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# STOCHASTIC ANALYSIS OF NONLINEAR MECHANICAL SYSTEM

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**Abstract:** *The majority of engineering problems and other areas are subject to certain types of uncertainties, and these affect the responses. The study about the stochastic methods have gained featured in recent years, with the intention of obtain answers that are more reliable. Among the existing techniques, to consider the uncertainties on problems, stand out the Monte Carlo method and Latin Hypercube method. It is worth mentioning that, the Latin Hypercube method is a variant of the Monte Carlo method and shows itself efficient for obtaining solution, even with a large number of random variables. Thus, this method has the advantage of reducing the required number of simulations to the obtainment of the results, having a smaller computational cost, when compared with the Monte Carlo. For this reason, this work has for objective the application of the Latin Hypercube method to a system with Nonlinear Dynamic Vibration Absorber and analyze the results. For this, initially will be described about the functioning of Latin Hypercube method. The following, is modeled a system of two degrees of freedom and presents a study of their behavior through the application of Fourth Order Runge-Kutta Method. Finally, with the application of the Latin Hypercube method, value ranges (Minimum, Medium and Maximum) of problem are obtained and analyzed, highlighting that, as closer is the minimum range of the maximum, the better the result.*

***Keywords:*** *Uncertainty, Monte Carlo Method, Latin Hypercube Method, Mechanical Systems*

## 1. INTRODUCTION

The phenomena that occur in nature or in human activities contain a certain inherent uncertainty, that is, such phenomena cannot be predicted with absolute precision and can be described through the words as uncertainty, randomness and stochasticity (Krüger, 2008).

The stochasticity can be related with errors in mathematical modeling, or variations of the physical parameters of the system or the excitation forces to which the system is subject and these can directly influence the response of a given system (Sampaio and Ritto, 2008).

With the advent of scientific computing, stochastic methods have stood out between scholars, seeking ever more precision in dynamic response of a system. These probabilistic methods have been widely used as a tool to obtain best configurations for engineering projects (Sampaio and Cataldo, 2010).

Another tool of great interest in engineering are called Dynamic Vibration Absorbers (DVAs). Due the fact that reduce the vibration levels in the response of a system is of great importance to have an efficient and reliable project. Thus, using a DVA can be obtained projects that, assures satisfactory conditions of performance and security, in this way, there is a decrease vibration level, since, the DVA is a device that when considered in structures, machines, among others equipment, it has the attenuation in amplitude of vibration (Fernandes, 2008).

The DVA consists in a mass coupled to a system by means of a spring and a damper, this has the purpose of absorbing part or all of the system vibration, i.e., it is tuned to vibrate at high amplitudes, absorbing the vibrational energy in the coupling point (Koronev and Reknikov, 1993).

The DVA may have linear or nonlinear characteristics. However, in recent years, there is growing interest in studying the DVA with nonlinear characteristics, due to its improved robustness, when compared with the linear absorber, based on the fact of linear DVA operate satisfactorily only in its tuning frequency (Borges, 2008).

In this work, initially describes the functioning of the Latin Hypercube method (HCL).Then is written the equations of motion of a vibrating system 2 GDL, consisting of a primary and a secondary system. Later, is obtained the behavior of the problem in question, using the numerical method known as Fourth Order Runge-Kutta (RK4).

Once, obtained the responses by RK4, is applied the HCL method considering three cases: First, a linear main system with linear absorber, after, linear main system with nonlinear absorber and finally, nonlinear system with nonlinear absorber. For the cases was considered as uncertain the stiffness parameters , for the purpose of verify the effects of uncertainty in both linear and nonlinear parameters, and analyzing those uncertainties still considering the system and absorber also as linear and nonlinear.

## 2. STOCHASTIC METHODS

The aleatory generation, also, known as random is defined as a simulation that has the numbers generation by raffle. The methods to realize this kind of generation are known as stochastic (Bogoni, 2009).

The stochastic methods are being widely used as a tool for consideration of randomness in various kinds of engineering problems. To perform stochastic simulation must consider one or more variables such as random, and may be, for example, some mechanical property of the material, thus, may be obtained best configurations for the problem in question (Sampaio and Ritto, 2008).

Among the stochastic techniques, there is the Monte Carlo Method (MC) and the Latin Hypercube Method (LHS) that is a simpler version of MC. These probabilistic methods perform simulations in that samples are generated for one or more random variables.

In 1949 appear the work entitled "The Monte Carlo Method", the article was written by Stanislaw Ulam and Metropolis, where previously already if known some concepts about the method, however, your employment in fact happened with the advent of computers, by it be a numerical technique (Bogoni, 2009).

This method searches a solution to a problem that is obtained through evaluation of random samples, being known as a stochastic technique used to consider variables as uncertain, it has wide applicability to various problems but, has the disadvantage of requiring several simulations for find the solution, thus, it has a high computational cost (Haldar and Mahadevan, 2000).

The LHS method is efficient to obtain the solution even with a large number of random variables. Thus, this method has the advantage of reducing the number of required simulations to obtain the results in comparison with MC. For this reason, in this paper, we used the LHS implemented in MATLAB® software.

The following describes the functioning of the Latin Hypercube Method (Thom *et al*., 2010):
Wish to be simulated values of a function $y = f(x_1, x_2, ..., x_k)$ where $f$ is a deterministic function and $x_1, x_2, ..., x_k$ are random variables. Therefore, we have the following steps:

1. Division of the interval of each $x_i$ at $n_i$ intervals;
2. For each variable $x_i$ and their $n_i$ intervals, randomly selects a value; Existing $n_i$ values for each random variable $k$;
3. Select $n_i$ where each value will appear once in $n_i$ combinations, for the first combination, randomly selects one of the values for each of the $k$ random input variables; For second combination is selected one of the values;
4. Evaluate the function for each $n_i$ combination, by generating $n_i$ values for function.

The LHS divides the intervals of possible values for each variable in tracks, randomly raffling a value that, is considered only once in the simulation process, this is, a sampling technique, which stratifies the probability distribution in n strata or parts performing random raffling (Fonte, 2011).

## 3. APPLICATIONS

Consider the vibration system the of two-degree-of-freedom, composed of the main system $(m_1, k_1, c_1)$ connected to the subsystem $(m_2, k_2, c_2)$, called Dynamic Vibration Absorber (DVA), whose model is shown in Figure 1.

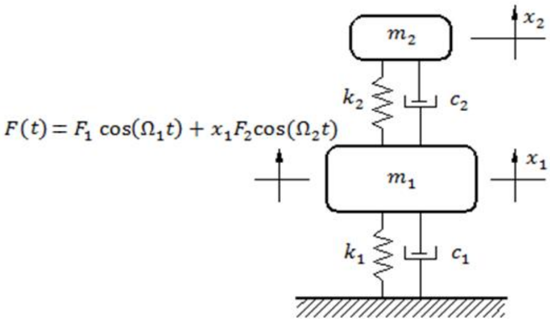


Figure 1. System with Nonlinear Dynamic Vibration Absorber

The equations of motion for the system are:

$$m_1\ddot{x}_1 + k_{11}x_1 + k_{12}x_1^2 + k_{13}x_1^3 + c_1\dot{x}_1 + c_2\dot{x}_1x_1^2 + c_3(\dot{x}_1 - \dot{x}_2) + k_{21}(x_1 - x_2) + k_{22}(x_1 - x_2)^2 + k_{23}(x_1 - x_2)^3$$
$$= f_1cos(\Omega_1 t) + x_1 f_2 cos(\Omega_2 t) \qquad (1)$$
$$m_2\ddot{x}_2 + k_{21}(x_2 - x_1) + k_{22}(x_2 - x_1)^2 + k_{23}(x_2 - x_1)^3 + c_3(\dot{x}_2 - \dot{x}_1) = 0$$

This vibrating system consists of springs with non-linear characteristics and it has damping. The main mass is excited by a force of kind $F(t)$, and $\Omega_j (j = 1,2)$ are excitation frequencies, $x_j$ are displacements of the main and secondary system, respectively, $\dot{x}_j$ and $\ddot{x}_j$ are derived from $x_j$ and $t$ the time. Values for the parameters are presented in the Tab. 1

Table 1. Parameter values of the system.

| Symbol | Variable | Value | Unit |
|---|---|---|---|
| $m_1$ | Mass of the main system | 10 | $kg$ |
| $m_2$ | Mass of the absorber | 0.8 | $kg$ |
| $k_{11}$ | Linear stiffness | 30.35 | $N/m$ |
| $k_{12}$ | Nonlinear stiffness | 0.3 | $N/m^3$ |
| $k_{13}$ | Nonlinear stiffness | 0.4 | $N/m^3$ |
| $k_{21}$ | Linear stiffness | 2.048 | $N/m$ |
| $k_{22}$ | Nonlinear stiffness | 0.1 | $N/m^3$ |
| $k_{23}$ | Nonlinear stiffness | 0.2 | $N/m^3$ |
| $c_1$ | Damping coefficient of the main system | 0.5 | $Ns/m$ |
| $c_2$ | Damping coefficients of the absorber | 0.2 | $Ns/m$ |
| $c_3$ | Coupling damping coefficient | 0.1 | $Ns/m$ |
| $f_1$ | Forcing amplitude | 6 | $N$ |
| $f_2$ | Forcing amplitude | 3 | $N$ |

From algebraic manipulations in Eq. (2) and assuming nonlinearity weak and small damping, we obtain:

$$\ddot{x}_1 + \omega_1^2 x_1 + \varepsilon\alpha_1 x_1^2 + \varepsilon\alpha_2 x_1^3 + \varepsilon\xi_1\dot{x}_1 + \varepsilon\xi_2\dot{x}_1x_1^2 + \varepsilon\xi_3(\dot{x}_1 - \dot{x}_2) - \varepsilon\alpha_3 x_2 + \varepsilon\alpha_4(x_1 - x_2)^2 + \varepsilon\alpha_5(x_1 - x_2)^3$$
$$= \varepsilon F_1 cos(\Omega_1 t) + \varepsilon x_1 F_2 cos(\Omega_2 t) \qquad (2)$$
$$\ddot{x}_2 + \omega_2^2(x_2 - x_1) + \varepsilon\beta_1(x_2 - x_1)^2 + \varepsilon\beta_2(x_2 - x_1)^3 + \varepsilon\xi_4(\dot{x}_2 - \dot{x}_1) = 0$$

where:

$$\omega_1^2 = \frac{k_{11} + k_{21}}{m_1}, \omega_2^2 = \frac{k_{21}}{m_2}, \alpha_1 = \frac{k_{12}}{m_1}, \alpha_2 = \frac{k_{13}}{m_1}, \alpha_3 = \frac{k_{21}}{m_1}, \alpha_4 = \frac{k_{22}}{m_1}, \alpha_5 = \frac{k_{23}}{m_1}, \xi_1 = \frac{c_1}{m_1}, \xi_2 = \frac{c_2}{m_1}, \xi_3 = \frac{c_3}{m_1},$$
$$\xi_4 = \frac{c_3}{m_2}, \beta_1 = \frac{k_{22}}{m_2}, \beta_2 = \frac{k_{23}}{m_2}, F_1 = \frac{f_1}{m_1}, F_2 = \frac{f_2}{m_1}. \qquad (3)$$

and $\varepsilon$ introduced to show weakness of a parameter.

The behavior of the main system and absorber will be analyzed through the numerical solution obtained by integration of Eq. (2) held by implementing the fourth order Runge-Kutta method in MATLAB® software (Yang *et al.*, 2005).

### 3.1 Numerical results and discussion

The figures 2 - 4 show the responses in the time domain and phase plans for the following cases:

- Linear main system with linear absorber, eliminating the nonlinear parameters, from both systems;
- linear main system with nonlinear absorber, eliminating the nonlinear parameters the main system and
- Nonlinear main system with nonlinear absorber.

In all cases it is considered simultaneous resonance, given by combining the primary resonance and parametric internal, i.e., $\Omega_1 \cong \omega_1, \Omega_2 \cong 2\omega_1$ and $\omega_2 \cong 3\omega_1$. The values of the parameters used are listed in Tab. 1, presented in Section 3.

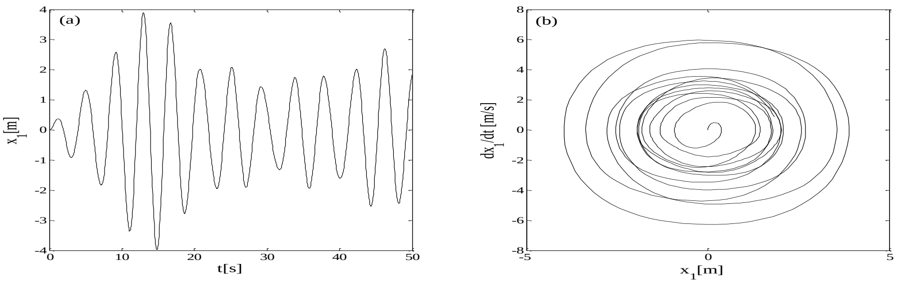


Figure 2. (a) Response with linear main system with linear absorber and (b) phase plane

In Figure 2 (a) we observe the response in the time domain and, Fig. 3 (b), the phase plane of linear primary system with linear absorber. In them, it is observed the presence of limit cycles and chaotic behavior.

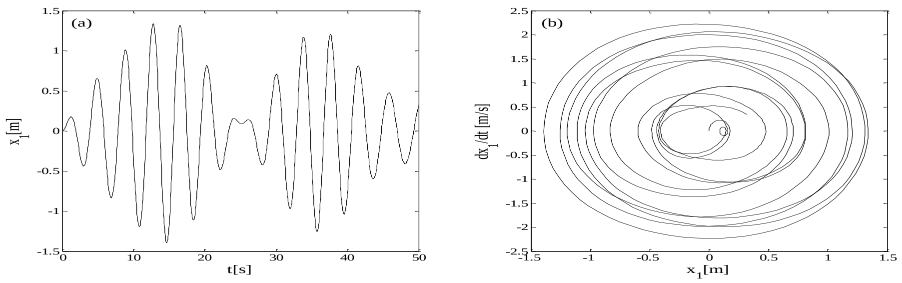


Figure 3. (a) Response of the linear system with nonlinear absorber and (b) phase plane at simultaneous resonance

The Figure 3 (a), show the response in the time domain linear main system with nonlinear absorber, exhibits low amplitude oscillations around the zero point, which are regular and irregular. Can be visually confirmed in the Fig. 3 (b), the phase plane has regular oscillations and they are represented by cycles limits that are intermittently interrupted by irregular intervals. Also, the chaotic wave motion will increase.

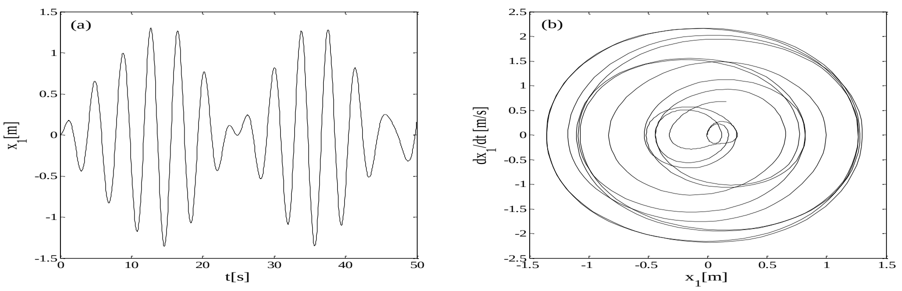


Figure 4. (a) Response of the nonlinear system with nonlinear absorber and (b) phase plane at simultaneous resonance

The Figure 4 (a) shows the response in the time domain and, Fig. 4 (b), phase plane of nonlinear main system with nonlinear absorber. Observed that this system also presents limit cycles and dynamic chaos.

Also observed that, in cases where input to the nonlinear absorber there was a significant attenuation of the amplitude response of the system in question, however, is increasing dynamic chaos (Sayed *et al.*, 2011).

## 4. INFLUENCE OF UNCERTAINTIES

The effects of uncertainty, for the cases cited in subsection 3.1, Figs. 7 - 9 are constructed by applying uncertainty (s) in (s) parameter (s) of Eq. (2), through the Latin Hypercube method. Based on the obtained solutions, we analyze the value ranges (minimum, average and maximum) in order to, check the effect of the uncertainty of the problem in question. For such simulations can be considered one or more parameters of the problem as random and has that consider a percentage of uncertainty.

### 4.1 Linear main system with linear absorber

The Figure 5 (a) shows the effect of the uncertainty in the linear stiffness parameter of the main system, $k_{11}$, and Fig. 5 (b) in the linear stiffness parameter of the absorber, $k_{21}$, and Fig. 5 (c) in the parameters $k_{11}$ and $k_{21}$, simultaneously, considering 10% of uncertainty in the displacement response $x_1$.

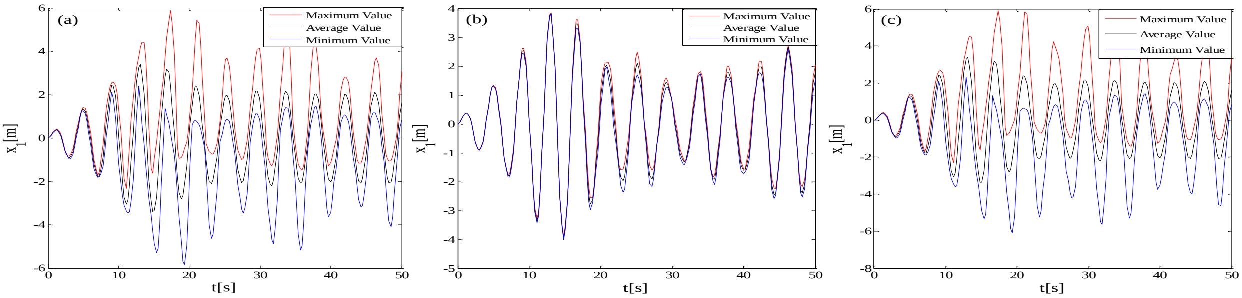


Figure 5. Effect of uncertainty for linear main system with linear absorber considering 10% uncertainty, (a) $k_{11}$, (b) $k_{21}$, (c) $k_{11}$ and $k_{21}$

In Figure 5 (a) is observed that even a small percentage of uncertainty of 10%, the value ranges (maximum, average and minimum) move away showing that the linear stiffness parameter of the main system $k_{11}$ is very sensitive the consideration of uncertainty. This does not apply for the linear stiffness parameter of the absorber $k_{21}$, Fig. 5 (b), where the value ranges are very close. Note that we consider the uncertainty, in this work, only the response of the main system, i. e., in the displacement $x_1$.

In Figure 5 (c) is considered the linear stiffness parameter of the main system as random, $k_{11}$, and of the absorber, $k_{21}$, simultaneously, for 10% of uncertainty in the displacement response $x_1$. The intention is to analyze the effect of uncertainty in the linear stiffness parameters of the system in question. Thus, it was observed that the value ranges keeps the evident distancing, as shown in Fig. 5 (a).

### 4.2 Linear main system with nonlinear absorber.

The Figure 6 (a) shows the effect of the uncertainty in the linear stiffness parameter of the secondary system, $k_{21}$. Fig. 6 (b) nonlinear stiffness parameters are considered, $k_{22}$ and $k_{23}$, simultaneously, considering 30% of uncertainty in the displacement response $x_1$.

In Fig. 6 (c) was considered the linear stiffness parameter $k_{21}$, and nonlinear, $k_{22}$ and $k_{23}$, of the absorber, simultaneously, with the purpose of analyze the effects of uncertainty in stiffness parameters, expressed in the Eq. (2) as a quadratic and cubic polynomial in the displacement of the absorber mass, $m_2$.

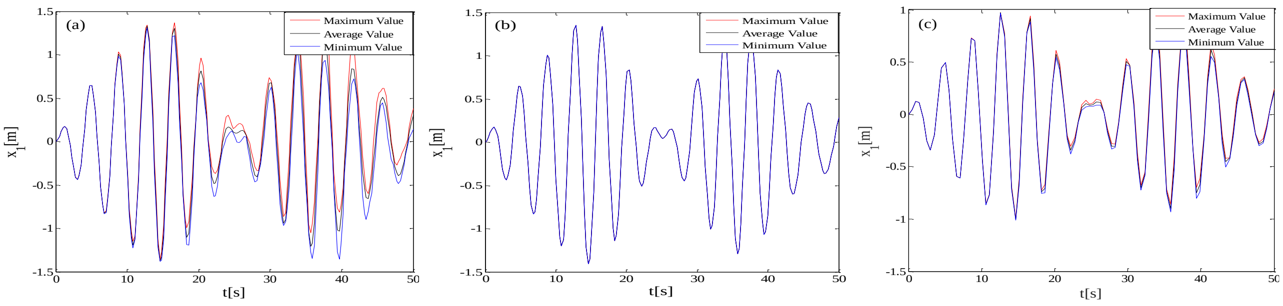


Figure 6. Effect of uncertainty for linear main system with nonlinear absorber considering 30% uncertainty (a) $k_{21}$ , (b) $k_{22}$ and $k_{23}$ , (c) $k_{21}$,$k_{22}$ and $k_{23}$

In Figure 6 (a) was confirmed again, the sensitivity of the linear stiffness parameter to consider uncertainty. In Fig. 6 (b) is verified that, even considering a percentage of high uncertainty, 30%, the nonlinear stiffness parameter of the absorber, there is still a small distance between the ranges of values. In Figure 6 (c) is considered as random, the linear and nonlinear stiffness parameters forming the quadratic and cubic polynomial on the ends of the spring displacement. In this case, also, it has a small distance between range of values (minimum, average and maximum).

### 4.3 Nonlinear main system with nonlinear absorber.

In Figure 7 (a) is considered as random, the linear stiffness parameter, $k_{11}$, and nonlinear $k_{12}$ and $k_{13}$, of main system, simultaneously, with the purpose of analyze the effects of uncertainty in stiffness parameters, expressed in the Eq. (2) as a quadratic and cubic polynomial in the displacement of the main mass, $m_1$.

In Fig. 7 (b) considers the randomness in the nonlinear stiffness parameters of the absorber, $k_{22}$ and $k_{23}$, simultaneously, and in Fig. 7 (c) considered in the linear stiffness parameters $k_{21}$, and nonlinear, $k_{22}$ and $k_{23}$, of the absorber, simultaneously, aiming to analyze the effects of uncertainty in stiffness parameters expressed in the Eq. (2) as a quadratic and cubic polynomial in the relative displacement of the spring ends.

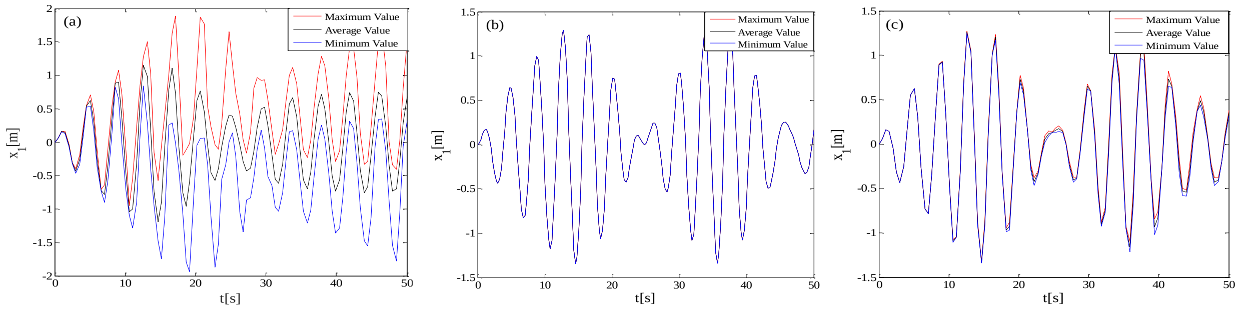


Figure 7. Effect of uncertainty for nonlinear main system with nonlinear absorber considering 10% uncertainty, (a) $k_{11}$,$k_{12}$ and $k_{13}$ , (b) $k_{22}$ and $k_{23}$ , (c) $k_{21}$,$k_{22}$ and $k_{23}$

In these figures, we evaluate the effects of uncertainty in stiffness parameters for the case where the main system and the absorber are nonlinear, considering 10% uncertainty in the displacement response $x_1$. In Fig. 7 (a) has again, the sensitivity to changes caused by factors external to the system (uncertainty). However, in Fig. 7 (b) is obtained result with values close to the original system and Fig. 7 (c) has a small distancing between the ranges of values.

## 5. CONCLUSION

In computational simulation, stands out the stochastic methods. This method has been widely used to solve many complex problems. This is because to consider some parameter of the project as random obtaining generally more precision in the responses. It is noteworthy that, among the stochastic methods, the best known and used are Monte Carlo and Latin Hypercube.

In this paper, we used the Latin Hypercube method, which is a variant of the Monte Carlo method. Also, studied the influences of uncertainty considered in linear stiffness parameters and nonlinear of the main system in the time domain in order to, improve the accuracy of the solution such systems. Accordingly, simulations were performed for the mechanical system of two degrees of freedom, especially three cases, the first being for the linear system and a linear absorber, the second for the nonlinear linear system and the absorber, and the third for the nonlinear system and nonlinear absorber.

Simulations show that, in the three cases, that when the uncertainty is considered in (s) parameter (s) of linear stiffness (es) obtains a significant distancing between the ranges of values, i.e., this (s) is (are) very sensitive consideration of uncertainty. However, for nonlinear stiffness parameters, it obtained small distancing between the ranges of values, generating a result very close to the original system values, even given a relatively high uncertainty value of 30%, which shows a higher robustness the nonlinear system as compared to linear.

However, it is emphasized that, the Latin hypercube method was shown to be effective in generating samples for the three cases discussed. The cases with nonlinear absorber, proved to be more robust in the consideration of uncertainty in stiffness parameters.

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