23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

REDUCTIONS METHODS APPLIED OF NONLINEAR MECHANICAL SYSTEMS

Lázaro Antônio da Fonseca Júnior
Daniel Ferreira Gonçalves
Romes Antonio Borges
Universidade Federal de Goiás – Regional Catalão
Avenida Dr. Lamartine Pinto de Avelar 1120, 75.704-020, Catalão, Goiás, Brasil
jrlazaro@live.com
dniellferreira2015@gmail.com
romes@ufg.br
Antônio Marcos Gonçalves de Lima
Universidade Federal de Uberlândia
Avenida João Naves Vieira 2121, 38.408-100, Uberlândia, Minas Gerais, Brasil
amglima@mecanica.ufu.br

Abstract. *The analysis of the structures subjected to non-linear phenomena is of great importance in modern engineering, a fact that is given by the constant search for efficient techniques and numerical-computational tools applied in analysis of automotive structures, aerospace, among others. In the search for greater precision, flexibility and reliability, dynamic systems, for the most part, are modeled which Finite Elements (FEM), numerical analysis technique designed to obtain approximate solutions to problems governed by differential equations. A range of systems modeled by finite element procedures may require a great processing ability of computers being feasible, then the use of templates reduction methods in which the degrees of freedom and consequently the order of the system matrices are reduced such as to preserve its dynamic characteristics. Thus, these methods provide a significantly lower computational cost and storage required in the review process, namely, Modal Basis and CA (Combined Approximations) methods. Here a numeric-computational study of the nonlinear mechanics systems. In order to validate the flexibility of the results and the computational gain obtained with the use of reduction methods, analyzes were performed in the time response, utilizing the numerical integration of the Newmark and Wilson Theta methods.*

Keywords: combined approximations method, modal basis methods, nonlinear systems, newmark method, wilson theta method.

1. INTRODUCTION

With advances in technology and the evolution of the design of structures, which are increasingly lighter, stronger and slender, subject to more excessive vibration levels, the use of computers as a troubleshooting tool becomes essential. Due to the great complexity involved in the equations, analytical solutions are not feasible or even not exist. Thus, it uses numerical methods for solving such equations, which are usually represented by nonlinear systems of equations, in order to obtain a more accurate response of the actual system. The finite element method, known simply as MEF is a numerical analysis technique designed to obtain approximate solutions to problems governed by differential equations.

The MEF has become a powerful tool for structural engineering analysis. However, applications of this method will depend in many cases as the actual physical model is represented by the mathematical model due to the complexity of engineering structures, and the fact that the approximate finite element model it contain modeling errors and can not represent real structures perfectly, especially for resources in the medium and high frequency, so, the MEF is a discretization process that aims to transform an infinite-dimensional problem in a finite-dimensional problem. The degree of approximation depends on the complexity of the problem, the discretization of the structure (mesh refinement), the type of finite element, the integration rule used, which consequently, as for a more refined mesh will be best results, however, the calculation time increases significantly with the refinement. (RADE, 2011; AZEVEDO, 2003)

Considering also that the structures involve some kind of nonlinearities, be they caused by force of acting, geometric, due to material or other nonlinearities, efficient computational techniques mean power significantly reduce computer storage and time required.

The major motivation for the study and development of model reduction techniques is to reduce the scale of the problem in question in order to get accurate results when compared to the original system, with a significant computational time gain.

The model reduction techniques have been widely used in the area of re-analysis and structural dynamics optimization, problems of structural vibration and buckling, damage detection, sensitivity studies and control parameters in nonstructural problems related to heat transfer and interaction fluid-structure.

However, due to truncation errors, the reduced model can not maintain all the characteristics of the complete model. Even for a frequency band of interest, they can not be exactly maintained at the reduced model, in the most of the model reduction techniques. For this reason, the most important point of the reduction of the model is to find the smallest model that contains the highest grade of the complete model information.

In this paper, we present the development of a numerical computational modeling, of a mechanical structure, a beam Euler-Bernoulli submitted to the application of harmonic loads, with the presence of no local linearity, in from of the coupling of the spring with characteristics nonlinear. Analyzes will be performed on the time domain, of the model complete and reduced by two reduction methods, namely: Modal Base (BM) and Combined approximations (CA).

2. MODELING THE TYPE STRUCTURE OF EULER-BERNOULLI BEAM WITH LOCAL NONLINEARITIES

This work was considered a beam Euler-Bernoulli type crimped onto free. The shear and the rotational inertia are neglected assuming that after deformation the cross sections remain flat and orthogonal to the axis of the beam. The displacements and rotations are considered small and the material have isotropic linear elastic behavior

Equation (1) represents the modeling of the structure described above, which can be found in. Reddy (1993), Bathe (1196), Crisfield (1997), Zienkiewicz *et al* (2005).

$$[M]_n \ddot{x}(t) + [D]_n \dot{x}(t) + [K]_n x(t) = f_n(t) \tag{1}$$

Where, $[M]_n$, $[D]_n$, $[K]_n \in R^{nxn}$, are the respective mass matrices, damping and stiffness of the model complete, and $x_n, \dot{x}_n, \ddot{x}_n \in R^{nxn}$, They are the respective vector displacement, velocity and acceleration, end, $f_n \in R^{nx1}$ is the vector of external forces applied to the structure. The structural damping matrix used in Eq. (1) is usually defined as a linear combination of the mass and stiffness matrices:

$$C = \alpha[M] + \beta[K] \tag{2}$$

Where α and β are real constants Rayleigh damping matrix defined to quantify the relationship between the mass and stiffness matrices. Particularly, the values are assumed $\alpha = 0$ and $\beta = 0.0005$.

Most engineering problems, have some nonlinearity, whether due to geometry of the material (which relates to the strain energy of the structure may be local (on only part of the structure) or global (contained throughout the structure), due to the material (such as viscoelastic materials), and also due to contact (the non-linear behavior is due to the dissipative nature, such as joints, contacts lubricated of machines).

In this article, it is coupled to Euler Bernoulli beam, a nonlinear spring, in order to verify the efficiency of reduction methods, for systems generated from local nonlinearity.

The governing equation for the nonlinear case is of the form:

$$[M]_n \ddot{x}(t) + [D]_n \dot{x}(t) + [K]_n x(t) + [KM]_n x^3(t) = f_n(t) \tag{3}$$

The matrix $[KM]_n$ is formed by two matrices, the first referring to the linear part of the stiffness of the spring and the second part relating to nonlinear spring stiffness, namely:

$$[KM]_n = [Kl]_n + [Knl]_n \tag{4}$$

One can also consider, that the nonlinear term, is represented by a non-linear actuation force, thus, the equation (3) can be rewritten as:

$$[M]_n \ddot{x}(t) + [D]_n \dot{x}(t) + [K]_n x(t) = f_n(t) - f_{nl}(t) \tag{5}$$

Where:

$$[KM]_n x^3(t) = f_{nl}(t) \quad 6$$

3. REDUCTION OF MODELS

The resolution of problems through reduced models is characterized by the removal of some degrees of freedom (referred to as secondary) of the complete model, maintaining a smaller set of degrees of freedom (referred to as primary). Thus, it is possible to increase the speed of computational model, allowing a greater number of simulations in a shorter period of time.

The general concept of reducing models applied in equation (5) is to find a smaller subspace of $T \in R^{(nxr)}$,with $r \leq n$ for the purpose to approximate as far as possible, the vector of space / displacement x .

However, the structural properties information such as mass, damping and stiffness can be changed, so the main challenge of the methods is to preserve the most of the dynamic characteristics of the main model. (Koutsovasilis *et al*. 2007).

The projection of the subspace in (5), a new differential equation of second order with smaller is obtained:

$$[M]_R \ddot{x}_R(t) + [D]_R \dot{x}_R(t) + [K]_R x_R(t) = f_R(t) - f_{nlR}(t) \quad 7$$

Where $[M]_R = T^T[M]T, [D]_R = T^T[D]T, [K]_R = T^T[K]T$ it is the reduced matrix dimensions system $R \times R$ e $F_R = T^T f$, $fnl_R = T^T f_{nl}$ are the reduced force vectors of dimensions $R \times 1$.

3.1 Modal Base (BM)

The method of modal basis is an extension of the method of Ritz bases, wherein the approximation of the matrix mass, stiffness and damping, and force vectors (linear and nonlinear), are obtained through the selection of certain eigenvectors of system.

The selection of which and how many eigenvectors will be selected depends on the influence of nonlinearity in the system.

Considering equation (1), without the presence of damping, given by the matrix ϕ is selected eigenvectors, truncated by the amount of desired columns.

In some cases, only the selection of certain eigenvectors are not sufficient to describe the system's behavior, in this case, the enrichment of such base is required. (Masson, *et al* (2006));

Gerges (2013) and De Lima *et al* (2015) also used Modal Base method for reduction of non-linear models.

This enrichment is done using the residue of the force that is acting on the system.

$$R = [K]_n^{-1} f_n \quad 8$$

This residue is added to the Ritz vectors, normalized by mass.

$$T = [\phi \quad R] \quad 9$$

Where T is the transformation matrix of Modal Base method.

3.2 Combined Approximations (CA)

The CA method, was developed by Kirsch (1991) for review of the sensitivity linear, nonlinear static and dynamic problems can be found in (Kirsch 1991, 2000, 2002, 2003)

According to Kirsch (2002), the approximate methods can be divided into global, local and combined approaches. Local approaches are very efficient, but they are only effective for small changes in design variables. Global approaches are more accurate for large changes in the structure, but they may require large computational effort during simulations.

The approach combined approximations (CA) developed by Kirsch, is based on the integration of various concepts and methods, including the series expansion, reduced basis, matrix factorization and Gram-Schmidt orthogonalization.

Thus, the CA method combines the efficiency of local approximations and the precision of the global approximation, thus generating efficient and accurate solution.

Chen et al (2000), Kirsch (2003), Rong et al (2003), Kirsch (2004) and Kirsch (2006), used the approach CA to eigenproblem reanalysis. According to Chen et al (2000), the AC method is more suitable for the lower vibration modes.

Gerges (2013), used the CA method in reducing nonlinear systems, getting good results as the approach normal system, as the computational gain.

In the method of CA reduction procedure using the eigenvector problem, it is assumed that the terms of the binomial series are used as basis vectors, to generate a reduced basis.

The transformation matrix is found by selecting linearly independent basis vectors, $r = r_1 + r_2 + \ldots + r_s$.

The basis vectors are determined by the binomial series terms:

$$r_1 = [K]^{-1} M\phi_r \tag{10}$$

$$r_i = -Br_{i-1} \qquad (i = 2,\ldots,s) \tag{11}$$

Where:

$$B = K + \Delta K \tag{12}$$

In ΔK corresponding to some modification in the structure, and ϕ_s corresponding to truncated matrix of auto vectors.
In this work, we used $r = r_1$.

4. NUMERICAL SIMULATIONS

The same was subjected to external forces type $f(t) = f_0 \sin(\omega t)$, with the frequency of excitation coincides with the first natural frequency of the beam, the part linear spring of stiffness equal to $200N/m$ and the part nonlinear on spring stiffness is $10^8\, N/m^3$. The beam's physical and geometrical properties are shown in Table 1.

Table 1. Beam's physical and geometrical properties.

	Aluminum beam	
	Length [m]	0.5
Geometrical properties	Width [m]	0.010
	Thickness [m]	0.003
Physical properties	Volume density [Kg/m^3]	2710
	Modulus of elasticity [MPa]	70e9

In both case, the response time was obtained using two numerical integration methods, the Newmark method and the method of Wilson Theta. (Gugliotta (2002); Zienkiewicz et al (2005); Rao 2008; Lulf (2014)).

All simulations were compiled in Matlab® software, with 1000 reviews each experiment. Thus, the percentage of time described in the tables below are an arithmetic average of time spent on each assessment.

In the first case, in the figure 1, considered the clamped-free beam and with the springs and force in the last node.

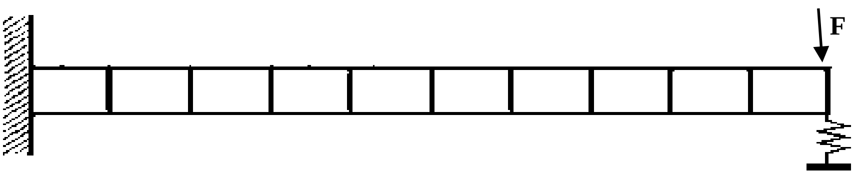
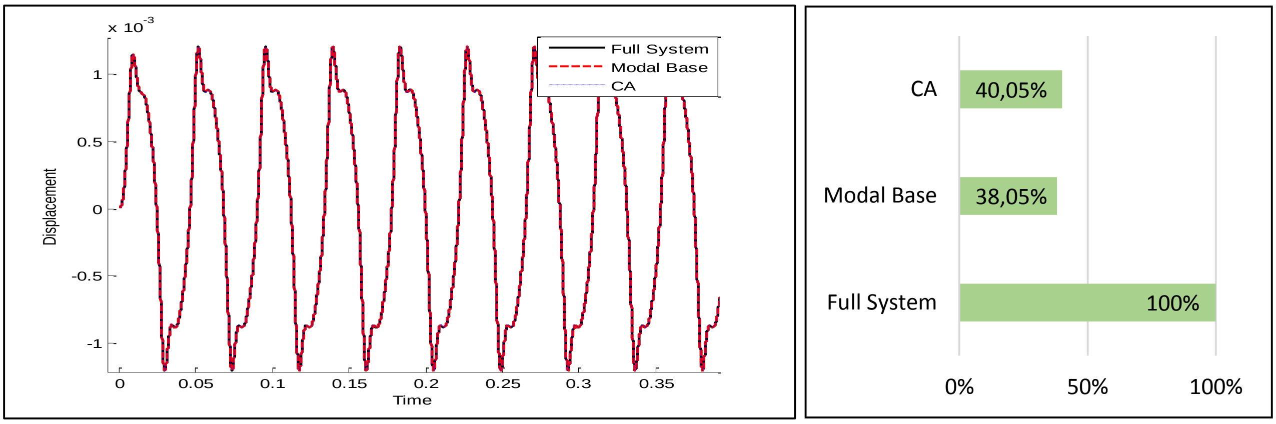


Figure 1: First case

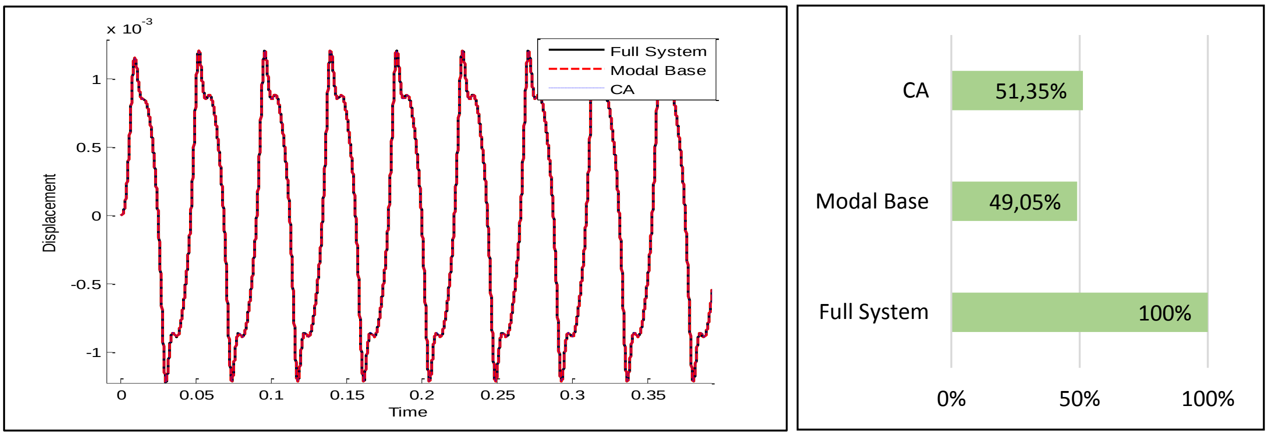
The figure 2 (a), the transient analysis was performed out using Newmark's method for complete system and to comparing the two reduction methods presented. And in figure 2 (b) relates the gain of computational time of reduction methods with the total time spent for the complete system.



(a) (b)

Figure 2: (a) Newmark Method for first case. (b) % of time of execution with Newmark Method.

In Figure 3 (a), the transient analysis was performed using the method of Wilson Theta, and Figure 3 (b) shows the gain of computational time of reduction methods with relation the full system.



(a) (b)

Figure 3: (a) Wilson Teta Method for first case. (b) % of time of execution Wilson Teta Method.

According to Figures 2 (a) and 3 (a), it can be seen that, both reduction method presented good results when compared to the original system, so much method and Newmark how much the method of Wilson Theta.

In addition to the good results presented in Figures 2 (a) and 3 (a), in Figures 2 (b) and 3 (b) are shown the percentage of time spent by the methods of reduction, as compared to the complete system, wherein, so much the Newmark method and by the Wilson Teta method, the computational times were reduced significantly.

For the second case, the spring was coupled in the penultimate node, as shown in figure 4.

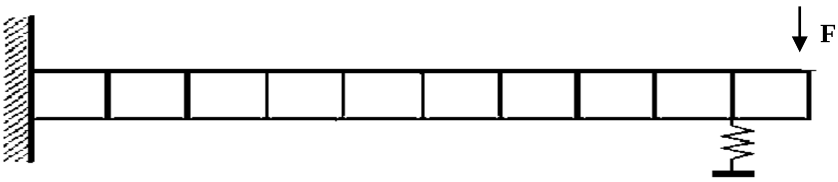
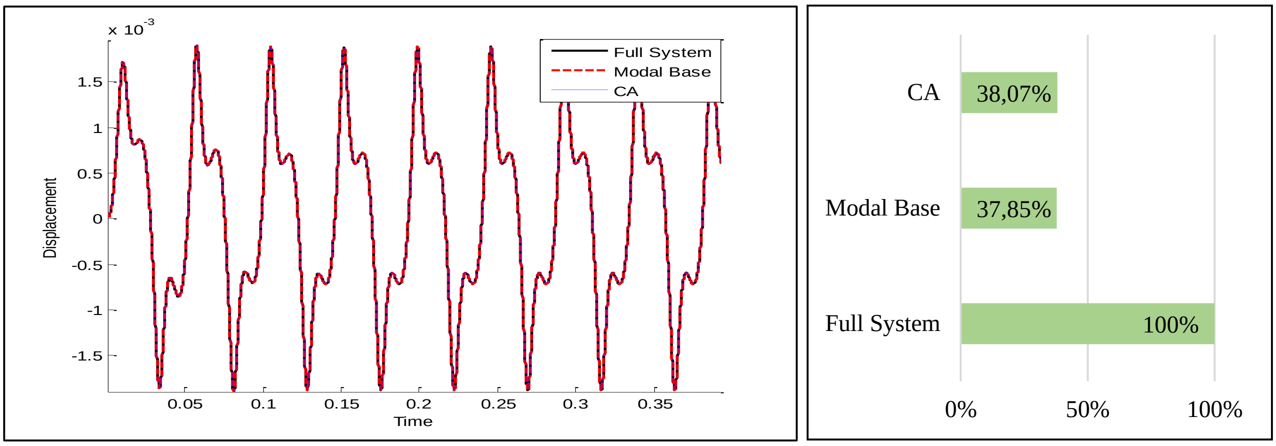


Figure 4: Second case

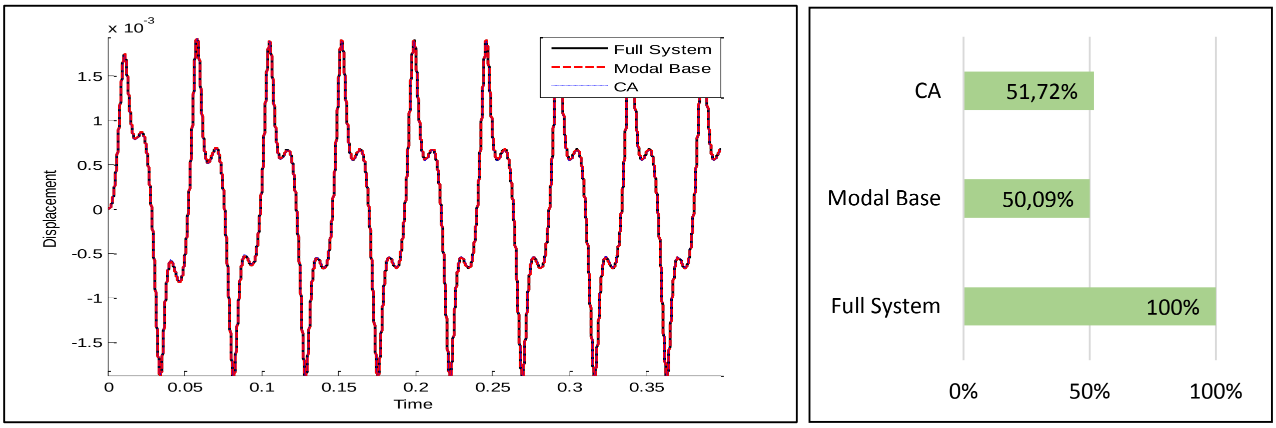
In Figure 5 (a), the transient analysis was performed using the Newmark method in order to compare the results obtained by reduction methods with the complete system, which is also expressed in Figure 5 (b), the percentage of time spent by each method of reduction, with respect to the time of complete system.



(a) (b)

Figure 5: (a) Newmark Method for second case. (b) % of time of Newmark Method

In Figure 6 (a), another method was used for transient analysis, the method of Wilson Theta as well as in Figure 6 (b) and presented the percentage of time of reduction of each method when compared to the complete system.



(a) (b)

Figure 6: (a) - Wilson Teta Method for second case. (b) % of time of execution with Wilson Teta Method.

According to Figures 2(a), 3(a), 5(a) and 6(a), we can see how accurate is the approximation using reduction methods, even with the presence of no local linearity, showed good results.

The computational gain obtained by using the CA method, and the mode-based method can be shown in the figures 2(b), 3(b), 5(b) and 6(b), in which the method of the modal basis, for the first case, the spent time 38.05% that normal system, savings of more than 60% of the time.

Also, as can be seen in figures 2(b), 3(b), 5(b) and 6(b), the Newmark method and Wilson Theta method demonstrate good results with respect to the model reduction methods, although the method has Newmark excelled in both cases, a computational gain greater than the method of Wilson Theta.

Attention should be paid to the case in which both reduction methods, the eigenvalues of matrix was truncated in the first three vibration modes for the method of Modal Base, and five modes for the CA method, which in these cases. They were sufficient, so that in both simulations, the method of modal basis, had a saving of time than the method CA.

5. CONCLUSION

In this work, a study was done using two model reduction methods, Modal Base and the CA.

To demonstrate the efficiency of both methods, two numerical simulations using a beam with local nonlinearity have been made.

Also two methods were used for numerical integration in time, the Wilson Theta and Newmark, in order to verify the behavior and the fidelity of the model results with reduction methods.

Reduction methods, can be understood as a multi-objective optimization problem, which seeks to minimize the simulation run time and maximize the approach of the reduction method with the original system.

Thus, from Figs. 2 (b), 3 (b), 5 (b) and 6 (b), it can be seen that good results obtained thereby minimizing the run time between 64% and 49% of the original system time.

How computational gain can be demonstrated using the method Newmark, which showed better results than Wilson Theta method.

How much the approximation, Figs 2 (a), 3 (a), 5 (a) and 6 (a) show the results both with the use of Newmark method as for the method of Wilson Theta, where both showed good results, maximizing the approach of reduction method with the original system..

With the efficiency of these methods, the next challenge and use such reduction methods in nonlinear geometrically systems with large displacement.

6. ACKNOWLEDGEMENTS

The authors are grateful to the UFG, FAPEG, CNPQ for financial support in terms of scholarship at the Post-Graduate Program in Modeling and Optimization as well as the Universidade Federal de Goiás - Catalão.

7. REFERENCES

Bathe, K. J., 1996. "*Finite element procedures"*. Upper Saddle River: Prentice Hall.

Chen, S. H., Yang, X. W., 2000. "*Extended Kirsch combined method for eigenvalue reanalysis".* AIAA J, 38, p. 927–30.

Crisfield, M. A., 1997. *"Non-Linear Finite Element Analysis of Solids and Structures"*. Vol 2, Jonh Wiley & Sons.

De Lima, A. M.G., Bouhaddi, N., Rade, D. A., Belonsi., 2015. *"A Time-Domain Finite Element Model Reduction Method for Viscoelastic Linear and Nonlinear Systems"*. Latin American Journal of Solids and Structures, 12, p. 1182 – 1201.

Gerges, Youssef., 2013. *"Méthods de Réduction de Modéles en Vibroacoustique non-linéaire"*. Doctoral thesis, Université de France – Comté.

Gugliotta, A., 2002. *"Elementi Finiti Parte IV"*. Politecnico di Torino.

Kirsch, U., 1991. *"Reduced basis approximations of structural displacements for optimal design".* J. AIAA 29, p. 1751 – 1758

Kirsch, U., 2002. *"Design-oriented analysis of structures".* Dordrecht: Kluwer Academic.

Kirsch, U., 2003. *"Approximate vibration reanalysis of structures."* AIAA J, 41, p. 504–11.

Kirsch, U., Bogomolni, M., 2004. *"Procedures for approximate eigenproblem reanalysis of structures".* IJNME, 60, p.1969–86.

Kirsch, U., Bogomolni, M., Sheinman, I., 2006. *"Nonlinear dynamic reanalysis of structures by combined approximations".* Computer methods in applied mechanics and engineering, 195(33-36), p. 4420–4432.

Koutsovasilis, P., Beitelschmidt, M., 2007. "Model reduction comparison for the elastic crankshaft mechanism". In *Proc. 2. Internacional Operational Modal Analysis Conference - IOMAC*, vol.1, pp. 95 – 106. Copenhagen.

Masson, G., Ait Brik, B., Cogan, S., Bouhaddi, N., 2006. "Component Mode Synthesis (CMS) based on an enriched Ritz approach for efficient structural optimization". *Journal of Sound and Vibration*, 296, p. 845 – 860.

Rao, S.S., 2008. *Vibraçoes Mecânicas*, Pearson Prentice Hall, São Paulo, 4ª edition.

Rong, F., Chen, S. H., Chen, Y. D., 2003. *"Structural modal reanalysis for topological modificsatioms with extended Kirsch method".* Comput Methods Appl Mech Eng, 192, p. 697–707.

Reddy, J. N., 1993. *"An Introduction to the Finite Element Method"*. McGraw-Hill.

Zienkiewicz, O.C., Taylor, R. L; Zhu, J.Z., 2005. "*The Finite Element Method: Its Basis and Fundamentals"*. Elsevier, Barcelona, 6ª edition.

8. RESPONSIBILITY NOTICE