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Asymptotic model of laminar and turbulent displacement flows in annular space

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Abstract. *In the construction of oil and gas wells the cementing process is an important step. Displacing fluids in downhole conditions and for long distances is a complex task, affecting several steps on well construction. Cementing gains relevance in the moment that fluid contamination compromises cement sheath integrity and consequently zonal isolation. Hence, fluid properties and process condition should be designed to minimize non-uniformities on the displacement front. A complete analysis of the annular flow that occurs during cementing is extremely complex, because of the presence of different liquids that often present non-Newtonian characteristic and the flow is three dimensional and transient. In some cases, the flow in the lower viscosity liquid phase may become turbulent. A complete model has a prohibitive high computational cost. Simplified models are available in the literature and are used by the oil industry in commercial simulation software for cementing. The available models are not able to accurately describe turbulent flow regime. In this work, we extend the lubrication based model in cylindrical coordinates developed before to include turbulent flow regimes and inner cylinder rotation.*

Keywords: *cementing process, lubrication model, turbulent flow*

1. INTRODUCTION

The process of drilling a new well is divided in different at some stages. One of these stages is the cementing process as showed at figure 1. After positioning the steel casing into the well-bore, it is necessary to fill the space that it left between the casing and rock wall. The cement has the following objectives: fill the space between the casing and well, to promote adhesion between the wall (rock) and the casing; provide mechanical support for the coating; and to isolate the formation to prevent fluid loss.

For the success of the cementing operation it is necessary a complete remove the drilling fluid or spacer fluid and not allow mixing between them. Issues or failures that occur during cementing can affect the hydraulic isolation of the well leading to the migration of gases or liquids from rock to annular and can cause severe productivity problems, but also endanger the safety of operation and cause environmental damage.

There are many studies in the literature on cementing. Some factors strongly influence the final outcome of the cementing process, among them there is the rheology of the fluids pumped, the geometry of the wells, the flow rate and the pumped volume of each fluid.

Bittleston *et al.* (2002) developed a model that considers the eccentricity of well and use a Cartesian coordinate system to represent the geometry of the annular space. An asymptotic method the 2D problem as a sequence of 1D problems. As discussed in work, the main focus was to resolve the problem with low computational cost without a major concern with the accuracy of the solution.

The work of Pelipenko and Frigaard (2004) was focused on viscoplastic fluids (Herschel-Bulkley). The model determines the areas where the stress is smaller than the yield stress and fluid does not flow. The model is tested for problems with steady-state fluid displacements in a reference frame attached to the interface. The eccentricity is taken to be constant and small.

Pina and Carvalho (2006) developed a model using the lubrication theory and a cylindrical coordinate system. The effect of curvature of the straight section of the annular was not neglected. Thus, the model produces very accurate results, for any radii ratio R_i/R_0. However, the analysis was restricted to the flow of a Newtonian fluid.

Gomes and Carvalho (2009) developed a model an asymptotic model based on the theory of lubrication to study the displacement of different liquids through an annular space with variable eccentricity along well. The model having a good accuracy of the results is greater for radii ratio close to the unity.

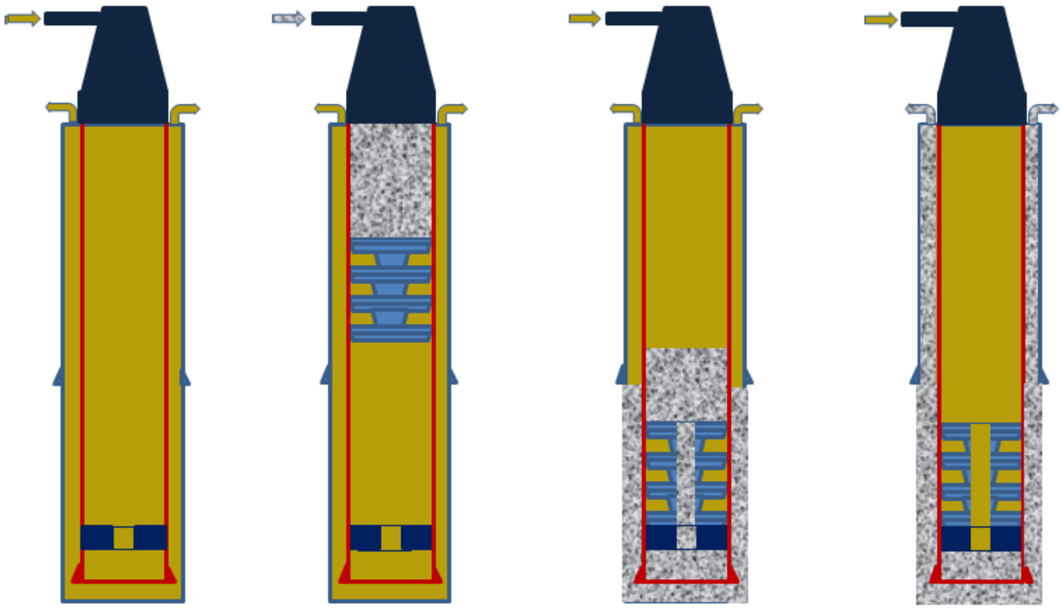

Figure 1. Draught of Cementing Process

2. MATHEMATICAL FORMULATION

To model the flow in the annular space formed by two cylinders with eccentricity varying along the axial direction, we chose to discard the curvature of the cylinder wall by adopting a Cartesian system of coordinates (z, y, x), where z is the coordinate in main flow direction (axial), y in the radial direction and x tangential direction. The origin of the coordinate system in each straight section of the well is located at the center of the smaller cylinder which has radius R_i. The position of the center of the outer cylinder is defined by the eccentricity $e(z) = (e_1^2 + e_2^2)^{0.5}$, where e_1 and e_2 are respectively, horizontal and vertical (orthogonal functions) eccentricity. The coordinate of the wall of the outer cylinder is defined in terms of the larger cylinder radius R_0 and the eccentricity:

$$\frac{\partial}{\partial z}\left[C_1\frac{\partial P}{\partial z} + C_2\right] + \frac{\partial}{\partial \theta}\left[C_3\frac{\partial P}{\partial \theta} + C_4\right] = 0 \tag{1}$$

Where the functions C_i are defined as a function of local geometry and liquid properties:

$$C_1 = \frac{1}{4\mu}\left[\frac{R_0^4 - R_i^4}{4} - \frac{\left(R_0^2 - R_i^2\right)\left(R_0^2 + R_i^2\right)}{2} + \frac{\left(R_0^2 - R_i^2\right)^2}{4\ln R_0/R_i}\right] \tag{2}$$

$$C_2 = -\frac{\rho g z}{4\mu}\cos\alpha\frac{d\alpha}{dz}\left[\frac{R_0^4 - R_i^4}{4} - \frac{\left(R_0^2 - R_i^2\right)\left(R_0^2 + R_i^2\right)}{2} + \frac{\left(R_0^2 - R_i^2\right)^2}{4\ln R_0/R_i}\right] \tag{3}$$

$$C_3 = \frac{1}{36\mu\left(R_0 + R_i\right)}\left[2R_0^4 + 2R_0^3R_i + 12R_0^2R_i^2\ln R_i/R_0 - 2R_0R_i^3 - 2R_i^4\right] \tag{4}$$

$$C_4 = \frac{\rho g\cos\alpha\cos\theta}{36\mu\left(R_0^5 + R_0^4R_i - 8R_0^3R_i^2 + 8R_0^2R_i^2 - R_0R_i^4 - R_i^5\right)} \tag{5}$$

2.1 Turbulent Flow

The concept of Equivalent Newtonian Viscosity used before by Gomes and Carvalho (2009) is extended to turbulent flows. The equivalent viscosity is the viscosity that a Newtonian fluid in laminar flow condition would have to have in order to yield the same local flow rate at a given pressure gradient of the flow in turbulent regime.

$$\overline{\mu} = \frac{R^2}{8}\left[(1-\kappa^2) - \frac{(1-\kappa^2)^2}{\ln(1/\kappa)}\right]\left(-\frac{\partial P}{\partial z}\right)^{\frac{1}{2}}\frac{1}{1-\kappa^2}\left[\frac{4(R-R_i)}{\rho f}\right]^{\frac{1}{2}} \tag{6}$$

The flow rate - pressure gradient relation of the turbulent flow is determined through the friction factor f. Here, we used the modified Blasius equation for annular flow.

$$\overline{f} = \frac{0.316}{(Re\,\Gamma)^{1/4}} \tag{7}$$

$Re \equiv \rho\overline{U}(R - R_i)/\mu$ is the local Reynolds number and Γ a geometric parameter, defined as:

$$\Gamma = \frac{1+\kappa^2}{(1-\kappa)^2} - \frac{1+\kappa}{(1-\kappa)\,\ln(1/\kappa)} \tag{8}$$

The pressure equation 1 becomes non linear. The coefficients C' are a function of the local velocity (pressure field).

3. RESULTS

The first example shown in this work was a comparison between the Laminar model and the Turbulent model. A vertical well with constant concentric and length of 1092 meters, external diameter $D_e = 0.161m$ and internal diameter $D_i = 0.089$. The radii ratio between cylinders is 0.4. Three Herschel-Bulkley fluids are pumped sequentially with constant flow rate of 7.2 bpm and the fluid properties according the table 1.

-	$K[Pa.s^n]$	$\rho[kg/m^3]$	$n[s^{-1}]$	$\tau_0[Pa]$
Fluid 1	0.1176	1102.40	0.7845	0.6548
Fluid 2	0.00453	742.9	1.0	0.0

Table 1. Fluids properties – Example 1

The results showed in figure 2 when consider a turbulent model have a good results (stable interface), which in principle was not expected, since the sequence of injected fluids did not obey the appropriate hierarchy criteria for both density and rheology.

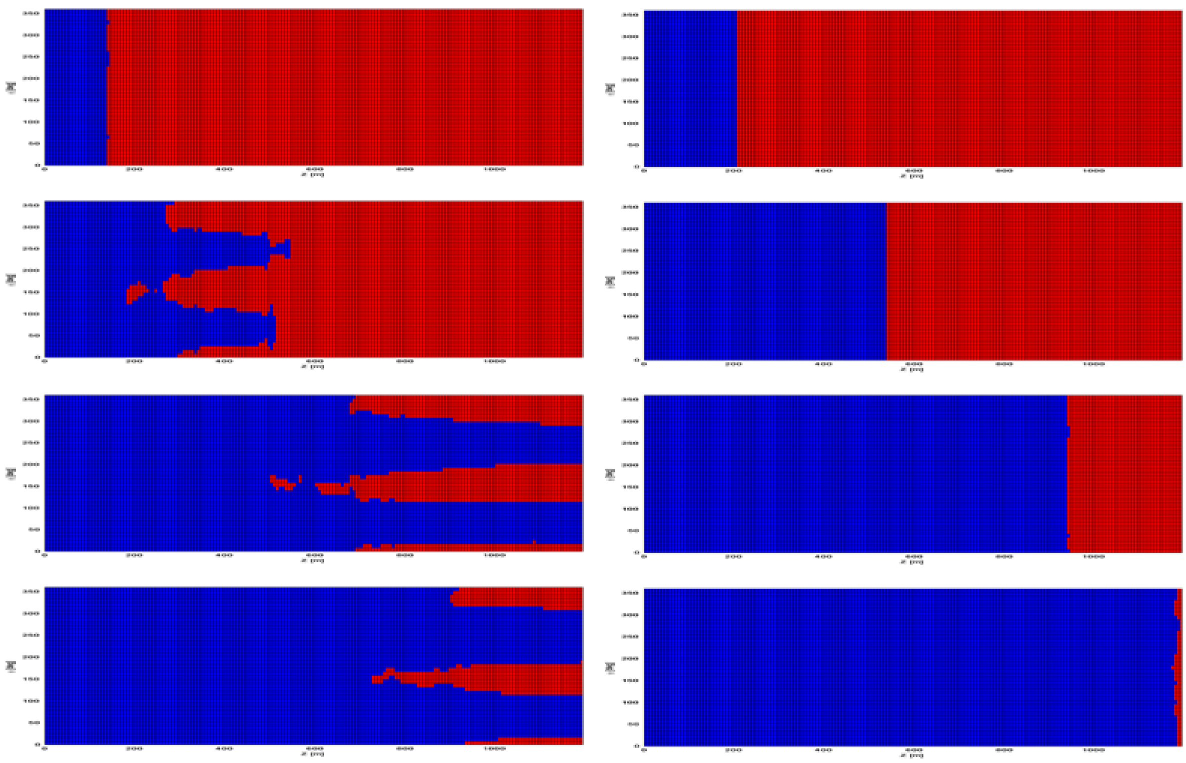

Figure 2. Simulation with laminar regime (left column) and with turbulent regime (right column)

This stabilization effect is according with experimental evidences reported by Aranha *et al.* (2012) for turbulent regime flow.

Another effect that can be stabilize the flow that the casing rotating during de cementing process, the next example illustrate this situation.

A vertical annular with $L = 200m$, 75% of stand-off. The flow rate was $0.0026m^3/s$ and were pumped two Newtonian fluids with properties described in table 2.

-	$\mu[Pa.s]$	$\rho[kg/m^3]$
Fluid 1	0.004	1100.0
Fluid 2	0.004	750.0

Table 2. Fluids properties – Example 2

In figure 3 can be observed in first column the pumped without the casing rotating occur with an instable interface and the when the casing moving the fluid interface was stabilized (second column).

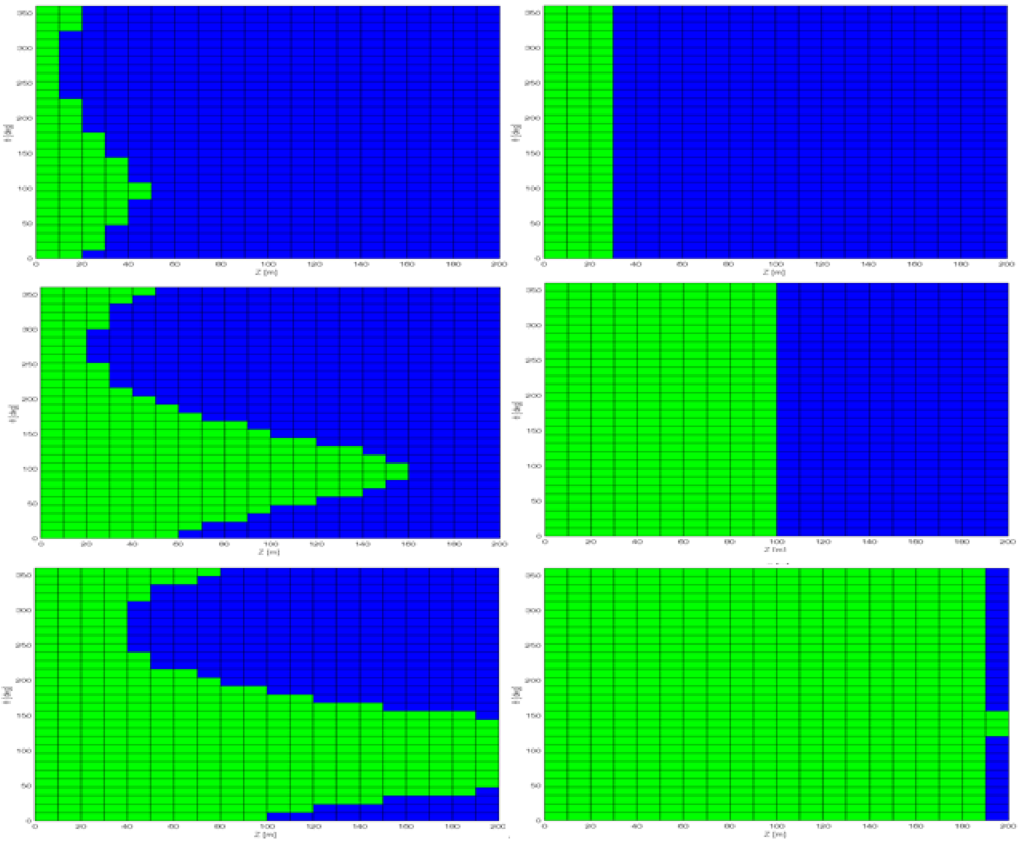

Figure 3. Simulation without rotation (left column) and with rotation, $\Omega = 5rd/s$ (right column)

4. FINAL REMARKS

The results show not only the importance of considering turbulent conditions in turbulent flows, but also the stabilization effect of turbulent flow, which agrees with experimental observations.

As well as the effect of turbulent flow regime stabilizes the rotation of the column also produces stabilize the flow.

5. REFERENCES

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