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UNBALANCE PARAMETERS IDENTIFICATION IN ROTATING MACHINES USING COMPUTATIONAL PROCEDURE

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Abstract. *This study aims to understand the main phenomena involved in rotating systems, indicate a calculation method and apply the estimation method of unbalance parameters in a hybrid model, where the response to unbalance is used through the implementation and application of a numerical model in a rotating system. The study addresses both numerical computer simulation, where the rotating system model as well as the response to the unbalance is generated by a computer program dynamic analysis of rotating systems, which development was conducted in MatLab®. The performance of the identification and the influence of the factors that influence the results are analyzed and discussed through the simulation. The parameter identification method is shown robust to changes in system configuration with noisy data and modeling errors. In general, the method has great practical potential, which is beneficial for the proper functioning of the machines in the field and encourages the application of the method on larger, more complex machines.*

Keywords: *Rotating Machinery, Rotordynamics, Unbalance Parameters and Identification Parameters.*

1. INTRODUCTION

In an industrial environment there are numerous sources of vibration, such as in motors, compressors, machine tools, rotating systems, among others. In many cases the presence of vibration cause formation of cracks, loosening of screws, mechanical and structural failure, excessive wear of bearings and in frequent maintenance machines, (Rao, 2003). Rotating machines, also called rotors, are considered one of the most important classes of equipment, and are used extensively in industries in general. As an example of these rotating systems is the use in power generation plants, combustion engines, aviation turbine, machine tool, turbo-compressors, etc. According to Pereira (2005), the large capacity of the rotors to generate mechanical energy comes from high speed, which their axes are submitted. Associated with this high speed are high loads due to inertia of its components and potential vibration problems (critical speed) and instability of the rotors. The forecast rotors behavior using mathematical models is relatively successful when compared to experimental measurements.

This work aims at applying and estimation method validation unbalance parameters and foundation, through the amplitude measures, proposed by Edwards *et al.* (2000) and Sinha *et al.* (2002) and implemented in a numerical model of rotating machine by Ubinha (2005). The rotation system, as well as the response function to the unbalance were implemented in the computer program of dynamic analysis for rotating machines X-Rotor, developed in MatLab platform.

2. THEORY

In a study of the dynamic behavior of rotors it must be considered not only the rotor but also the bearings and its foundation structure in its analysis. During the dynamic behavior of rotating systems must be determined the interaction among its components, (Cavalca *et al.*, 2004). Rotordynamics is a tool of great importance for solving and preventing potential problems from development to operation and equipment maintenance. During the rotating machine maintenance or development the response to unbalance has great importance as a prediction tool of operating characteristics and obtaining parameters such as critical speeds and unbalanced mass location, also called eccentricity, through the vibration amplitudes caused by the forces that act in the set, (Ubinha, 2005).

Due to the huge advancements of computing resources and consequently important advances in mathematical modeling techniques, it is possible to accurately check the critical speed, the response to unbalance and the speed rotor stability limit.

2.1 Mathematical modeling

In this section it will be shown the implemented formulation in the X-Rotor program in order to obtain the response to a rotating assembly unbalance. The X-Rotor program is a software based on the Finite Element Method (FEM) and developed in Matlab® environment for dynamic analysis of rotating machines. For an initial analysis of the unbalanced force acting on a rotor it will be considered a simplified rotor, since a simplified model holds many characteristics of a more complex model. The set comprises a mass disk *m* coupled to a flexible axis and leaning on two bearings as the Fig. (1).

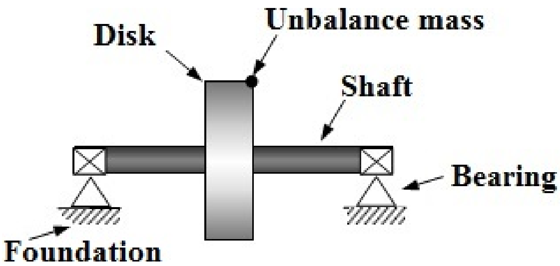


Figure 1. Simplified Representation (Jeffcott Rotor)

According Sinha *et al.* (2002), the residual unbalance is distributed in a discrete manner through the rotor, this is, in several planes. The amplitude of each unbalance plane is given by $m_d \cdot r$, which together with the phase angle can be represented by a complex plane, according to the Fig. (2).

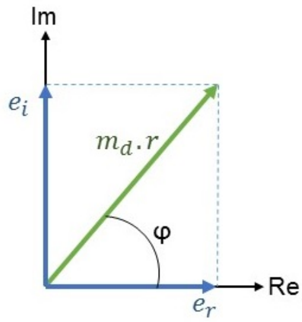


Figure 2. Unbalance amplitude in complex form

Thus, the unbalance vector can be represented:

$$d = (m_r \cdot r) \cdot e^{j\varphi} = e_r + je_i \tag{1}$$

being e_r a real component and e_i an imaginary component of the unbalance amplitude. Similarly, the unbalance force can be represented in complex form according to Fig. (3).

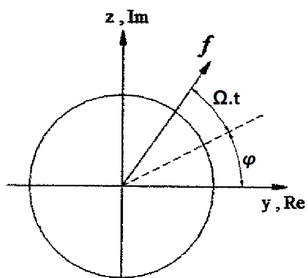


Figure 3. Unbalance force in complex form

From the representation, we can write the unbalance force as:

$$f = F_n \cdot e^{j(\Omega \cdot t + \varphi)} = F_n \cdot \cos(\Omega \cdot t + \varphi) + jF_n \cdot sen(\Omega \cdot t + \varphi) \tag{2}$$

being $F_n = m \cdot r \cdot \Omega^2$.

Force component toward y:

$$\begin{aligned} f_y &= F_n \cdot cos(\Omega \cdot t + \varphi) = F_n \cdot [cos(\Omega \cdot t) \cdot cos(\varphi) - sin(\Omega \cdot t) \cdot sin(\varphi)] \Rightarrow \\ &= F_n \cdot cos(\Omega \cdot t) \cdot cos(\varphi) - F_n \cdot [sin(\Omega \cdot t) \cdot sin(\varphi)] \Rightarrow \\ &= f_{yc} \cdot cos(\Omega \cdot t) + f_{ys} \cdot sin(\Omega \cdot t) \end{aligned} \tag{3}$$

where, $f_{yc} = F_n \cdot cos(\varphi)$ and $f_{ys} = -F_n \cdot sin(\varphi)$.

And toward z there is:

$$\begin{aligned} f_z &= F_n \cdot sin(\Omega \cdot t + \varphi) = F_n \cdot [sin(\Omega \cdot t) \cdot cos(\varphi) + cos(\Omega \cdot t) \cdot sin(\varphi)] \Rightarrow \\ &= F_n \cdot sin(\Omega \cdot t) \cdot cos(\varphi) + F_n \cdot [cos(\Omega \cdot t) \cdot sin(\varphi)] \Rightarrow \\ &= f_{zc} \cdot sin(\Omega \cdot t) + f_{zs} \cdot sin(\Omega \cdot t) \end{aligned} \tag{4}$$

where, $f_{zc} = F_n \cdot sen(\varphi)$ and $f_{zs} = F_n \cdot cos(\varphi)$.

Starting from the generic differential equation of the movement of a rotor we have:

$$[M_r] \cdot \ddot{u}_r + (C_r + G_r) \cdot \dot{u}_r + [K_r] \cdot u_r = f_r \tag{5}$$

considering that $[M_r], [C_r], [G_r]e[K_r]$ are respectively the matrices of, mass, gyroscopic, damping and stiffness of the rotor. The displacement vector is given by u_r and the external forces vector is f_r.

Whereas the system is linear, it is proposed a solution in permanent basis. In order to develop the identification method it is needed that the matrix movement equation be in complex form, so we can obtain the unbalance responde function (FRD). According to Hong and Park (1997), we can write the force and displacement vectors as follows:

$$f_r = F \cdot e^{j\Omega \cdot t} \tag{6}$$

$$u_r = u_f \cdot e^{j\Omega \cdot t} + u_f \cdot e^{j\Omega \cdot t} \tag{7}$$

Replacing the Eq. (6) and the Eq. (7) and its derivatives at Eq. (5), we obtain:

$$(-\Omega^2 \cdot M_r + j\Omega \cdot (C_r + G_r) + K_r) \cdot u_f \cdot e^{j\Omega \cdot t} + (-\Omega^2 \cdot M_r - j\Omega \cdot (C_r + G_r) + K_r) \cdot u_b \cdot e^{-j\Omega \cdot t} = F \cdot e^{j\Omega \cdot t} \tag{8}$$

Thus:

$$\text{at t} = 0 \rightarrow \; e^{j\Omega \cdot t} = 1 \qquad \text{and} \qquad e^{-j\Omega \cdot t} = 1 \tag{9}$$

$$\text{at t} = \frac{\pi}{2 \cdot \Omega} \rightarrow \; e^{j\Omega \cdot t} = j \qquad \text{and} \qquad e^{-j\Omega \cdot t} = -j \tag{10}$$

Thus, writing an equation for each time point we have:

$$(-\Omega^2 \cdot M_r + j\Omega \cdot (C_r + G_r) + K_r) \cdot u_f + (-\Omega^2 \cdot M_r - j\Omega \cdot (C_r + G_r) + K_r) \cdot u_b = F \tag{11}$$

$$j(-\Omega^2 \cdot M_r + j\Omega \cdot (C_r + G_r) + K_r) \cdot u_f - j(-\Omega^2 \cdot M_r - j\Omega \cdot (C_r + G_r) + K_r) \cdot u_b = jF \tag{12}$$

Multiplying the Eq. (12) by $-j$ and adding to Eq. (11), we have:

$$(-\Omega^2 \cdot M_r + j\Omega \cdot (C_r + G_r) + K_r) \cdot u_f = F \qquad \text{or} \qquad Z_r \cdot u_f = F \tag{13}$$

Thus, the Eq. (13) represents an expression for obtaining the FRD in the complex form. The Z_r matrix is the dynamic stiffness matrix of the rotor. Applying Euler's formula we reach the ratios between u_f, u_c and u_s below:

$$
\begin{aligned}
u_r &= u_f \cdot e^{j\Omega \cdot t} + u_b \cdot e^{-j\Omega t} \Rightarrow \\
&= u_f \cdot (cos(\Omega \cdot t)) + (jsin(\Omega \cdot t)) + u_b \cdot (cos(\Omega \cdot t)) - (jsin(\Omega \cdot t)) \Rightarrow \\
&= (u_f + u_b) \cdot cos(\Omega \cdot t) + j(u_f - u_b) \cdot sin(\Omega \cdot t)
\end{aligned} \tag{14}
$$

Comparing the Eq. (14) with a linear system and of solution in permanent basis $u_r = u_c \cdot cos(\Omega \cdot t) + u_s \cdot sin(\Omega \cdot t)$ we can conclude that:

$$
u_c = u_f + u_b \qquad \text{and} \qquad u_s = j(u_f - u_b) \tag{15}
$$

Handling the Eqs. (15) we have:

$$
u_f = \frac{1}{2} \cdot (u_c - ju_s) \qquad \text{and} \qquad u_b = \frac{1}{2} \cdot (u_c - ju_s) \tag{16}
$$

Thus, based on the u_c and u_s results obtained from the X-Rotor software, then is calculated the complex response to the uf unbalance, throught the Eqs. (16), in order to proceed with the identication method. In practice, according Dias jr. and Allemang (2000), the u_r vector is directly measured and the response to the u_f unbalance corresponds to the portion of the forward whirl from the response.

2.2 Parameters Identification

One of the great challenges of science has been modeling systems similar to the physical phenomena observed, in other words, which can mathematically represent some features of the phenomenon observed, (Aguirre *et al.*, 1998).

In a general way, the identication methods of dynamic parameters (stiffness and damping) differ about the domain, time or frequency and about the type of exciting force used. Such methods require the input signal (force) and output (displacement, velocity, acceleration) measurement of the dynamic system.

Therefore the unknown parameters are calculated based on the ratio between the exciting and response forces, (Tiwari and Vyas, 1997). The unbalance identification parameters made through the amplitude measurement of a rotor system has been a widely used technique in the analysis of rotordynamics.

2.2.1 Identification Method

In this section it will be shown the mathematical formulation proposed by Edwards *et al.* (2000) and implemented in the computer program X-Rotor. The identification method proposed is based on the amplitude response measurement of the unbalance. For this methodology is considered that the bearings are fully rigid, in other words, there is no relative displacement between the rotor and the foundation. In the formulation implemented, we consider the action of the forces due to the initial warping or bending of the axis.

The X-Rotor program provides an accurate model and accurately determines the system FRD, so we can obtain the amplitudes and its respectives frequencies at the bearing points.

Based on this method, the rotating machines can be divided into three main parts: rotor, bearings and foundation. These parts called sub-system will be represented as dynamic stiffness matrix. These sub-systems can be divided into degrees of freedom (DOF) and connected according to the Eqs. (17) a (19) below:

$$
\begin{bmatrix} Z_{r,ii} & Z_{r,ib} \\ Z_{r,bi} & Z_{r,bb} \end{bmatrix} \cdot \begin{pmatrix} u_{r,i} \\ u_{r,b} \end{pmatrix} = \begin{pmatrix} f_{r,i} \\ -f_{r,b} \end{pmatrix} \longrightarrow \text{Rotor} \tag{17}
$$

$$
\begin{bmatrix} K_b & -K_b \\ -K_b & K_b \end{bmatrix} \cdot \begin{pmatrix} u_{r,b} \\ u_{f,b} \end{pmatrix} = \begin{pmatrix} f_{r,f} \\ -f_{f,b} \end{pmatrix} \longrightarrow \text{Bearing} \tag{18}
$$

$$
\begin{bmatrix} Z_{f,bb} & Z_{f,bi} \\ Z_{f,ib} & Z_{f,ii} \end{bmatrix} \cdot \begin{pmatrix} u_{f,b} \\ u_{f,i} \end{pmatrix} = \begin{pmatrix} f_{f,b} \\ 0 \end{pmatrix} \longrightarrow \text{Foundation} \tag{19}
$$

The first subscripts r, b and f are related to the rotor sub-systems, bearings and foundation, respectively, the second subscript (i) is the internal DOFs of the rotor and foundation, and the second subscript (b) is the connected DOFs. Grouping the Eqs. (17) to (19), we obtain the global equation system:

$$
\begin{bmatrix} Z_{r,ii} & Z_{r,ib} & 0 & 0 \\ Z_{r,bi} & Z_{r,bb} + K_b & -K_b & 0 \\ 0 & -K_b & K_b + Z_{f,bb} & Z_{f,bi} \\ 0 & 0 & Z_{f,ib} & Z_{f,ii} \end{bmatrix} \cdot \begin{pmatrix} u_{r,i} \\ u_{r,b} \\ u_{f,b} \\ u_{f,i} \end{pmatrix} = \begin{pmatrix} f_{r,i} \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{20}
$$

The Equation (20) represents the general equation of moviment for the rotor-bearing-foundation system previously described. Considering that the foundation structure is directly connected to the bearings and admitting the bearings are fully stiff citeEdwards et al, 200, the Eq. (20) will be reduced to:

$$\begin{bmatrix} Z_{r,ii} & Z_{r,if} \\ Z_{r,fi} & (Z_{r,ff} + Z_f) \end{bmatrix} \cdot \begin{pmatrix} u_{r,i} \\ u_{r,f} \end{pmatrix} = \begin{pmatrix} f_{r,i} \\ 0 \end{pmatrix} \tag{21}$$

The Figure (4) represents the FEM model of the experimental lab bench of the Structures and Machines Dynamics Laboratory (LDEM) at State University of Campinas, which is shown in Fig. (5).

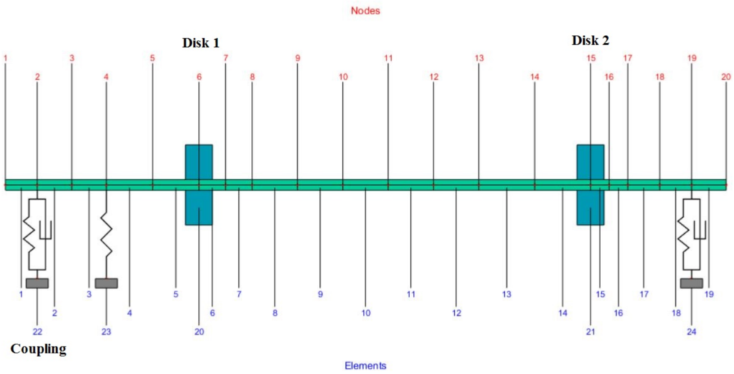


Figure 4. Rotor Finite Element Model

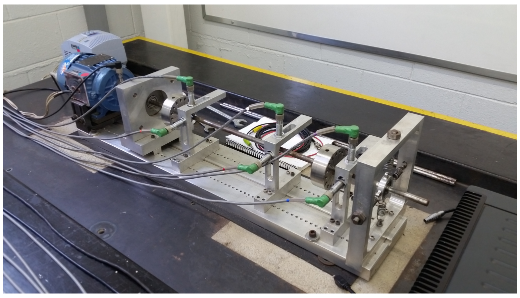


Figure 5. Experimental Lab Bench picture - LDEM

For the identication system, it is used a model of an axis with two discs assembly leaning on two stiff bearings with flexible foundation. The movements are considered only in orthogonal planes XZ and YZ.
In order to validate the identification method used, starting from the matrix system from Eq.(21) by Ubinha (2005), so we have:

$$Z_{r,ii} \cdot u_{r,i} + Z_{r,if} \cdot u_{r,f} = f_{r,i} \tag{22}$$

$$Z_{r,fi} \cdot u_{r,i} + (Z_{r,ff} + Z_f) \cdot u_{r,f} = 0 \tag{23}$$

Using the Eq.(22), we isolate the $u_{r,i}$ component, replacing it at Eq.(23), so we have:

$$Z_{r,fi} \cdot (Z_{r,ii}^{-1} \cdot f_{r,i} - Z_{r,ii}^{-1} \cdot Z_{r,if} \cdot u_{r,f}) + (Z_{r,ff} + Z_f) \cdot u_{r,f} = 0 \tag{24}$$

Expanding the Eq. (24) and rearranging the left side of equality with unknown terms and right side with the known terms according Edwards *et al.* (2000), so the resulting equation is:

$$Z_f \cdot u_{r,f} + Z_{r,fi} \cdot (Z_{r,ii}^{-1} \cdot f_{r,i}) = (Z_{r,fi} \cdot Z_{r,ii}^{-1} \cdot Z_{r,if} - Z_{r,ff}) \cdot u_{r,f} \tag{25}$$

Identifying the terms, follows:

$$Z_f \cdot u_{r,f} = S; \tag{26}$$

$$(Z_{r,fi} \cdot Z_{r,ii}^{-1} \cdot Z_{r,if} - Z_{r,ff}) \cdot u_{r,f} = P; \tag{27}$$

$$Z_{r,fi} \cdot Z_{r,ii}^{-1} \cdot f_{r,i} = Z_{r,fi} \cdot Z_{r,ii}^{-1} \cdot T \cdot \Omega^2 \cdot e = R \cdot e \tag{28}$$

Wherein S, P and R contains the terms collected in frequency measured. Assuming that the e vector from the Eq. (28) contains the system unbalance amplitude, the vector of forces $f_{r,i}$ can be defined as:

$$f_{r,i} = \Omega^2 \cdot T \cdot e \tag{29}$$

For identication proceedings the Eq. (29) needs to be manipulated, it is proposed that the product between the Z_f matrix (unknown) and the $u_{r,f}$ vector (measured) is:

$$S = Z_f(\Omega) \cdot u_{r,f}(\Omega) = w(\Omega) \cdot v \tag{30}$$

The v vector parameters depend on the shape of the dynamic stiffness matrix specified for the foundations, and the corresponding matrix w contains the terms related response in each frequency measured. Thus, by making use of the Eqs. (26) to (28) and the Eq. (30) we can write as follows:

$$W \cdot v + R \cdot e = P \qquad \text{or} \tag{31}$$

$$\begin{bmatrix} W & R \end{bmatrix} \cdot \begin{pmatrix} v \\ e \end{pmatrix} = P \tag{32}$$

The W, R and P matrices are known by the finite element model of rotor and the measurements, v and e are the unknown terms. Thus, it can be built a linear system of algebraic equations and by using numerical methods it determines the foundation parameters and the unbalance of the rotating assembly.

3. COMPUTER SIMULATION

For the identification of unbalance parameters there are some considerations to be made in order to extract the most accurate results. Among them, we can mention the methods used for the assembly of linear systems of matrix W, solution methods of these systems and the characteristics of frequency bands.

In his work Sinha *et al.* (2002) considers three methods for assembling the arrays W. The first method uses the frequency band division of the FRD in a single band, in other words, all amplitude measurements on each frequency are used. The method 2 is virtually various combinations of method 1, so, it divides in FRD n frequency bands and the method 3 is a combination of the methods 1 and 2, wherein the method is used to identify the first unbalance parameters and the method 2 for establishment of the model.

For the mathematical methods used to solve linear problems MatLab® was used, thus, a linear type system $A \cdot x = y$ can be solved using the inverse matrix of A coefficients, as Eq. 33.

$$\begin{aligned} A \cdot x = y \Rightarrow \\ x = inv(A) \cdot y \end{aligned} \tag{33}$$

According to Hanselman *et al.* (2003), this is most direct method but less appropriate, since the matrix A is generally rectangular. The most highly recommended procedure is to use the left division, Eq.(34). In this case the MatLab® uses the LU decomposition to solve this problem.

$$x = A \setminus y \tag{34}$$

The variations of the frequency band characteristics allowed to select the best band configuration, so, the best number of selected points, the best position of the FRD band and the optimal bandwidth. According to Ubinha (2005) testing the various combinations showed good results, a caveat in the case of varying the number of selected points where the most accurate result was 2 selected points instead of 15 points.

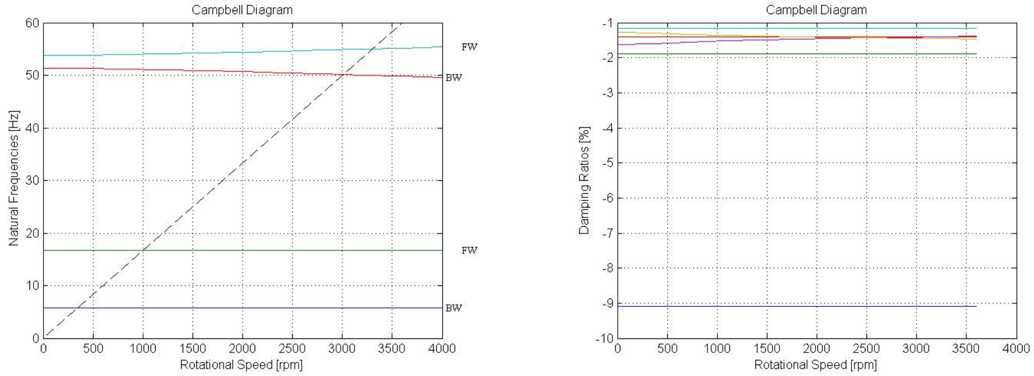
In addition, among the methods used to assemble the matrix W the best was the method 1, because the other methods besides not present a consistently better set of solutions also have an additional difficulty to determine which of the sets of best solutions it represents the physical parameters.

Given these observations, this section will display the results of computer simulation of dynamic analysis of rotors considering a frequency band with two selected points. The variation of the position and the width of the frequency band proved to be indifferent.

For the simulations they are considered, according to figs. (5) and (4), the coupling in the element 22, the isotropic bearing in Element 23 and the anisotropic bearing in Element 24, (Mesquita, 2004) and (Sousa, 2010).

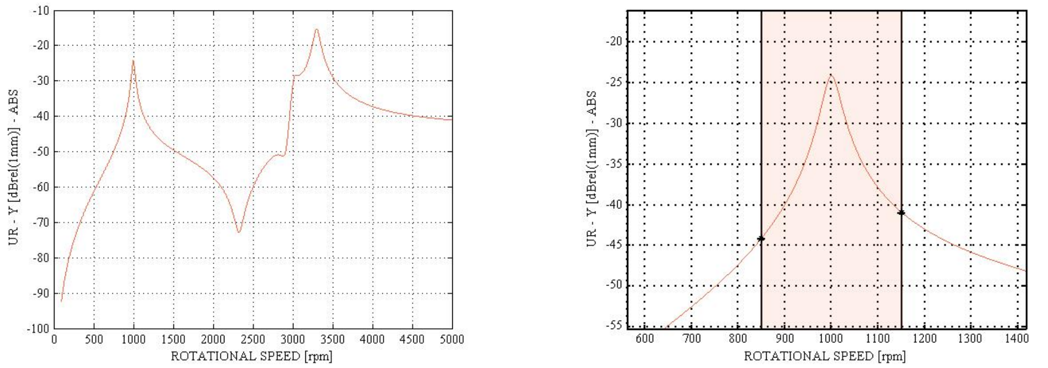
The Figures 6 illustrates Campbell diagrams for this system by damping graph will be considered for the analyzes a maximum damping ratio of around 9%, Fig. 6(b). Using the response to unbalance the system, illustrated in Fig. 7(a), a frequency band is selected for the representation of the respective amplitudes and frequencies as shown in Fig. 7(b). From this selection results are obtained if the identification of the desired parameters in the Table 1.

From these considerations, the results are:



(a) Natural frequencies (b) Damping Ratios

Figure 6. Campbell Diagrams



(a) FRD (b) Position of the band and location of points in FRD

Figure 7. Unbalance Response Function (FRD)

Table 1. The results analyzed foundation structure

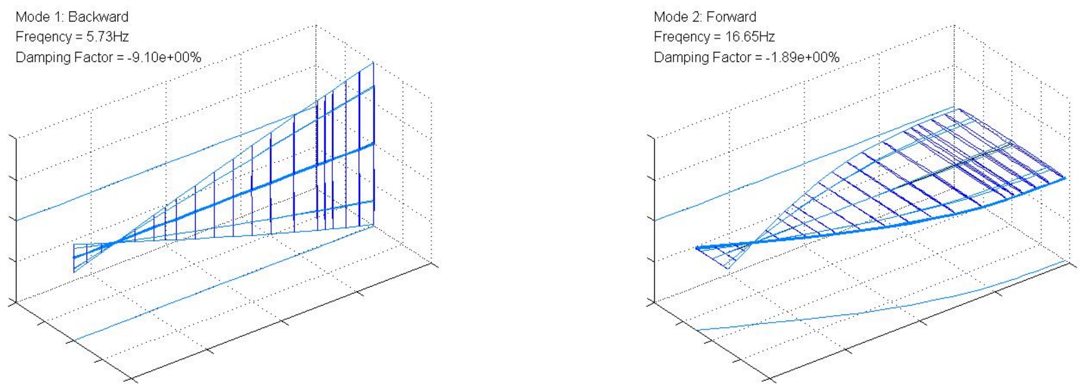
Parameters	**Vr**	**Ve**	**Error [%]**
Kyy_c	100	7,575	92,4
Kzz_c	100	1024	-924
Cyy_c	10	10,13	-1,3
Czz_c	10	13,11	-31,1
Kyy_i	1000000	996700	0,3
Kzz_i	1000000	947400	5,3
Kyy_a	10000	9978	0,2
Kzz_a	1000	913,2	8,7
Cyy_a	4	4,024	-0,6
Czz_a	5	5,068	-1,4
$Umbalance_i$	0,00001	0,000009867	1,3
$Umbalance_a$	0,00001	0,00001024	-2,4
$Phase_i$	0	0	0
$Phase_a$	0	0	0

Wherein: **Vr** = real value of the reference model; **Ve** = estimated value for the model; subscript **c** refers to the coupling; subscript **i** refers to isotropic bearing and subscript **a** refers to the bearing anisotropic. The parameter units are: stiffness (K) - *[N/m]*; damping (C) - *[N.s/m]*; $Umbalance$ - *[Kg.m]* and $Phase$ - *[degrees]*.

4. CONCLUSIONS

The results obtained for the examined rotation, as shown in Tab. 1 were fairly accurate for the parameters of stiffness and damping of the bearings, and relatively accurate for the parameters of stiffness and damping of the coupling. This is due to the fact that it was chosen in the frequency band the region where the first natural frequency of the assembly towards Y. The Figures 8(a) and 8(b) illustrate the shapes of the rotor operating mode where the critical frequency (backward whirl) is 345 rpm in the direction Z and the critical frequency (forward whirl) is 1000 rpm in the direction Y. This may have been a factor algorithm accuracy loss. Another reason may be the inaccuracy in obtaining the coupling parameters due to this element has in its structure polymeric materials (rubber), making the algorithm accuracy.

The method of identification used in this study, which identifies the parameters of a rotating machines from the rotor unbalance response, proved in fact to be an applicable method on real machines. The simulation presented using X-Rotor program aims to analyze the robustness of the algorithm by experimental data (Mesquita, 2004).



(a) First Mode (b) Second Mode

Figure 8. Vibration Modes

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