# A DOUBLE-LOOP CONTROL APPROACH APPLIED TO DAMAGE-TOLERANT ACTIVE CONTROL

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***Abstract.*** *This paper presents a double-loop control technique for vibration control of a smart flexible structure subject to damage. Damage-tolerant active control is an adaptation of fault-tolerant control that includes damage information in the controller design. Controllers are typically designed using damage-free structure models, without considering due changes, despite damage usually implies progressive severity increase and consequent performance deterioration. To overcome this control limitation, an additional adaptive controller is proposed to compose a double-loop control system in order to mitigate damage effects. In this work, a linear-quadratic regulator is designed for a healthy structure, together with an adaptive controller that ensures an acceptable performance level, compensating eventual model changes. An adaptive law updates online the controller parameters, based on state-tracking technique. A tall building model is used as a case study structure, including health and damage conditions, to examine the control strategy. Results show the methodology effectiveness here proposed.*

***Keywords:*** *damage-tolerant active control, active vibration control, adaptive control, LQR, smart structure.*

## 1. INTRODUCTION

Performance and robustness are two important features in any method to be used in control system design. However, these two properties are opposite to each other and are directly influenced by the control model accuracy (Chomette *et al.*, 2008). Structural damage causes changes in the structure dynamics, conducting the control system to the loss of performance and/or robustness. Thus, regular controllers may not be indicated for structures subject to damage that require an acceptable performance level such as aircrafts, spacecrafts, robotic systems, and smart buildings subject to seismic events (Genari *et al.*, 2015a). Therefore, a good design of structural active control should have the following characteristics: ability to automatically accommodate structural damage and ability to keep the overall system stability and acceptable performance in case of damage (Mechbal and Nóbrega, 2015). Structural control with these characteristics is referred to as damage-tolerant active control (DTAC) (Mechbal and Nóbrega, 2012).

In this paper, a control methodology based on double-loop control is presented, applied to vibration reduction of a flexible structure subject to damage. The first controller is designed to attend previously defined performance and robustness requirements for the healthy plant, using a regular linear-quadratic regulator (LQR). The second controller aims to ensure satisfactory closed-loop performance even if the structure suffers damage. A reconfigurable technique is adopted to design this damage controller, based on a state-tracking method, which updates online the controller parameters. A case study simulation of a smart tall building is conducted in order to examine the proposed double-loop control methodology. Results confirm the technique effectiveness.

## 2. REGULAR CONTROLLER: LQR APPROACH

The state-space model of a flexible structure can be described as:

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

in which $x(t)$ represents the state vector, $u(t)$ is the control vector, $y(t)$ is the measurement vector, and $w(t)$ represents the disturbance vector. Matrices $A$, $B$, and $C$ are the state matrix, the input matrix, and the output matrix, respectively. Matrix $A$ is not necessarily stable, but the pair $(A,\ B_2)$ is considered controllable.

The optimal regulator for the system represented by Eq. (1) and Eq. (2) is given by (Ogata, 2009):

$$u(t) = -K_{lqr}x(t), \tag{3}$$

where the gain $K_{lqr}$ is calculated to reduce vibrations provoked by disturbance $w(t)$. $K_{lqr}$ is obtained from the minimization of the following quadratic cost function:

$$J = \int_0^{\infty} (x^T M x + u^T R u)dt,$$

in which $M$ is a positive-semidefinite matrix ($M \geq 0$) and $R$ is a positive-definite matrix ($R > 0$).

The solution of the LQR problem is (Preumont, 2011):

$$K_{lqr} = R^{-1}B_2^T\bar{P},$$

where $\bar{P}$ is a positive-definite matrix obtained from the solution of the algebraic Riccati equation:

$$\bar{P}A + A^T\bar{P} + M - \bar{P}B_2R^{-1}B_2^T\bar{P} = 0. \tag{4}$$

Substituting Eq. (3) in Eq. (1), the closed-loop system becomes:

$$\dot{x}(t) = \tilde{A}x(t) + B_1w(t)$$
$$y(t) = Cx(t),$$

where $\tilde{A} = (A - B_2K_{lqr})$.

## 3. DOUBLE-LOOP METHODOLOGY: LQR AND DAMAGE CONTROLLER

The signal controller $u(t)$ of Eq. (1) is obtained from two controllers: $u(t) = u_1(t) + u_2(t)$. The first controller is designed for the healthy structure to attend previously defined performance indexes, based on the LQR approach. The second controller is designed to mitigate damage effects in the structure dynamics, automatically acting only when the structure suffers damage.

Considering both the controllers, the system represented by Eq. (1) and Eq. (2) becomes:

$$\dot{x}(t) = \tilde{A}x(t) + B_1w(t) + B_2u_2(t) \tag{5}$$
$$y(t) = Cx(t).$$

The ideal fixed-gain control law $u_2(t)$ is considered as:

$$u_2(t) = K_x^T x(t), \tag{6}$$

and substituting Eq. (6) into Eq. (5), the closed-loop system with both controllers is given by:

$$\dot{x}(t) = (\tilde{A} + B_2K_x^T)x(t) + B_1w(t). \tag{7}$$

Considering the closed-loop reference model as:

$$\dot{x}_r(t) = A_rx_r(t) + B_1w(t) \tag{8}$$
$$y_r(t) = C_rx_r(t),$$

then the match condition is obtained by comparing Eq. (8) to Eq. (7):

$$A_r = (\tilde{A} + B_2K_x^T). \tag{9}$$

### 3.1 Adaptive law

The real control law is defined from the ideal fixed-gain control law of Eq. (6):

$$u_2(t) = \hat{K}_x^T x(t),$$

in which $\hat{K}_x$ is the estimative of $K_x$. Thus, the closed-loop system is rewritten as:

$$\dot{x}(t) = (\tilde{A} + B_2\hat{K}_x^T)x(t) + B_1w(t). \tag{10}$$

The closed-loop state tracking dynamic is obtained from Eq. (8), Eq. (9), and Eq. (10):

$$\begin{aligned}\dot{e}_x(t) &= \dot{x}(t) - \dot{x}_r(t)\\ &= (A_r - B_2K_x^T + B_2\hat{K}_x^T)x(t) - A_r x_r(t)\\ &= A_r e_x(t) + B_2 \triangle K_x^T x(t),\end{aligned} \tag{11}$$

where $\triangle K_x = (\hat{K}_x - K_x)$.

To analyse whether the state-tracking error converges asymptotically to zero, the following Lyapunov function is proposed:

$$V(e_x, \triangle K_x) = e_x^T P e_x + tr([\triangle K_x^T T_x^{-1} \triangle K_x]), \tag{12}$$

where $T_x = T_x^T > 0$ is the adaptation rate, and $tr$ represents the trace of a matrix. Furthermore, $P = P^T > 0$ satisfies the following algebraic Lyapunov equation:

$$PA_r + A_r^T P = -Q, \tag{13}$$

in which $Q = Q^T > 0$.

Then, the derivative of Eq. (12), evaluated using Eq. (11), is given by:

$$\begin{aligned}\dot{V}(e_x, \triangle K_x) &= \dot{e}_x^T P e_x + e_x^T P \dot{e}_x + 2tr(\triangle K_x^T T_x^{-1} \dot{\hat{K}}_x)\\ &= -e_x^T Q e_x + 2tr(\triangle K_x^T [x e_x^T P B_2 + T_x^{-1} \dot{\hat{K}}_x]).\end{aligned}$$

If the adaptive law is selected as:

$$\dot{\hat{K}}_x = -T_x x e_x^T P B_2,$$

the selected gain makes $\dot{V}(e_x, \triangle K_x)$ negative semidefinite, $\dot{V}(e_x, \triangle K_x) = -e_x^T Q e_x \leq 0$. Thus, the state-tracking error asymptotically tends to zero, i.e., $\lim\limits_{x\to\infty} \|x(t) - x_r(t)\| = 0$.

### 3.2 State estimation

The law that updates the control gain $\hat{K}_x$ is written in function of the known parameters $T_x$, $P$, and $B_2$. The unknown vectors $e_x(t)$ and $x(t)$ are continuously estimated online from an unknown input observer. This observer is fundamental to provide information about damage effects in the structure dynamics.

The model of a flexible structure subject to damage can be considered as:

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) + F f(t) \tag{14}$$

$$y(t) = Cx(t), \tag{15}$$

where $F$ is an unknown matrix, and $f(t)$ represents perturbations in the structure model.

The unknown input observer representation for the structure described by Eq. (14) and Eq. (15) is proposed as:

$$\begin{aligned}\dot{z}(t) &= Nz(t) + Ly(t) + Gu(t)\\ \hat{x}(t) &= z(t) - Ey(t),\end{aligned}$$

in which $z(t)$ is the observer state vector, and matrices $E$, $G$, $L$, and $N$ are chosen so that the estimated state $\hat{x}(t)$ converges to $x(t)$ asymptotically. An output error vector, also known as residual vector, is defined as:

$$r(t) = y(t) - \hat{y}(t).$$

Then, the estimation error dynamic is given by:

$$\begin{aligned}\dot{e}_e(t) &= \dot{x}(t) - \dot{\hat{x}}(t)\\ &= Ax(t) + B_2 u(t) + B_1 w(t) + F f(t) - \dot{z}(t) + E\dot{y}(t)\\ &= Ne_e(t) - (NH + LC - HA)x(t) - (G - HB_2)u(t) + HB_1 w(t) + HF f(t),\end{aligned}$$

where $H = I + EC$, with $I$ being an identity matrix with appropriate dimensions. To make the error dynamics independent of $x(t)$, $u(t)$, $w(t)$, and $f(t)$, the following equations should be satisfied:

$$HB_1 = B_1 + ECB_1 = 0 \tag{16}$$

$$HF = F + ECF = 0$$

$$G - HB_2 = 0 \tag{17}$$

$$NH + LC - HA = 0.$$

Matrix $E$ can be obtained from the solution of Eq. (16) as follows:

$$E = -B_1(CB_1)^{\dagger}, \tag{18}$$

where $(CB_1)^{\dagger}$ is the Moore-Penrose inverse of $(CB_1)$.
The solution of Eq. (17) for $G$ can be written as:

$$G = B_2 + ECB_2. \tag{19}$$

To obtain stable poles, matrix $N$ is defined as:

$$N = A_1 - KC, \tag{20}$$

in which $A_1 = HA$, and $L = K - NE$. If $(A_1, C)$ is observable, matrix $K$ can be computed by a pole placement routine. Otherwise, $K$ is obtained from the observable part of $(A_1, C)$ (Chen and Patton, 1999). Thus, the estimation error dynamics become:

$$\dot{e}_e(t) = Ne_e(t),$$

where the estimate error converges to zero when $N$ is stable. Moreover, the residue signal is rewritten in the following augmented form:

$$r(t) = (I + CE)y(t) - Cz(t). \tag{21}$$

### 3.3 Damage detection

Under the presence of damage the system model can be rewritten as:

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) + \triangle Ax(t) + \triangle Bu(t)$$
$$y(t) = Cx(t) + \triangle Cx(t),$$

where $\triangle A$, $\triangle B$, and $\triangle C$ represent the model perturbations caused by damage. The unknown input observer is designed to make the estimation error converges asymptotically to zero despite the perturbations and disturbance, so that the residue signal in Eq. (21) remains close to zero. When the structure suffers damage, the residue signal in Eq. (21) changes drastically. Thus, it is possible to define a convenient threshold to determine damage occurrence.

The damage controller is designed to mitigate damage, based on the closed-loop reference system obtained from the healthy structure with the LQR controller. Thus, when the structure suffers damage, the damage controller creates control signals to minimize the difference between the damaged structure and the reference model. In other words, the damage controller acts to isolate the damage effects over the structure dynamics, diminishing them over the control system performance.

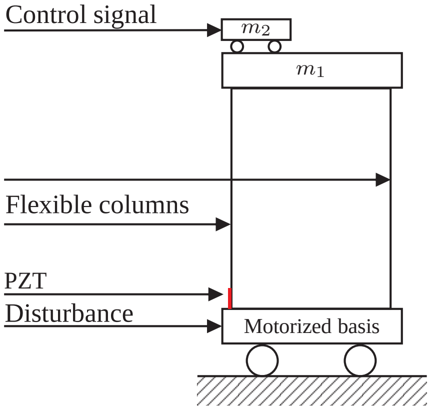
## 4. RESULTS

This section presents the double-loop control methodology validation. Initially, a vertical structure that simulates a tall building is introduced. Then, the LQR controller is designed for lateral vibration control, using an unknown input observer in order to estimate the state vector. After that, the damage controller is designed to mitigate damage effects in the vertical structure. Finally, damage is provoked in the structure and the double-loop methodology is examined for damage tolerance.

### 4.1 Flexible structure

The vertical structure is presented in Fig. 1a and can be divided into three main parts: basis, flexible system that simulates a tall building and active mass driver (AMD). The basis comprises a lead screw mechanism coupled to the mechanical structure. This way, rotational movements of a DC motor are converted into linear movements of the basis. The structure comprises the mass $m_1$ supported by four aluminum columns attached to the basis and there is a piezoelectric element glued onto one column to measure lateral vibration. The active mass $m_2$ is moved linearly by another DC motor using a similar lead screw mechanism. The control system is designed to minimize vibrations due to disturbance caused by the basis movement. For this purpose, a control signal is used in order to create a specific movement of the AMD to counterbalance seismic forces. Figure 1b shows the block diagram that describes the system.

(a) Vertical Structure




(b) Block diagram

Figure 1: Structure setup

The vertical structure model was experimentally identified by Genari *et al.* (2015b) and its state-space representation is:

$$\dot{x}(t) = \begin{bmatrix} -0.4282 & 15.6400 & 0 \\ -15.640 & -0.4282 & 0 \\ 0 & 0 & -20.5940 \end{bmatrix} x(t) + \begin{bmatrix} 0.0942 \\ 0.2234 \\ 0.1334 \end{bmatrix} w(t) + \begin{bmatrix} -0.0157 \\ -0.0214 \\ -0.0175 \end{bmatrix} u(t) \tag{22}$$

$$y(t) = \begin{bmatrix} -2.1785 & 2.0789 & 6.8361 \\ 0 & 0 & 1 \end{bmatrix} x(t), \tag{23}$$

where two states represent one mode of the structure and one state describes the dynamics of the two similar DC motors. Moreover, $y_2(t)$ is added into the model to analyse the motor dynamic behaviour.

### 4.2 LQR and observer design

The LQR is designed to reduce vibrations caused by the disturbance $w(t)$. Thus, matrices $M$ and $R$ are chosen as:

$$M = \begin{bmatrix} 7.5 & 0 & 0 \\ 0 & 7.5 & 0 \\ 0 & 0 & 7.5 \end{bmatrix}, \text{ and } R = 0.005,$$

in which the gain is obtained through the solution of Eq. (4):

$$K_{lqr} = \begin{bmatrix} -18.8411 & -24.9591 & -0.6203 \end{bmatrix}. \tag{24}$$

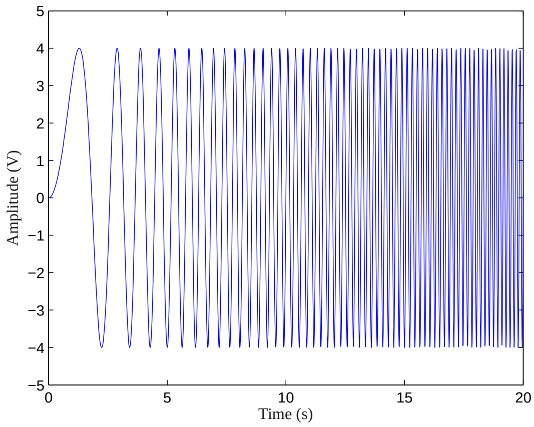
In this paper, the unknown input observer estimates the states used in the feedback. The observer matrices are calculated from Eq. (18)-(20), using the model matrices of Eq. (22) and Eq. (23):

$$E = \begin{bmatrix} -0.0794 & -0.0090 \\ -0.1884 & -0.0215 \\ -0.1124 & -0.0128 \end{bmatrix}, L = \begin{bmatrix} -126.3872 & 1153.3528 \\ -132.3349 & 1192.3006 \\ -003.0782 & 0031.6216 \end{bmatrix}, G = \begin{bmatrix} -0.0052 \\ 0.0034 \\ -0.0027 \end{bmatrix}, \text{ and}$$

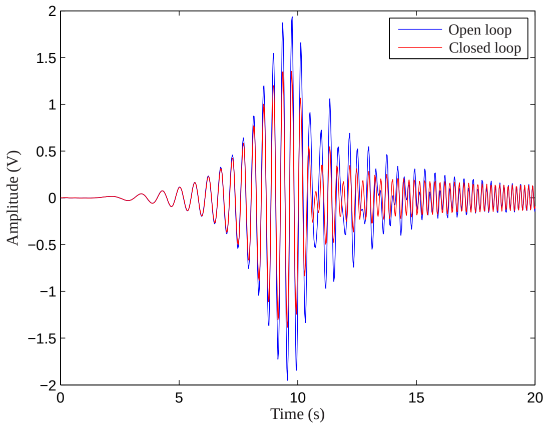
$$N = \begin{bmatrix} -312.5707 & 318.6925 & -152.5521 \\ -312.4368 & 295.0663 & -214.5766 \\ 026.9745 & -18.4203 & -111.1978 \end{bmatrix},$$

where $K$ in Eq. (20) is chosen such that the poles of $N$ are six times faster than the $A$ poles.

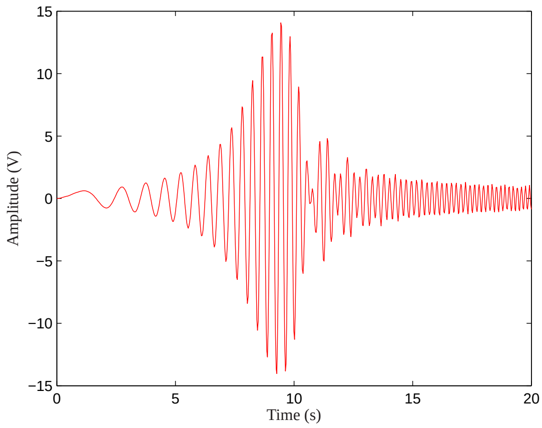
To analyse the control performance, the disturbance, presented in Fig. 2a, is considered as a chirp signal with frequency between $0$ and $38$ rad/s, amplitude of $4$ V, $20$ seconds of period, and sampled at $100$ Hz. The structure responses in open loop and closed loop are compared in Fig. 2b. The controller reduced the lateral vibrations $y_1(t)$ by almost $31\%$, generating the control signal presented in Fig. 2c with values between $15$ V and $-15$ V (plant bounds). Similar results were obtained using the $H_\infty$ methodology by Genari *et al.* (2015b).




(a) Disturbance $w(t)$




(b) Lateral vibrations




(c) LQR control signal

Figure 2: Healthy structure vibration control

### 4.3 Damage controller

The damage controller is designed using only the healthy structure model. Matrices $Q$ and $T_x$ are chosen with the highest gain over the states $x_1$ and $x_2$. Moreover, matrix $P$ is obtained from the solution of Eq. (13). Thus, matrices $Q$, $P$, and $T_x$ are given by:

$$Q = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; P = \begin{bmatrix} 6.0911 & 0.0369 & -0.0001 \\ 0.0369 & 5.7858 & -0.0041 \\ -0.0001 & -0.0041 & 0.0243 \end{bmatrix}, \text{ and } T_x = 550 \times \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

After that, the adaptive gain law can be written as:

$$\dot{\hat{K}}_x = -550 \times \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{x} e_x^T \begin{bmatrix} -0.0963 \\ -0.1242 \\ -0.0003 \end{bmatrix},$$

where the state vector is estimated using the unknown input observer. The double-loop control law is given by:

$$u(t) = -K_{lqr}\hat{x}(t) + \hat{K}_x\hat{x}(t),$$

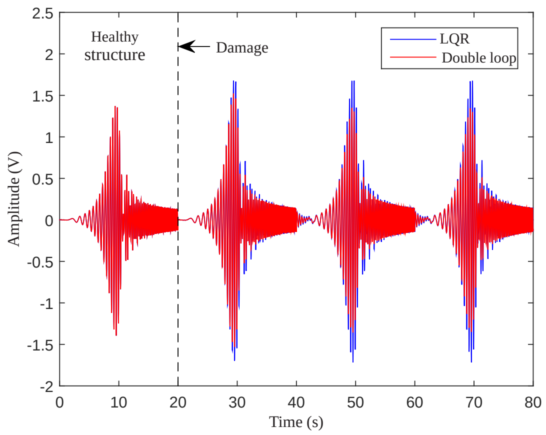
in which $K_{lqr}$ is the gain of Eq. (24) calculated for the healthy structure.

Structural damage may significantly alter dynamic response, due to changes in stiffness, mass, or energy dissipation (Sohn *et al.*, 2004). In this way, a novel model is needed to represent the new structure dynamics caused by damage. Here, damage is induced in the structure to provoke vibration increase. Moreover, the induced damage does not cause natural frequency shift, meaning that the mass change is counterbalanced by the stiffness change. The vibration increase is simulated by reducing the structural damping in 50%, representing the removal of two aluminum columns of the vertical structure. Mass changes alters matrix $B_2$ in 10%. Finally, a variation of 10% in matrix $C$ is also considered a damage effect. Thus, the model matrices that represent the damaged structure are given by:

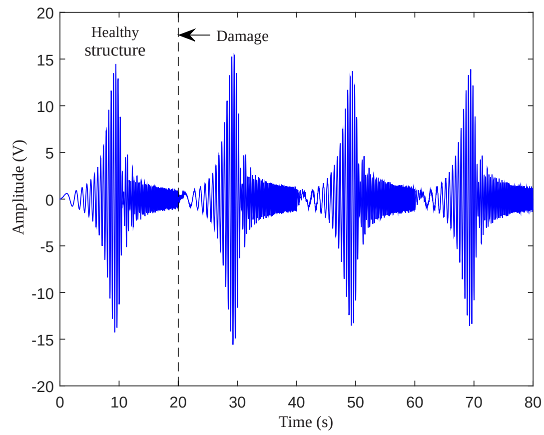
$$A_d = \begin{bmatrix} -0.4282 \times 0.5 & 15.6400 & 0 \\ -15.640 & -0.4282 \times 0.5 & 0 \\ 0 & 0 & -20.5940 \end{bmatrix}, \; B_{2d} = \begin{bmatrix} -0.0157 \times 1.1 \\ -0.0214 \times 1.1 \\ -0.0175 \times 1.1 \end{bmatrix}, \text{and}$$

$$C_d = \begin{bmatrix} -2.1785 \times 1.1 & 2.0789 \times 1.1 & 6.8361 \times 1.1 \\ 0 & 0 & 1 \end{bmatrix}.$$

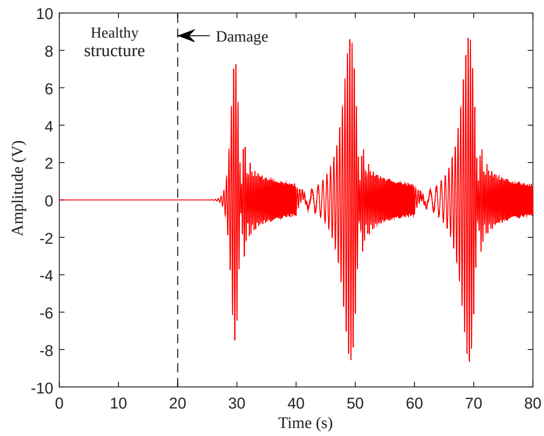
A damage controller is applied to attenuate the damage effect in the regular closed-loop system. The simulated results in the time domain are shown in Fig. 3. The same chirp signal used to analyse the LQR controller is considered as disturbance. The closed-loop response of the healthy structure is shown during the first period. Damage appears in the second period, provoking the visible vibration increase of 22% in the regular closed-loop system. Results shown in Fig. 3a represent vibration reduction gain due to the second controller signal addition. It can be seen that the damage controller contributes quickly to mitigate the damage effect in the structural vibration. Furthermore, Fig. 3c shows that the damage control signal $u_2(t)$ increases with damage effects. In Fig. 3b, the nominal controller decreases its action, returning to the levels when the structure was healthy.




(a) Lateral vibrations




(b) LQR control signal contribution in the double loop




(c) Adaptive control signal contribution in the double loop

Figure 3: Damaged structure vibration control

### 4.4 Damage detection

The damage detection is performed using the residue signal represented by Eq. (21). In Fig. 4, the residue signal of the closed-loop systems is compared. Both methodologies have rapid change in the residue signal amplitudes when damage appears at $20\,s$. Thus, a threshold can be used to determine damage occurrence in the structure. Moreover, the double-loop methodology conducts the structure response signal to the reference output signal. In other words, the damage controller is able to restore the original performance, reducing and isolating the damage effects over the structure dynamics.

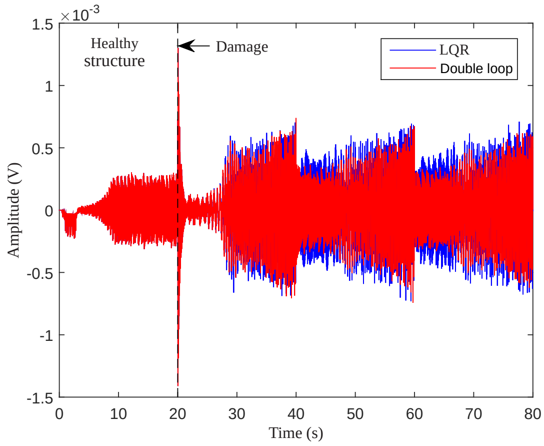



Figure 4: Residue signal

## 5. CONCLUSION

A control methodology for damage-tolerant active control was presented in this paper, based on a double-loop state-tracking control approach. In the outer loop, a LQR controller provides robustness in relation to the disturbance. The second controller, which is reconfigured online, has the objective to mitigate damage effects. The damage controller goal is to reduce the plant output measured error, considering the reference model output, which corresponds to the healthy closed-loop response. Simulation results show the effective vibration reduction of a structure submitted to damage. As future objectives, the methodology will be experimentally validated in the vertical structure.

## 6. ACKNOWLEDGEMENTS

The author Helói F. G. Genari gratefully acknowledges the financial support received from CNPq and CAPES ("Bolsista da CAPES - Proc. n° 12337/13-7").

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