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FATIGUE DAMAGE ESTIMATION USING FREQUENCY DOMAIN APPROACH

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Abstract. *Structures and mechanical components, such as wind turbines, car chassis, or offshore structures, are often submitted to random loading. This loading makes the stress vary with time, and resulting in the failure of the structure, when the stress level is high enough. The failure caused by alternating stress is called fatigue. The fatigue is the main mode of mechanical components failure and the durability of the component must be determined during the project. The damage estimation can be calculated in the time domain through the stress cycle counting, or in the frequency domain through the estimation of the distribution of probability of a certain stress level. The spectral approach presents a lower computing effort and best understanding of the dynamic behavior. This paper presents the frequency domain approach for the calculation of damage estimation of a cantilever plate with a hole, using numeric simulations through the combination of the available tools in the Abaqus® and nCode® softwares. The spectral analysis results are compared with simulations in the time domain by the Rainflow method, with the objective of evaluating the frequency with the time approaches. The results show that the methodology used in the frequency domain leads to solutions that are equivalent to the time domain approach, but with a lower computing cost.*

Keywords: *random fatigue, spectral fatigue, damage*

1. INTRODUCTION

Albert performed the first studies on metal fatigue around 1830 for the mining industry (Suresh, 1998). The traditional approach of the fatigue damage consists in calculations in the time domain, in which the stress time history is used for the cycles counting and for later damage accumulation calculation. This method needs dynamic analysis usually through Finite Element Method (FEM), which require great computing effort that makes it not feasible (Halfpenny, 1999).

An alternative approach on the frequency domain uses the spectral response of the structure called frequency response function (FRF), instead of the dynamic response. In the frequency approach, the loadings are described through its power spectral densities (PSD). That makes it presents great computing improvement (Mrsnik, *et al*., 2013). One of the advantages of this methodology is in the fact that the FEM analysis needs to be performed only once to obtain the FRF, as different analysis can be done only by substituting the loading PSD (Halfpenny, 1999).

Many spectral methods were developed with the purpose of obtaining better correlation with experimental data and with the time domain method (Mrsnik, *et al.*, 2013; Benasciutti and Tovo, 2005), being the Dirlik method the most used. However, methods like Zhao-Baker and Benasciutti-Tovo seem to present results that are closer to the experimental ones (Nieslony, *et al.*, 2012; Benasciutti and Tovo, 2006).

In this work, the Dirlik method is applied for the calculation of fatigue damage estimation of a cantilever plate with a hole. The random load is an experimental automotive acceleration signal. The FRF of the structure is obtained via FEM numeric simulation in the Abaqus® software and the fatigue calculation is done in nCode®. The results are compared with the solution by the Rainflow method obtained via nCode® in the time domain, with the objective of evaluating the possible restrictions of the method.

2. RANDOM PROCESSES

A Gaussian $X(t)$ random method is described in the time domain through its mean μ_X and the autocorrelation R_X :

$$\mu_X = E[X(t)] \tag{1}$$

$$R_X(\tau) = E[X(t)X(t+\tau)] \tag{2}$$

where τ is the increment for the correlation and $E[\ ]$ is the probabilistic expected value. Equation (3) defines the bilateral PSD $S_X(\omega)$ of the process in the frequency domain through applying the Fourier transform of the autocorrelation function.

$$S_X(\omega) = \int_{-\infty}^{\infty} R_X(\tau) e^{-i\omega\tau} d\tau \tag{3}$$

The PSD used for the damage fatigue estimation is unilateral, as it considers only the positive frequencies due to its physical meaning. Equation (4) defines the one-sided PSD.

$$G_X(\omega) = \begin{cases} 2S_X(\omega) & \omega \geq 0 \\ 0 & \omega < 0 \end{cases} \tag{4}$$

Through the PSD, it is possible to define bandwidth parameters and spectral moments that describe the process. Equation (5) defines the spectral moments.

$$m_n = \int_0^{\infty} \omega^n G_X(\omega) d\omega \tag{5}$$

where n is the moments index. For the spectral fatigue calculation, the n moments corresponding to 0, 1, 2, and 4 are used, being 0, 2 and 4 the process variance and its derivatives in time domain.

$$m_0 = \sigma_X^2 \tag{6}$$

$$m_2 = \sigma_{\dot{X}}^2 \tag{7}$$

$$m_4 = \sigma_{\ddot{X}}^2 \tag{8}$$

Equation (9) represents the peak expectation

$$E[P] = \sqrt{\frac{m_4}{m_2}} \tag{9}$$

The irregularity factor γ is a bandwidth parameter used to describe the shape of the PSD.

$$\gamma = \frac{m_2}{\sqrt{m_0 m_4}} \tag{10}$$

3. FREQUENCY RESPONSE FUNCTION

Figure 1 represents a mass, spring, and damping system with 1 degree of freedom (DOF) under an external excitation force. Equation (11) defines the system movement based in the Newton's second law, in which m is the mass, c is the damping, k is the stiffness, and $f(t)$ is the applied force.

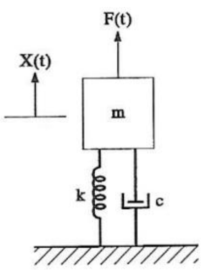


Figure 1. System with 1 DOF

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{11}$$

Equation (12) presents the complex form of the motion equation.

$$\left(-m\omega^2 + ic\omega + k\right)Xe^{i\omega t} = Fe^{i\omega t} \tag{12}$$

where X is the movement amplitude and F is the force amplitude. The transfer function $H(\omega)$, also known as the frequency response function (FRF), is defined as the displacement caused by the application of a unitary force.

$$H(\omega) = \frac{1}{\left(-m\omega^2 + ic\omega + k\right)} \tag{13}$$

Equation (14) defines the response of the 1 DOF system in the frequency domain, by using the Fourier transform.

$$X(\omega) = H(\omega)F(\omega) \tag{14}$$

For a system with n DOFs, Eq. (15) corresponds to Eq. (11) in the matrix form.

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{c}\dot{\mathbf{x}} + \mathbf{kx} = \mathbf{f} \tag{15}$$

where $\mathbf{m}$ is the mass matrix, $\mathbf{c}$ is the damping matrix, $\mathbf{k}$ is the stiffness matrix and $\mathbf{f}$ is the force vector. Equation (15) is composed by a system of coupled equations that are difficult to solve. To avoid having to deal with this issue, a modal superposition method is used, in which a change of coordinates uncouples the system.

The modal superposition primarily requires the modal solution of the system, defining the modes and frequencies of the structure. For that, Eq. (15) is rewritten ignoring the damping terms and the external force.

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{kx} = \mathbf{0} \tag{16}$$

Considering $\mathbf{x}$ as the solution for a harmonic movement, Eq. (16) is written as:

$$\left[-\omega^2\mathbf{m} + \mathbf{k}\right]\boldsymbol{\varphi} = \mathbf{0} \tag{17}$$

Equation (17) defines an eigenproblem, in which ω^2 is the eigenvalue and $\boldsymbol{\varphi}$ is the associated eigenvector. With the solution for Eq. (17) it is possible to define the modal matrix $\boldsymbol{\Phi}$.

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_n \end{bmatrix} \tag{18}$$

To solve the forced vibration problem, a change of variable is used:

$$\mathbf{x} = \boldsymbol{\Phi}\mathbf{q} \tag{19}$$

where $\mathbf{q}$ is the modal coordinates vector. Substituting Eq. (19) in Eq. (15) and pre-multiplying by $\boldsymbol{\Phi}^T$ the movement uncoupled equation is found.

$$\boldsymbol{\Phi}^T\mathbf{m}\boldsymbol{\Phi}\ddot{\mathbf{q}} + \boldsymbol{\Phi}^T\mathbf{c}\boldsymbol{\Phi}\dot{\mathbf{q}} + \boldsymbol{\Phi}^T\mathbf{k}\boldsymbol{\Phi}\mathbf{q} = \boldsymbol{\Phi}^T\mathbf{f} \tag{20}$$

Simplifying Eq. (19) gives:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{Kq} = \mathbf{F} \tag{21}$$

Where $\mathbf{M}$ is the modal mass matrix, $\mathbf{C}$ is the modal damping matrix, $\mathbf{K}$ is the modal stiffness matrix, and $\mathbf{F}$ is the modal force vector. The solution of the system is defined in the frequency domain by Eq. (22)

$$\mathbf{x}(\omega) = \mathbf{H}(\omega)\mathbf{F}(\omega) = \left[-\omega^2\mathbf{m} + i\omega\mathbf{c} + \mathbf{k}\right]\mathbf{F}(\omega) \tag{22}$$

4. FATIGUE DAMAGE IN THE FREQUENCY DOMAIN

Random loadings as the wave movements, turbulences, and road irregularities are best described in a statistical form. They can be represented by a PSD of excitation (Petrucci and Zuccarello, 2004). In this way, the fatigue damage can be estimated in the frequency domain through the spectral methods, such as the ones listed in Mrsnik, *et al.*, 2013.

The approach in the frequency domain is more computing efficient when the aim is to calculate the fatigue damage of a component through the finite elements method, as it needs a frequency response analysis instead of a dynamic analysis in the time domain (Mrsnik, *et al.*, 2013). The response PSD is proportional to the loading PSD, in which the proportionality is defined by the frequency response function of the structure. Thus, the analysis by MEF determines this matrix, which can be used to analyze many excitation PSDs.

However, for the fatigue damage calculation, the response PSD does not provide enough information about the stress history, making it necessary to know the probability distribution of the stress amplitudes (Nieslony and Macha, 2007). For the cumulative damage calculation, the spectral methods do not allow the direct utilization of the Palmgren-Miner rule, as they do not have defined cycles, becoming necessary to use the stress peak distribution, as stated in Eq. (23)

$$n(\sigma_a) = E[P] \cdot T \cdot p(\sigma_a) \tag{23}$$

where $n(\sigma_a)$ is the number of stress amplitude cycles, σ_a, $E[P]$ is the peak expectation, T is the time of observation, and $p(\sigma_a)$ is the probability density function (PDF) of stress amplitude. Using Eq. (23) it is possible to write the Palmgren-Miner rule as:

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = \frac{E[P] \cdot T \cdot p(\sigma_a)}{N_i} \tag{24}$$

with N_i being the total number of stress amplitude cycles σ_a until the failure.

Some spectral methods were proposed to solve the fatigue problem in the frequency domain. The way the stress PDF is expressed defines the difference between the methods. The first proposed solutions concentrated in problems of narrowband spectrums, in which the stress PDF is defined by a Rayleigh distribution, as in Eq. (26). This method presents good results when applied to narrowband spectrums, but is too conservative for wideband processes (Benasciutti and Tovo, 2006).

$$p_{NB}(\sigma_a) = \frac{\sigma_a}{m_0} e^{\left(\frac{-\sigma_a{}^2}{2m_0}\right)} \tag{25}$$

Wideband methods reduce the conservatism for this class of problems, being the Dirlik method the most used one. The Dirlik method defines through an empirical approximation that the Rainflow stress amplitude distribution is defined by the combination of an exponential distribution and two Rayleigh distributions. This approximation is obtained after many Monte Carlo simulations for different stress amplitude histories (Nieslony and Macha, 2007). Equation (26) defines the stress amplitude PDF, in which Z is the normalized stress amplitude and D_1, D_2, D_3, Q, and R are the model adjustment parameters.

$$p_{DK}(\sigma_a) = \frac{1}{\sqrt{m_0}} \left(\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-z^2}{2}} \right) \tag{26}$$

$$D_1 = \frac{2(x_m - \gamma^2)}{1+\gamma^2} \tag{27}$$

$$D_2 = \frac{1 - \gamma - D_1 + D_1{}^2}{1-R} \tag{28}$$

$$D_3 = 1 - D_1 - D_2 \tag{29}$$

$$Q = \frac{1,25(\gamma - D_3 - D_2 R)}{D_1} \tag{30}$$

$$R = \frac{\gamma - x_m - D_1^{\ 2}}{1 - \gamma - D_1 + D_1^{\ 2}} \tag{31}$$

$$x_m = \frac{m_1}{m_0}\sqrt{\frac{m_2}{m_4}} \tag{32}$$

5. CASE STUDY

For the fatigue damage in the frequency domain, it is necessary to perform a frequency response analysis by the finite elements method through the modal superposition technique using the Abaqus® software in order to obtain the transference function. The resulting FRF needs to be exported to nCode®, which generates the stress PSD and calculates the damage by using the Dirlik method. Figure 2 describes the performed analysis flowchart.

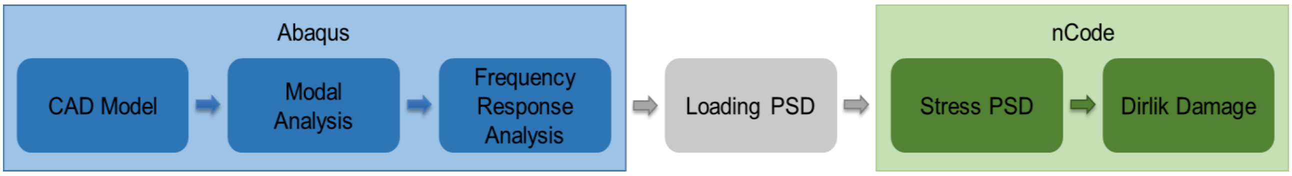


Figure 2. Sequence analysis

Figure 3 describes the plate with a hole used for the application of the spectral method. The thickness of the plate is 0.5 mm.

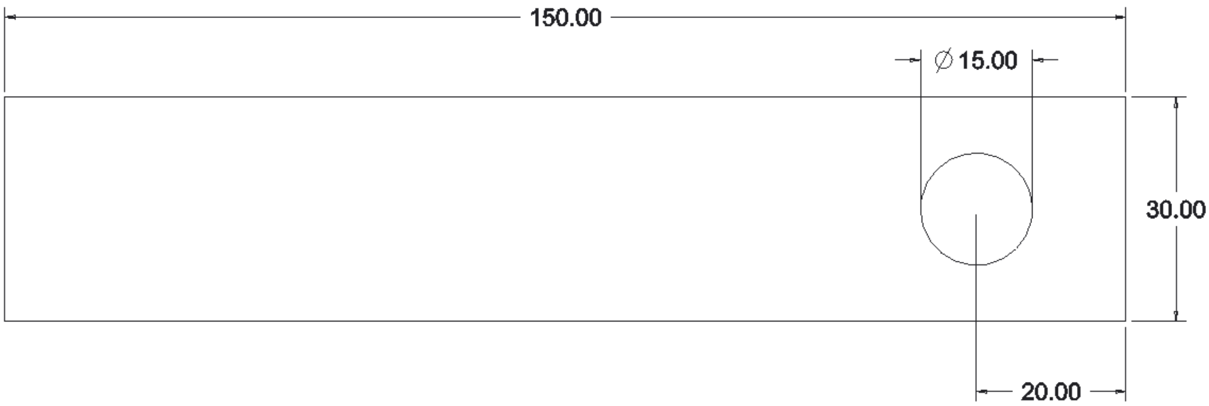


Figure 3. Geometry of the plate. Dimensions in mm

The mesh generated for the finite elements analysis uses linear quadrilateral shell elements through a discretization of 1152 nodes and 1056 elements. Table 1 describes the mechanical properties of the material of the plate.

Table 1. Mechanical properties.

Density	7850 kg/m³
Young’s Modulus	210 GPa
Poisson’s Ratio	0.3
Yield Strength	250 MPa

A modal superposition analysis generates the FRF. Thus, it is firstly necessary to obtain the first natural modes and frequencies of the plate. Table 2 describes the first 5 modes.

Table 2. Natural frequencies and mode shapes.

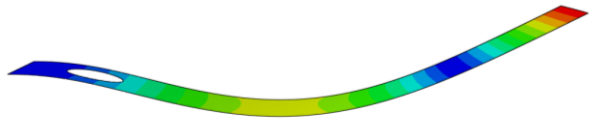
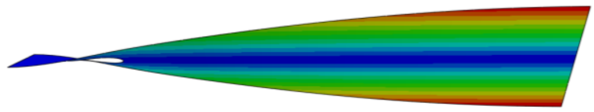
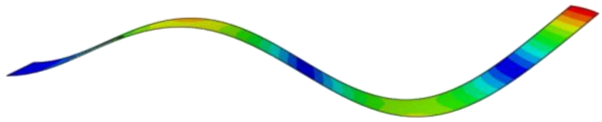
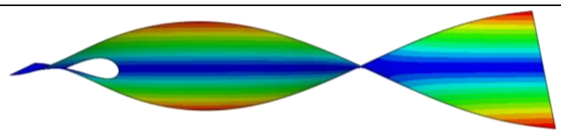
Frequency	**Mode**
16.8 Hz	
114.9 Hz	
178.2 Hz	
331.1 Hz	
553.7 Hz	

Figure 4 describes the FRF of the critical node of the plate that is found in the side of the hole, as this is a region of stress concentration. The frequency interval used is from 0 to 200 Hz. The base excitation is applied in the constrained region on the right edge, as seen in Fig. 3. The direction of the load is normal to the surface of the component and equal to 9.81 m/s². On Fig. 4 it is possible to verify the existence of stress peaks close to the first and second frequencies of the plate. This happens because they are bending frequencies, similar to the applied loading. The third frequency is a torsion mode and does not have a peak in the curve, as the applied loading does not tend to cause this phenomenon in the plate.

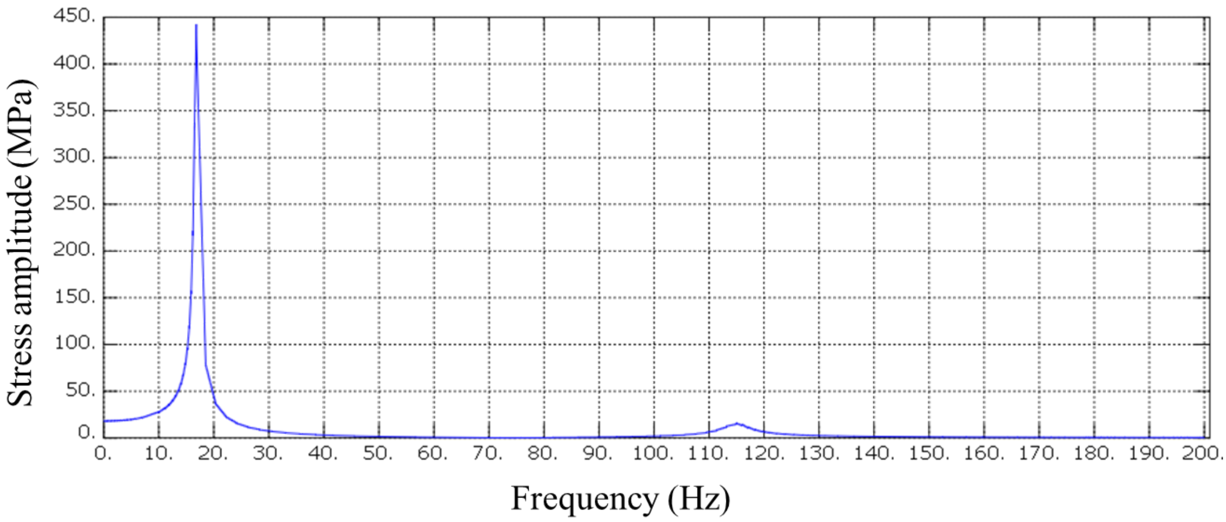


Figure 4. Frequency response function of critical node

At nCode®, the acceleration PSD is applied in the same region where the loading was defined in the frequency response analysis. Figure 5 defines the loading PSD used, which represents an automotive wideband real spectrum. Figure 5 describes the stress PSD of the critical node.

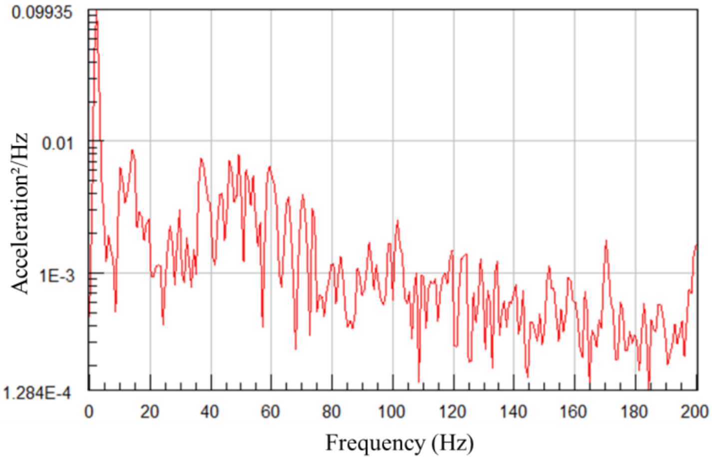


Figure 5. Loading PSD

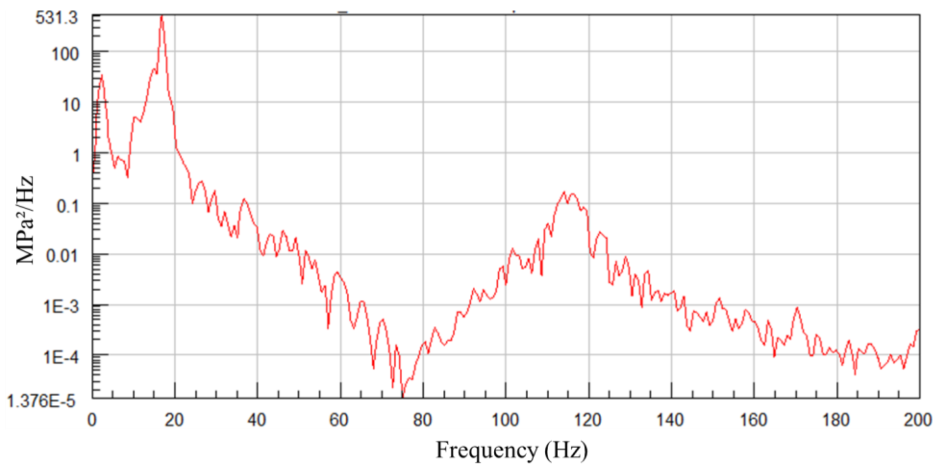


Figure 6. Stress PSD

Figure 7 describes the damage distribution found in the simulation by the Dirlik method. The maximum damage found in the critical node is of 2.997E-10.

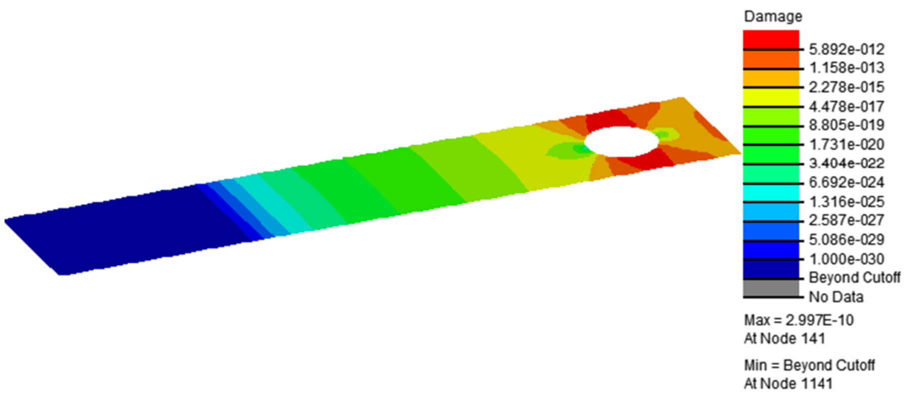


Figure 7. Damage distribution – Frequency domain

An analysis in the time domain was performed through a dynamic simulation using FEM, in order to verify the results found. With this result, the stress time history for each node was obtained and the calculation of damage was performed through the Rainflow cycle counting method. Figure 8 describes the damage distribution in the time domain. The maximum damage found is 3.131E-10.

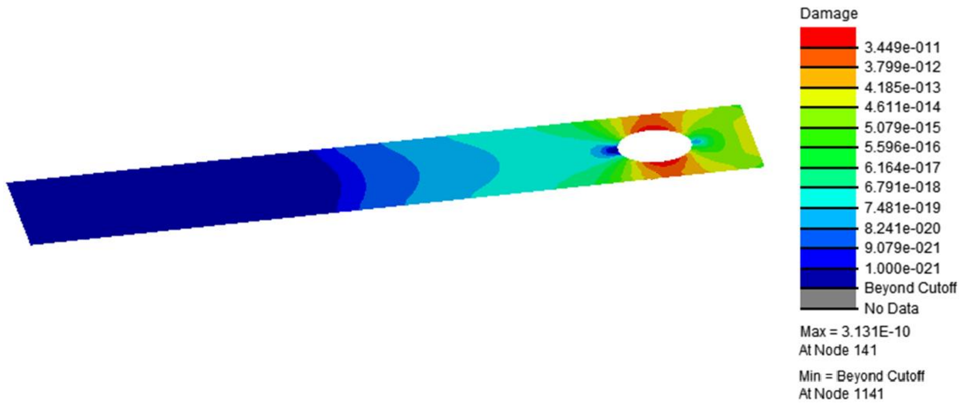


Figure 8. Damage distribution – Time domain

6. CONCLUSIONS

Considering the performed simulations, the spectral approach for the calculation of fatigue damage estimation is a viable alternative to the time domain methodology. The maximum damage found was of 2.997E-10 in the frequency domain and 3.131E-10 in the time domain. This means that the result in the time domain was 4.47% higher.

Even though both approaches present similar results, one factor makes the frequency domain methodology very attractive. The issue of numeric simulation by FEM requires two distinct types of analysis, one for each methodology used. For the time domain, an explicit dynamic analysis must be performed. For the frequency domain, a frequency response analysis must be done. Due to the characteristics of each analysis, the solution time for each model are very distinct. Using the same computer, the dynamic time domain analysis took 1 hour, 25 minutes, and 31 seconds to finish, while the frequency response analysis took 41 seconds. The great complexity of the dynamic analysis causes this difference. For this reason, the calculation of fatigue damage estimation in time domain is not feasible for complex structural projects, while the spectral approach is viable.

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