23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

WAVE PROPAGATION IN A ONE-DIMENSIONAL STRUCTURE USING WAVELET SPECTRAL FINITE ELEMENT

H.V. Cantanhêde, heliocs@fem.unicamp.br
J.M.C. dos Santos, zema@fem.unicamp.br
Departamento de Mecânica Computacional, Faculdade de Engenharia Mecânica, Universidade Estadual de Campinas,Caixa Postal 6122, 13083-970, Campinas, Brazil.

Abstract. *In the present paper, a study of wave propagation in an elementary rod is made using the wavelet based spectral finite element (WSFE) method. Computing the exact values of the connection coefficients and derivative operators of Daubechies compactly supported wavelets over a finite interval, the wave equation has been transformed from time to the wavelet domain. This reduces the partial differential equation to coupled ordinary differential equations, which are decoupled using eigenvalue analysis. The WSFE technique is very similar to the Fourier based spectral finite element (FSFE) except that it uses compactly supported Daubechies. The wave equations are reduced to ordinary differential equations using Daubechies compactly supported, orthonormal, scaling functions for approximations in time. Unlike FSFE formulation with periodicity assumption, the wavelet-based method allows imposition of initial values and thus is free from wrap around problems. This helps in analysis of finite length undamped structures, where the FSFE method fails to simulate an accurate response. Numerical experiments are performed to extract the wave characteristics, namely the spectrum and dispersion relation for these waveguides, and the results are compared with the ones from the literature.*

Keywords: *wavelets, daubechies, spectral finite element, wave propagation, elementary rod...*

1. INTRODUCTION

In the last twenty years, wavelets are being effectively used for signal processing and solution of differential equations. Several studies have been developed in this area (Amaratunga and Williams, 1993; Dahmen, 2001). The application of wavelets in mechanics can be understood from two ways, first, the analysis of mechanical responses for extraction of modal parameters, damage measures, de-noising, etc., and second, the solution of the differential equations governing the mechanical system. Though the wavelet transform has been proved to be very effective in all the above cases, there is no comprehensive text on the use of wavelet transform for solution of mechanics problems. The fact that Daubechies wavelets (Daubechis, 1992) possess several appealing properties such as orthogonality, compact support, exact representation of polynomials up to a certain degree, and an ability to represent functions at different resolutions, has resulted in the extensive use of wavelets in signal processing (Williams and Amaratunga, 1997), electromagnetics (Fujii and Hoefer, 2003), and computational fluid and solid mechanics (Qian and Weiss, 1993). Among these applications, the use of Daubechies compactly-supported wavelets for spectral analysis of elastic wave propagation was suggested and has mainly been pursued by (Mitra and Gopalakrishnan, 2005). This approach is referred to as conventional wavelet-based spectral finite element method in this paper.

2. DAUBECHIES COMPACTLY SUPPORTED WAVELETS

In this section a brief review of orthogonal basis of Daubechies wavelets is made. (Daubechis, 1998, 1992).The Wavelets, $\psi_{j,k}(t)$ form compactly supported orthonormal bases for $\mathbf{L^2(R)}$. The wavelets and associated scaling functions $\varphi_{j,k}(t)$ are obtained by translation and dilation of single functions $\psi(t)$ and $\varphi(t)$ respectively.

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \quad j,k \in \mathbf{Z} \tag{1}$$

$$\varphi_{j,k}(t) = 2^{j/2}\varphi(2^j t - k), \quad j,k \in \mathbf{Z} \tag{2}$$

The scaling function $\varphi(t)$ is obtained from the dilation or scaling equation,

$$\varphi(t) = \sum_k a_k \varphi(2t - k) \tag{3}$$

and the wavelet function $\psi(t)$ is found as

$$\psi(t) = \sum_k (-1)^k a_{1-k} \varphi(2t - k) \tag{4}$$

where a_k are the filter coefficients and they are fixed for specific wavelet or scaling function basis. For compactly supported wavelets only a finite number of a_k are nonzero. It is necessary to impose some restrictions on the scaling function to obtain the filter coefficients a_k. The restrictions are (1) - The area under scaling function is normalized to one, Eq. (5), (2) -The scaling function $\varphi(t)$ and its translates are orthonormal, Eq. (6), (3) wavelet function $\psi(t)$ has M vanishing moments in Eq. (7).

$$\int_{-\infty}^{\infty} \varphi(t)dt = 1 \tag{5}$$

$$\int_{-\infty}^{\infty} \varphi(t)\varphi(t+k)dt = \delta_{0,k}, \quad k \in \mathbf{Z} \tag{6}$$

$$\int_{-\infty}^{\infty} \psi(t)t^m dt = 0, \quad m = 0, ...M \tag{7}$$

The number of vanishing moments M denotes the order N of the Daubechies wavelet, where $N = 2M$. The translates of the scaling and wavelet functions on each fixed scale j form orthogonal subspaces,

$$V_j = 2^{j/2}\varphi(2^j t - k); \quad j \in \mathbf{Z} \tag{8}$$

$$W_j = 2^{j/2}\psi(2^j t - k); \quad j \in \mathbf{Z} \tag{9}$$

such that V_j forms a sequence of embedded subspaces

$$\{0\}, \dots, \subset V_{-1}, \subset V_0, \subset V_1, \dots, \subset \mathbf{L^2}(\mathbf{R}) \tag{10}$$

Further details of wavelet formulation is in (Daubechis, 1998, 1992).

3. REDUCTION OF WAVE EQUATIONS TO ODES

3.1 Longitudinal wave equation for rods

According to (Doyle, 1989) the governing differential wave equation of an isotropic rod is given as

$$EA\frac{\partial^2 u}{\partial x^2} - \eta A\frac{\partial u}{\partial t} = \rho A\frac{\partial^2 u}{\partial t^2} \tag{11}$$

where, E, A, η and ρ are the Youngs modulus, cross sectional area, damping ratio and density respectively. $u(x,t)$ is the axial deformation. Let $u(x,t)$ be discretized at n points in the time window $[0 \quad t_f]$. Let $\tau = 0, 1, ..., n-1$ be the sampling points, then

$$t = \Delta t\tau \tag{12}$$

where, Δt is the time interval between two sampling points. The function $u(x,t)$ can be approximated by scaling function $\varphi(\tau)$ at an arbitrary scale as

$$u(x,t) = u(x,\tau) = \sum_k u_k(x)\varphi(\tau - k), \quad k \in \mathbf{Z} \tag{13}$$

where,u_k are the approximation coefficient at a certain spatial dimension x. Substituting Eq. (12) and Eq. (13) in Eq. (11) we get,

$$EA\sum_k \frac{\mathrm{d}^2 u_k}{\mathrm{dx}^2}\varphi(\tau - k) - \frac{\eta A}{\Delta t}\sum_k u_k\varphi'(\tau - k) = \frac{\rho A}{\Delta t^2}\sum_k u_k\varphi''(\tau - k) \tag{14}$$

Taking inner product on both sides of Eq. (14) with $\varphi(\tau - j)$, where $j = 0, 1, ..., n-1$ we get

$$EA\sum_k \frac{\mathrm{d}^2 u_k}{\mathrm{dx}^2}\int \varphi(\tau-k)\varphi(\tau-j)d\tau - \frac{\eta A}{\Delta t}\sum_k u_k\int \varphi'(\tau-k)\varphi(\tau-j)d\tau = \frac{\rho A}{\Delta t^2}\sum_k u_k\int \varphi''(\tau-k)\varphi(\tau-j)d\tau \tag{15}$$

The translates of scaling functions are orthogonal i.e.

$$\int \varphi(\tau - k)\varphi(\tau - j)d\tau = 0 \quad for \quad j \neq k \tag{16}$$

Using Eq. (16), the Eq. (15) can be written as n simultaneous ODEs

$$EA\frac{\mathrm{d}^2 u_j}{\mathrm{dx}^2} - \frac{\eta A}{\Delta t}\sum_{k=j-N+2}^{j+N-2}\Omega^1_{j-k}u_j = \frac{\rho A}{\Delta t^2}\sum_{k=j-N+2}^{j+N-2}\Omega^2_{j-k}u_k \quad j = 0, 1, ..., n-1 \tag{17}$$

$$EA\frac{\mathrm{d}^2 u_j}{\mathrm{dx}^2} = \sum_{k=j-N+2}^{j+N-2}\left(\frac{\eta A}{\Delta t}\Omega^1_{j-k} + \frac{\rho A}{\Delta t^2}\Omega^2_{j-k}\right)u_k \quad j = 0, 1, ..., n-1 \tag{18}$$

where, N is the order of the Daubechies wavelet as discussed earlier. Ω^1_{j-k} and Ω^2_{j-k} are the connection coefficients defined as

$$\Omega^1_{j-k} = \int \varphi'(\tau - k)\varphi(\tau - j)d\tau \tag{19}$$

$$\Omega^2_{j-k} = \int \varphi''(\tau - k)\varphi(\tau - j)d\tau \tag{20}$$

For compactly supported wavelets, Ω^1_{j-k} and Ω^2_{j-k} are nonzero only in the interval $k = j - N + 2$ to $k = j + N - 2$. The details for evaluation of connection coefficients for different orders of derivative are given by (Beylkin, 1992). The forced boundary condition associated with the governing differential given by Eq. (11) is

$$F = AE\frac{\partial u}{\partial x} \tag{21}$$

where, $F(x,t)$ is the axial force applied. $F(x,t)$ can be approximated similarly as $u(x,t)$ in Eq. (13)

$$F(x,t) = F(x,\tau) = \sum_k F_k(x)\varphi(\tau - k), \quad k \in \mathbf{Z} \tag{22}$$

Substituting Eq. (13) and Eq. (22) in Eq. (21) and taking the inner product with $\varphi(\tau - j)$ we get,

$$F_j = AE\frac{\mathrm{du_j}}{\mathrm{d}x} \quad j = 0, 1, ..., n-1 \tag{23}$$

4. BOUNDARY VALUE PROBLEM

4.1 Periodic boundary condition

In this approach, the function $u(x,t)$ is assumed to be periodic in time. Considering the discretized $u(x,\tau)$ to be periodic with time period t_f in the Eq. (18) , the unknown coefficients u_j on LHS are taken as

$$u_{-1} = u_{n-1}, \quad u_{-2} = u_{n-2}, ... \quad u_{-N+2} = u_{n-N+2} \tag{24}$$

Similarly the unknown coefficients on RHS, i.e., u_n, u_{n+1},...,u_{n+N-2} are equal to u_0, u_1,...,u_{N-2} respectively. With the above assumption, the coupled ODEs given by Eq. (18) can be written in matrix form as

$$\left\{\frac{\mathrm{d}^2\mathrm{u_j}}{\mathrm{d}x^2}\right\} = \left(\frac{\eta A}{AE}\Gamma^1 + \frac{\rho A}{AE}\Gamma^2\right)\{u_j\} \tag{25}$$

where, Γ^1 and Γ^2 are n x n circulant connection coefficient matrices. For a circulant matrix Γ^1 (Davis, 1975), the eigenvalues are

$$\alpha_j = \sum_{k=-N+2}^{N-2}\Omega^1_k e^{-2\pi ijk/n} \qquad j = 0, 1, ..., n-1 \tag{26}$$

and the corresponding orthonormal eigenvectors v_j, $j = 0, 1, ..., n-1$ are

$$(v_j)_k = \frac{1}{\sqrt{n}}e^{-2\pi ijk/n}, \qquad k = 0, 1, ..., n-1 \tag{27}$$

for, Γ^1, $\Omega^1_p = -\Omega^1_{-p}$ for $p = 1, 2, ..., N-2$ and $\Omega^1_0 = 0$ and we can write $\alpha_j = i\lambda_j$ where

$$\lambda_j = -\frac{2}{\Delta t}\sum_{k=1}^{N-2}\Omega^1_k \sin\left[\frac{2\pi kj}{n}\right] \qquad j = 0, 1, ..., n-1 \tag{28}$$

the second order connection coefficient matrix Γ^2 can be evaluated independently (Beylkin, 1992), they can also be writte as

$$\Gamma^2 = [\Gamma^1]^2 \tag{29}$$

The spectral element formulation , involves eigenvalue analysis. This is done to diagonalize the matrix in Eq. (25) and decouple the ODEs. For periodic boundary condition, these eigenvalues are known analytically and hence decreases the computational cost. However, the solutions obtained are same as those obtained using FSFEM and possess several problems such as wrap around due to assumed periodicity of the solution.

4.2 Non-periodic boundary condition

In this section the non-periodic solution of the boundaries are treated using wavelet extrapolation method for Daubechies compactly supported wavelets. The detail of the formulation is given in (Amaratunga and Williams, 1995; Williams and Amaratunga, 1997).

5. FREQUENCY DOMAIN ANALYSIS

For periodic solution, the wavelet transformation given by Eq. (13) can be written as matrix equation where U_j, φ_j are the values of $u(x,\tau)$ and $\varphi(\tau)$ at $\tau = j$. For such circulant matrix, (Mitra and Gopalakrishnan, 2005) can be replaced by a convolution relation which can be written as

$$\left\{\tilde{U}_j\right\} = \left\{\tilde{K}_{\varphi j}\tilde{u}_j\right\} \tag{30}$$

where $\left\{\tilde{U}_j\right\}$, $\{\tilde{u}_j\}$ are FFT of $\{U_j\}$ and $\{u_j\}$ respectively. $\tilde{K}_{\varphi j}$ is FFT of $K_\varphi = 0, \varphi_1, \varphi_2, ..., \varphi_{N-2}...0.$ which is the first column of the scaling function matrix. Similarly, in Eq. (25) the matrix Γ^1 is also a circulant matrix and thus it can be written as

$$\left\{\frac{\mathrm{d}^2\tilde{\mathrm{u}}_\mathrm{j}}{\mathrm{d}x^2}\right\} = \left(\frac{\rho A}{AE}\right)\left\{\tilde{K}^2_{\Omega j}\tilde{u}_j\right\}. \tag{31}$$

where $\tilde{K}_{\Omega j}$ are the FFT of $K_\Omega = \Omega^1_0, \Omega^1_{-1}, ...\Omega^1_{-N+2}, ..., \Omega^1_{N-2}, ..\Omega^1_1$ which is the first column of the connection coefficient matrix Γ^1:
Substituting Eq. (30) in Eq. (31) we get

$$\left\{\frac{\mathrm{d}^2\tilde{\mathrm{U}}_\mathrm{j}}{\mathrm{d}x^2}\right\} = \left(\frac{\rho A}{AE}\right)\left\{\tilde{K}^2_{\Omega j}\tilde{U}_j\right\}. \tag{32}$$

It can be easily seen that the FFT coefficients $\tilde{K}_{\Omega j}$ are equal to the eigenvalues $i\lambda_j$ of the matrix Γ^1 given by Eq. (28). Thus Eq. (32) can be written as

$$\left\{\frac{\mathrm{d}^2\tilde{\mathrm{U}}_\mathrm{j}}{\mathrm{d}x^2}\right\} = \left(\frac{\rho A}{AE}[\lambda_j]^2\right)\left\{\tilde{U}_j\right\} \qquad j = 0, 1, ..., n-1. \tag{33}$$

It should be mentioned here that, by relating the Eq. (33), it can be observed that the transformation given by Eq. (37) is similar to FFT for periodic WSFE formulation. In FSFEM, the transformed ODEs are of the form

$$\left\{\frac{\mathrm{d}^2\tilde{\mathrm{U}}_\mathrm{j}}{\mathrm{d}x^2}\right\} = \left(\frac{\rho A}{AE}[\omega_j]^2\right)\left\{\tilde{U}_j\right\} \qquad j = 0, 1, ..., n-1. \tag{34}$$

where

$$\omega_j = \frac{2\pi j}{n\Delta t} \tag{35}$$

6. SPECTRAL ELEMENT FORMULATION

6.1 Spectral element for rod

In Figure 1 shows the spectral rod element with two nodes and one longitudinal degree of freedom (dof) $\hat{u}_j$ and nodal axial load $\hat{F}_j$ at each node.

Figure 1. Spectral element for rod with nodal forces and displacements.

The formulation of 2x2 elemental dynamic stiffness matrix is explained as follows. The ODEs obtained by decoupling the Eq. (25) can be written as

$$\frac{\mathrm{d}^2\hat{\mathrm{u}}_\mathrm{j}}{\mathrm{d}x^2} = \left(\frac{\eta A}{AE}\lambda_j + \frac{\rho A}{AE}\lambda_j^2\right)\{\hat{u}_j\} \quad j = 0, 1, ..., n-1 \tag{36}$$

where

$$\hat{u}_j = \Phi^{-1}u_j \tag{37}$$

Similarly, the force boundary condition given by Eq. (23) can be written as

$$\hat{F}_j = \frac{\mathrm{d}\hat{\mathrm{u}}_\mathrm{j}}{\mathrm{d}x} \quad j = 0, 1, ..., n-1 \tag{38}$$

where

$$\hat{F}_j = \Phi^{-1}F_j \tag{39}$$

The exact interpolating function obtained by solving Eq. (36) are

$$\hat{u}_j(x) = A_j e^{-ikx} + B_j e^{-ik(L-x)} \tag{40}$$

where, k is the wavenumber corresponding to the axial mode

$$k = \left(\frac{\eta A}{AE}\lambda_j + \frac{\rho A}{AE}\lambda_j^2\right)^{\frac{1}{2}} \tag{41}$$

According to (Doyle, 1989), we have that $[\hat{K}^e]$ is the elemental dynamic stiffness matrix. It is numerically solved for $j = 0, 1, ...N-1$, the nodal displacements are found,$\{\hat{u}^e\}$, given the nodal forces, $\left\{\hat{F}^e\right\}$,, and the boundary conditions imposed correctly. The equation that relates the nodal displacements with the nodal forces is,

$$\left\{\hat{F}^e\right\} = [\hat{K}^e]\{\hat{u}^e\} \tag{42}$$

7. NUMERICAL EXPERIMENTS

The results were obtained using the MATLAB 2014, note in Figure 2 that for a given sampling rate Δt, λ_j exactly matches ω_j up to a certain fraction of Nyquist frequency $f_{nyq} = \frac{1}{2\Delta t}$, Thus similar to FSFE, WSFE can be used directly for studying frequency-dependent characteristics like spectrum and dispersion relation but up to a certain fraction of f_{nyq}: This fraction is dependent on the order of basis and is more for higher-order basis.

Through the method they were obtained WSFEM spectrum dimensional relationships for a waveguide rod element, with $\Delta t = 1\mu s$, $(f_{nyq} = 500kHz)$, and $\Delta t = 2\mu s$, $(f_{nyq} = 250kHz)$ are shown in Figure 3 and Figure 4 , respectively. The comparison with FSFEM results in each case, shows that the non-dimensional wavenumber $k1h$ obtained using periodic WSFEM is exact up to a fraction p_N of f_{nyq}.That the above mentioned fraction p_N varies only with order of basis N and is independent of the problem. For $N = 22$, $p_N \approx 0,6$, while for $N = 6$, $p_N \approx 0,36$. Thus, for $\Delta t = 1\mu s$, wave characteristics can be obtained for a frequency range of $(f_{nyq} = 500kHz)$ using FSFEM and for a frequency range of $(f_{nyq} = 300kHz)$ using WSFEM with $N = 22$.

The rod structure is excited by a sinusoidal force type tone burst with five cycles and frequency of 60 kHz as shown in Figure 5, the Figure 6 shows the input signal in the frequency domain, the rod has length 5 m, modeled with thicknes $0,02$ m,width $0,02$ m, where in the area is $A = tw$, modulus of elasticity 210 GPa and density 7850 kg/m^3. From

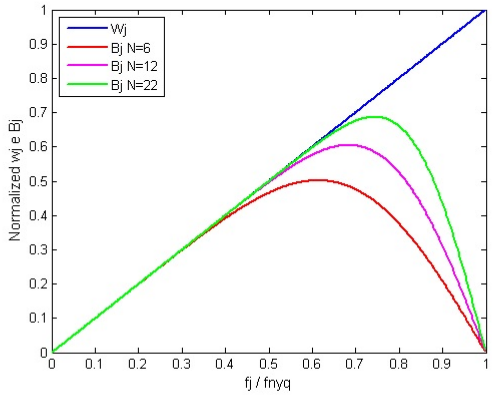

Figure 2. Comparison of ω_j and β_j for different order (N) of basis

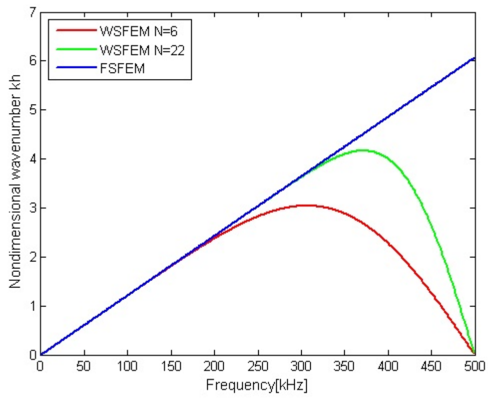

Figure 3. Wave number k_1h for sampling rate $\Delta t = 1\mu$s.

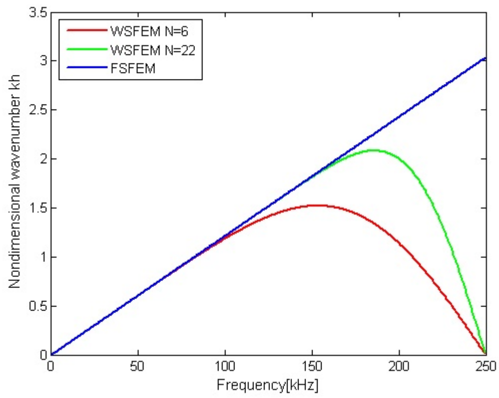

Figure 4. Wave number k_1h for sampling rate $\Delta t = 2\mu$s.

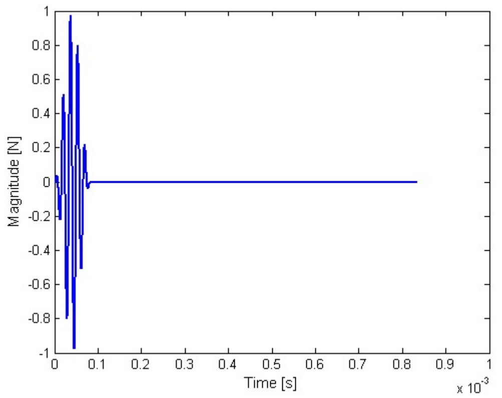

Figure 5. Input signal in time.

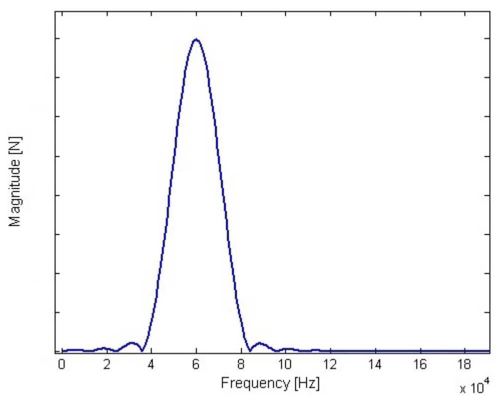

Figure 6. Input signal in frequency.

the excitation and dynamic stiffness matrix it was possible to obtain the accelerations and displacements in frequency and then in time domain. From the found shifts and accelerations, it is possible to find through the functions form the values anywhere on the bar.

The Figure 7 and Figure 8 illustrates the accelerations obtained at the nodes 1 and 2, of the studied element, there is some convergence of the WSFEM and FSFEM methods, studied in this article.

In Figure 7 is observed acceleration in the excitation node (Node 1), it is clear that there is a reflection on node 2, between 0,3 μs and 0,4μs. In the Figure 8 the accelerations obtained is compared for the two nodes, obtained by the method FSFEM, was inserted in estruturra one hysteretic damping coefficient, the initial excitement will have a loss of amplitude as it propagates the rod.

Already in Figure 7 and Figure 8 are shown the accelerations in the nodes 1 and 2, respectively, also notes there was some similarity between the WSFEM and FSFEM methods

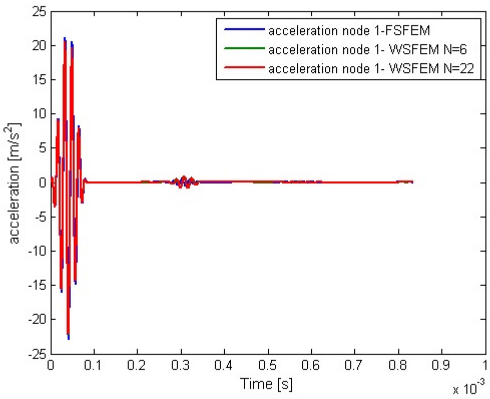

Figure 7. aceleration node 1.

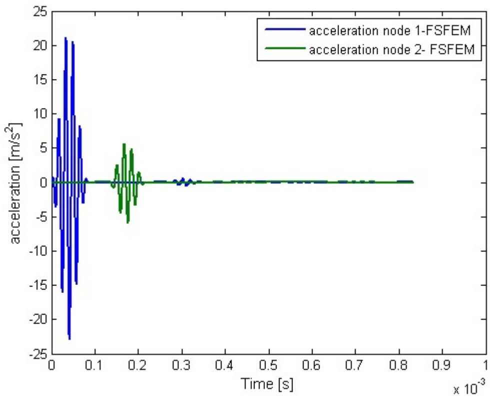

Figure 8. aceleration node 1 and node 2.

In the Figure 9 is plotted frequency response function (FRF) for the analyzed structure, results were compared for different wavelet order Daubechies base, as used in the case the orders $N = 6$ and $N = 22$, where FRF for $N = 22$ had values similar to FSFEM method for an analyzed frequency band where the frequency band is equal to the fraction found at the p_n calculating the wave number obtained previously.

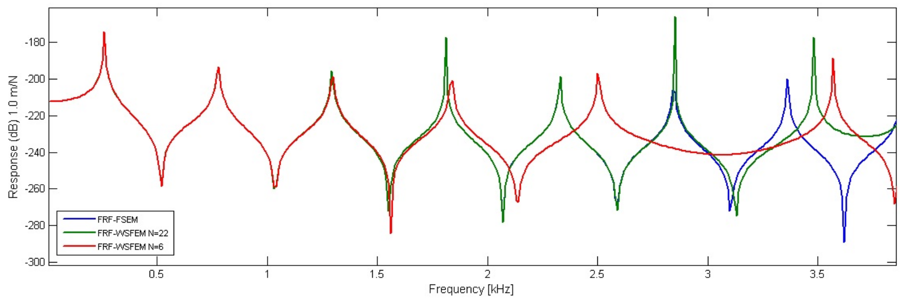

Figure 9. FRF for rod.

8. CONCLUSIONS

The novelty of the spectral element developed in this paper is that it uses wavelet transform to reduce the PDEs to ODEs, a correspondence is established between FSFEM and WSFEM for determining the wave characteristics, that is the spectrum and dispersion relation for a given waveguide. It is found that the WSFEM predicts exact wave behaviour up to a threshold frequency which is a fraction of the Nyquist frequency. The sampling time rate required in WSFEM can be a priori determined depending on the excitation frequency and order of bases to avoid the introduction of spurious modes in the analysis.The wavelet-based spectral formulation for elastic wave propagation in a one dimensional structure problem was demonstrated with sucess.This wavelet based approach retains all the advantages of FSFEM, while it removes the problems associated with Fourier transform.The results were compatible with the results found in the literature, WSFEM method becomes an alternative for the calculation of wave propagation in structures, but its disadvantage is the mathematical formulation that is not trivial compared with other methods, which makes its implementation is not so easy.

9. ACKNOWLEDGEMENTS

The authors are thankful to Unicamp and to CNPq/CAPES for funding this research.

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