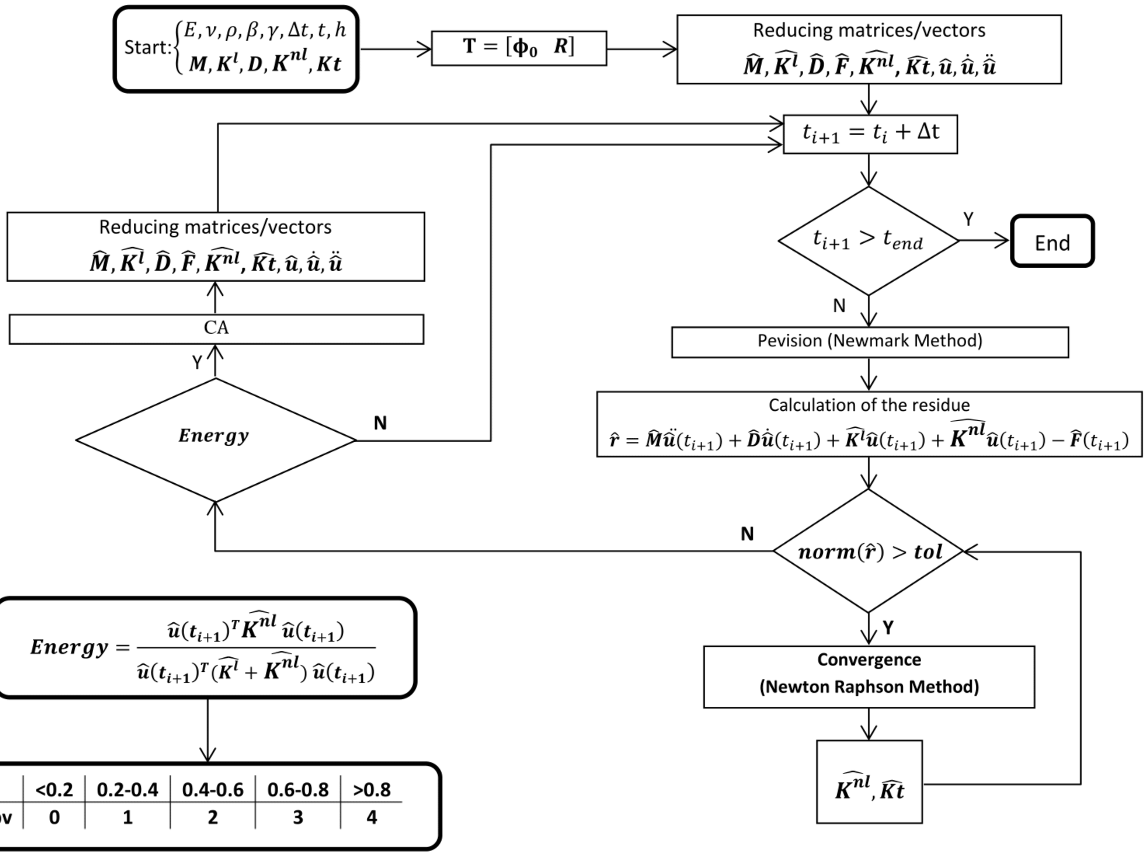
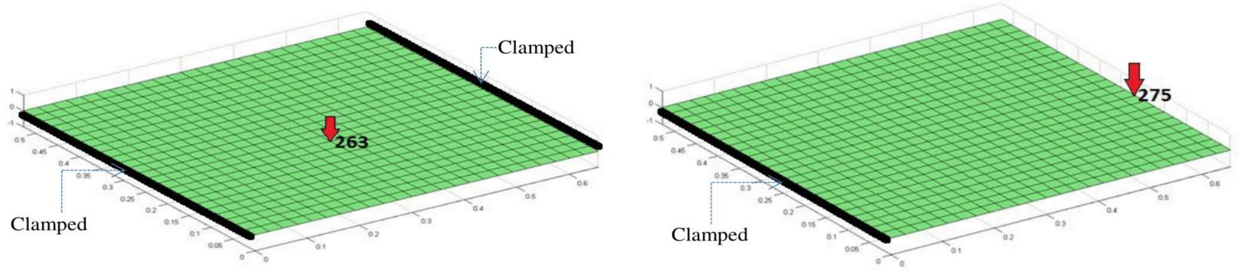



Figure 2: Diagrama de blocos do processo numérico usado para obter a resposta do sistema.

## 4. SIMULATIONS AND RESULTS

In applications, the numerical tests were performed using the system shown in Fig. 4, a rectangular plate composed of aluminum clamped on two sides, Fig. 4a, and clamped on one sides, Fig. 4b. The *EF* model is made with 576 elements, 4 nodes per element and five degrees of freedom per node, developed in accordance with Section 2. The dimensions of the simulated plate expressed in meters were 0.624, 0.527 and 0.0015 length, width and thickness, respectively, with a Young's modulus E=70x10$^9$PA, Poisson's ratio $\nu = 0.33$ and bulk density $\rho = 2700$ kg/m$^3$.

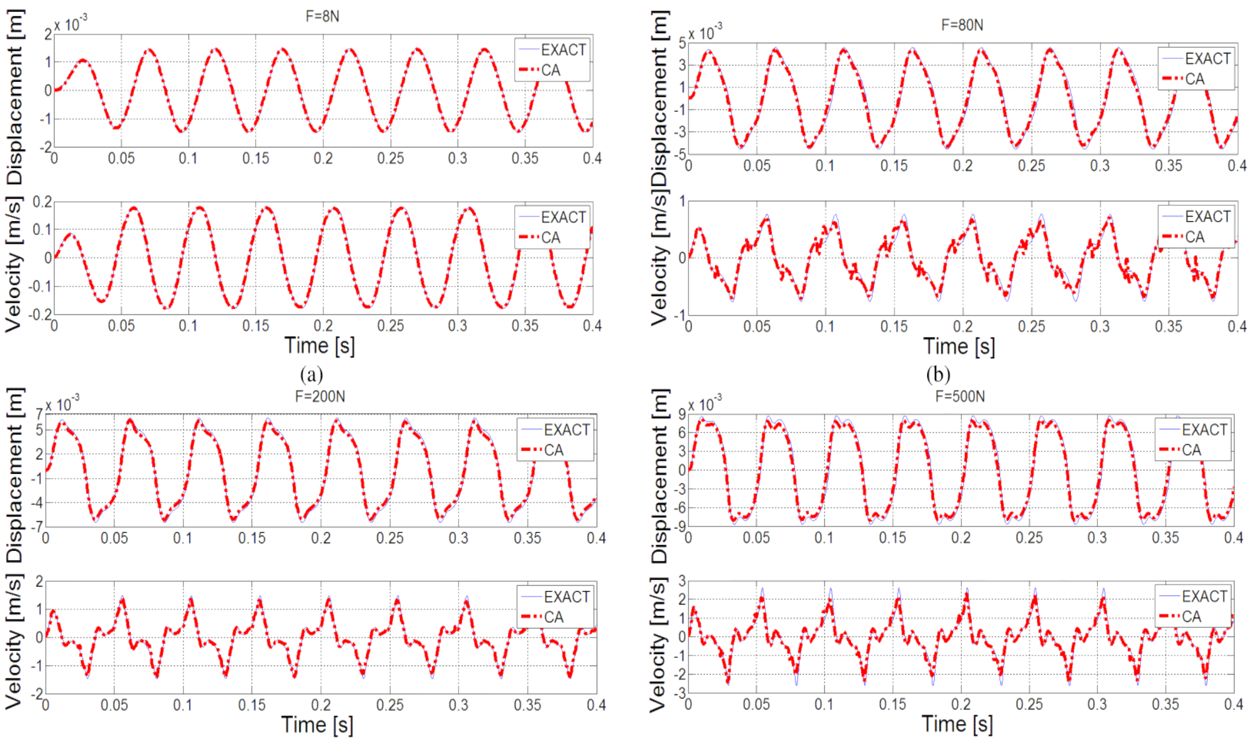
Figure 4 illustrates the representation of the EF of plate together with the boundary conditions to be simulated, as well as application point of the excitation force, $p(t) = Fsin(2\pi ft)$, and amplitudes are shown in Figs. 5 and 7, where $f = 20Hz$ for the simuled situation in Fig. 4a and $f = 3Hz$, the case of Fig. 4b.




(a) (b)

Figure 4: Boundary conditions simulated in aluminum plate.

Figure 5 illustrates the results displacements and velocitys of point indicated by the point of application of harmonic forces and the boundary condition indicated in Fig. 4a, for the case that the amplitude of the harmonic forces were 8N, 80N, 200N and 500N.




(c) (d)

Figure 5: Fields of displacements and velocity determined via Exact method and CA.

By observing Fig. 5 is possible to perceive that the responses via CA method for simulated situations for 8N, 80N and 200N force levels were satisfactory for both the case in which the system behavior was predominantly linear, Fig. 5a, and for cases in which the system is subject to nonlinearities due to the gradual increase of the external forces, Fig. 5b-d. In the case of Fig. 5d it is possible to see that CA method presented some difficulty in predicting the parameters analyzed, suggesting that the CA method becomes effective under certain conditions of nonlinearity, as suggested by analysis of Fig. 6a. Figure 6a it is possible find a purely linear response to the excitation force levels 1N and 8N and, as the external force levels evolves clearly noticed, the emergence and development of non-linearity inherent in the system.

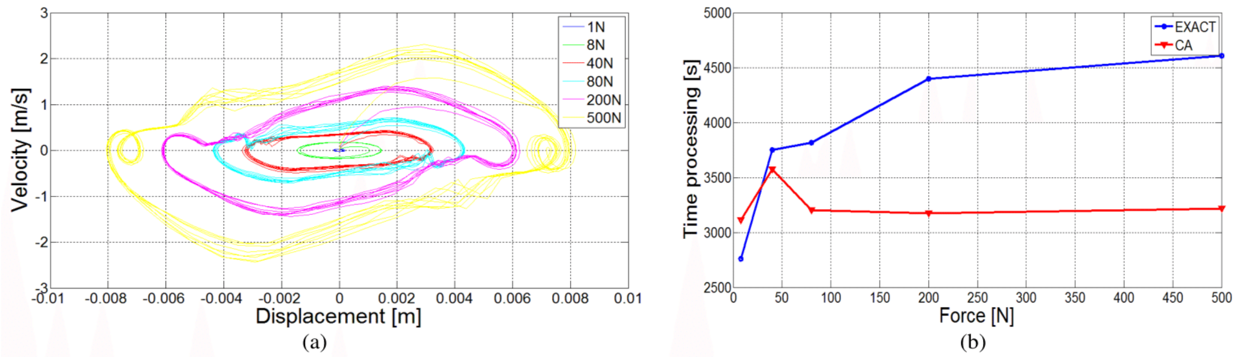



Figure 6: (a) Nonlinear evolution; (b) Processing time.

Figure 6b shows the processing time compared between the exact methods, using Exact, and the CA method, in face of Fig. 6b it is possible to observe a gain of computational time as the plate incorporates greater nonlinear influence due to the increase the external forces.

Figure 7 shows the responses obtained from the harmonic excitation of 1N and 8N for amplitudes (illustrated condition in Fig 4b), in both cases is possible to perceive the presence of geometric nonlinearity, as illustrated as Fig. 8a. The Fig. 8b represents a comparison of the processing times among exact methods and CA, in the levels of forces analyzed (in the presence of nonlinearities) the processing time using the CA method was always less, as was already seen in Fig. 6b. In the case of Fig. 8b you can see that the CA method, also shows difficulty of predicting the analyzed measures, showing that this method becomes effective under certain conditions of nonlinearities, as suggested by the analysis of Figs. 6a and 8a.

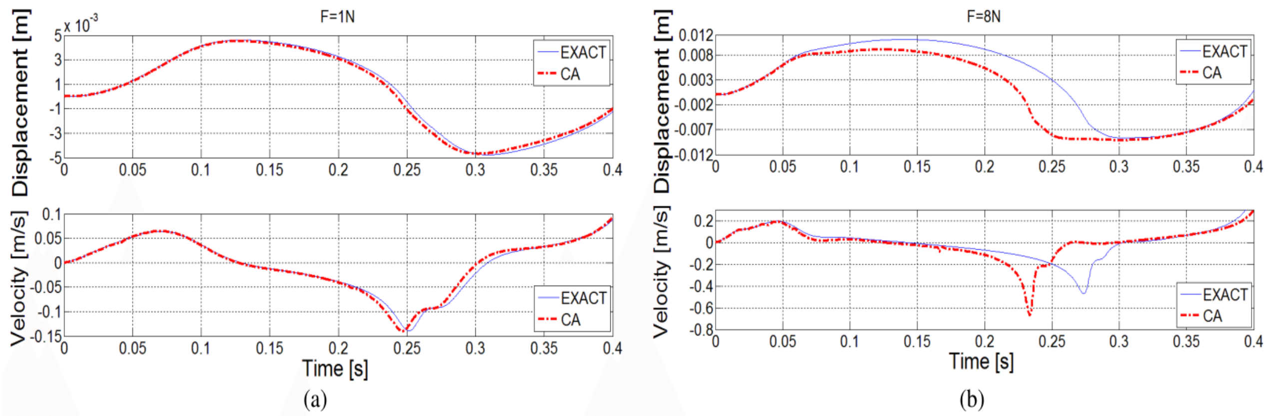



Figure 7. Fields of displacement and velocitydetermined via Exact method and CA.

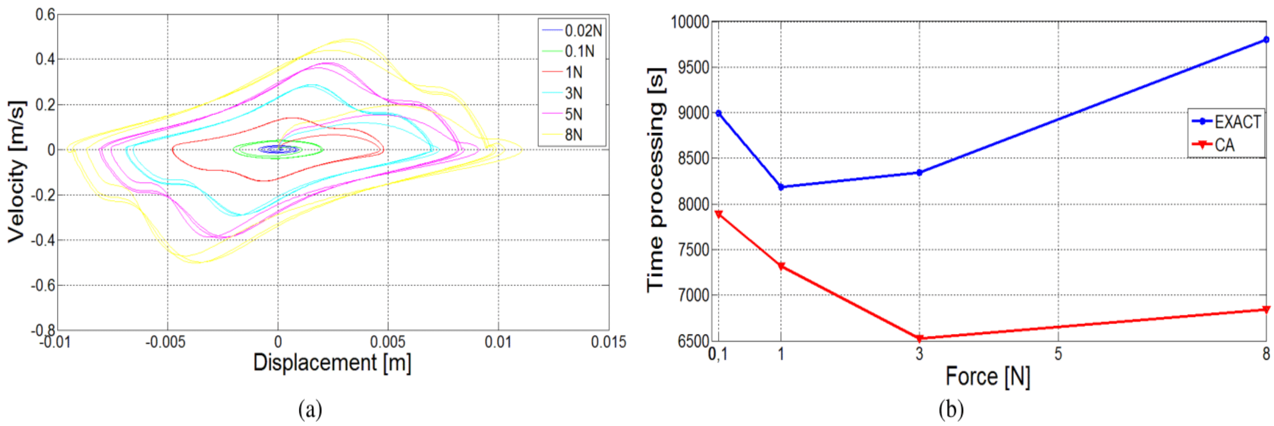



Figure 8: (a) Nonlinear evolution; (b) Processing time.

## 5. CONCLUSION

Based on the data obtained through the simulations carried-out in the previous section, the CA method has proven to be efficient in the analysis of nonlinear problems treated in the time domain. From the results, it is observed an improvement in the time calculation of 5% to 30% approximately and it can increase according to the introduction of nonlinearities. If considered those simulations in which there was the introduction only of the nonlinearities, the gain in time processing was around 20.6% and, 15.8% if considered the linear and nonlinear conditions. In both cases, see Fig. 2, the CA method has shown to be efficient under high conditions of nonlinearities, which means that the amplitudes of movement of the plate is bigger than three times its thickness but do not overpass four times that value, see Fig. 5c,d and Fig. 7b. Besides, it is possible to say that the CA method keeps its stability under linear and nonlinear conditions, since the deformations of the material remains purely elastic, showing a good precision when approximating the displacement, velocity and acceleration fields. Furthermore, its efficiency is based on the fact that only one exact analysis is performed. It is important to remark that the efficiency and precision of the calculus can be controlled by the quantity of information initially considered.

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