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VALIDATION OF THE MODIFIED HDD EQUATION FOR THIN POROUS MEDIA BASED ON EXPERIMENTAL RESULTS

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Abstract. *The classical pressure-drop equation for porous media flow—the Hazen-Dupuit-Darcy equation, or simply HDD—relates the pressure-drop per unit of porous medium length of an isothermal, incompressible, porous medium saturating, Newtonian fluid flow in terms of the two constitutive quantities K and C it defines, which are known as the permeability and the form coefficient of the porous medium, respectively. It has been noted that the HDD equation becomes ambiguous for cases in which the pressure-drop caused by the porous medium no longer bears relation with the porous medium length, such as for the entire class of porous media that are sufficiently thin in the flow direction. To account for pressure-drops on this class of porous media, a modified HDD equation was proposed by Naaktgeboren* et al. *(2012) based on numerical results of laminar flow through porous media of simplified (collapsed) porous medium geometry and vanishing length. The proposed modified HDD equation is written in terms of modified K and C parameters, namely, K_m and C_m, that are themselves composed of a linear combination of two pressure-drop terms, one being dependent and the other, independent of porous medium length. This work attempts validating the modification of the HDD equation introduced by Naaktgeboren* et al. *(2012) with experimentally measured pressure-drops of air flowing across porous metallic foams of various lengths due to Baril* et al. *(2008) by means of least squares data modeling. The analysis shows that the modification of the HDD equation proposed by Naaktgeboren* et al. *(2012) is valid for the chosen experimental data-set. Moreover, expected values of Darcian, porous medium core permeability and form coefficients can be recovered from the thin porous media pressure-drop measurements and the data model hither employed.*

Keywords: *Pressure-Drop, Thin Porous Media, Modified HDD Equation, Model Validation, Data Modeling*

1. INTRODUCTION

Pressure-drop predictions for saturated porous medium flows of an isothermal, incompressible, Newtonian fluid are usually made by the classical Hazen-Dupuit-Darcy (HDD) equation, also known as the Darcy-Forchheimer equation (Lage, 1998), namely

$$\frac{\Delta P}{e} = \frac{\mu}{K}\bar{u} + \rho C \bar{u}^2, \tag{1}$$

where ΔP is the pressure-drop in Pa, e is a length in m, μ is the fluid kinetic viscosity in $\mathrm{Pa \cdot s}$, $\bar{u}$ is the average fluid velocity in $\mathrm{m/s}$, given by the ratio $\bar{u} = Q/A$ of volumetric flow rate, Q, in $\mathrm{m^3/s}$, and the (usually clear) flow passage area, A, in $\mathrm{m^2}$; and K and C are the two porous medium constitutive parameters defined by the HDD equation known as the porous medium permeability and form coefficient, respectively. The units of K and C are $\mathrm{m^2}$ and $\mathrm{m^{-1}}$, respectively.

According to the classical HDD equation, complete saturated porous medium hydraulic characterization is done by determining K and C, which is usually done experimentally by means of measurements of Q, A, ΔP, and e, once fluid properties are known (Antohe *et al.*, 1997; Narasimhan and Lage, 2003, 2004; Nield *et al.*, 1999).

When experiments are carried out such that the pressure taps are placed within the porous medium, ΔP is taken as the measured pressure-drop, while e, is taken as the distance between the pressure taps. On the other hand, when experiment setups have the pressure taps placed outside the porous medium, ΔP maintains its meaning as the measured pressure-drop, while e is reinterpreted to mean the enclosed porous medium length.

One can expect that placing the pressure taps outside the porous medium actually captures pressure-drop effects due to (i) flow adjustments necessary to enter the porous medium, (ii) porous medium flow, and (iii) flow adjustments upon exiting the porous medium, i.e., $\Delta P_{(i)} + \Delta P_{(ii)} + \Delta P_{(iii)}$. Only contribution (ii) entails a clear, unambiguous pressure-drop per unit of porous medium length relation—$\Delta P_{(ii)}/e$ follows the HDD model and is *not* a function of e. Conversely, while contributions (i) and (iii) may develop along certain clear fluid and porous medium lengths (Naaktgeboren *et al.*, 2004), they are, in fact, generally unrelated to porous medium length. Moreover, they are expected to exist for porous media of *any* length and appear on measurements whenever pressure taps are placed outside the porous medium. Their lack of porous medium e dependence causes $(\Delta P_{(i)} + \Delta P_{(iii)})/e$ to be likely proportional to $1/e$.

Whenever porous media core pressure-drop contribution *overwhelms* entrance and exit effects, $\Delta P_{(ii)} \gg (\Delta P_{(i)} + \Delta P_{(iii)})$, one can argue that $(\Delta P_{(i)} + \Delta P_{(iii)})/e$ becomes negligible in comparison with $\Delta P_{(ii)}/e$; however, since the degree of porous medium core pressure-drop dominance is linearly proportional to e, such porous media must be *overwhelmingly long*. Therefore, the issue being discussed narrows down the application of the classical HDD equation only to porous media core pressure-drops—the ones measured by placing pressure taps inside the core of the porous medium—or by overwhelmingly long porous media, despite being usually associated with "thin" porous media.

Naaktgeboren *et al.* (2005, 2012) have pointed out that the original Darcy experiment (Darcy, 1856) has been carried out with the pressure taps located outside the porous medium, and modern experimental studies are commonly carried out with either pressure tap configuration, as if porous medium core pressure-drop is always assumed to dominate over entrance and exit effects. They have investigated the aspect of a minimum porous medium length beyond which the core contribution becomes the largest one; thus, at that length, the inlet and exit effects pressure-drop effects would *match* the porous medium core pressure-drop effects—thus allowing investigators to apply their own definitions of porous medium core pressure-drop "dominance"—a concept that can be application dependent, for instance, by means of maximum allowed uncertainty, or just simply arbitrary.

Baril *et al.* (2008) have carried out an experimental validation study of the existence of porous medium entrance and exit pressure-drop effects with air flow through porous metallic foam samples of various lengths and placing pressure taps outside the porous samples. They have indeed observed the presence of inlet and exit pressure-drop contributions that in some cases were larger than the total pressure-drop imposed by the porous medium core. In their study, they have also defined and determined a critical porous medium thickness, beyond which the entrance and exit pressure-drop were judged insignificant, and have determined this thickness to be proportional to the specimen pore size, for the tested samples, and gave the figure of 50 times the pore size for the critical thickness of the tested specimens. No entrance and exit pressure-drop model equation was presented or dealt with by Baril *et al.* (2008) for the dependence of $\Delta P/e$ in terms of the porous medium length.

In the works of Naaktgeboren *et al.* (2005, 2012), a model equation was proposed in terms of a modification to the classic HDD equation as to account for porous medium to clear fluid entrance and exit pressure-drop effects. Modifications to the classical HDD equations have been proposed whenever novel physical effects are contemplated. For instance, Narasimhan and Lage (2001) have proposed a modification so as to account for fluid viscosity non-uniformities arising from wall heating interactions. Despite being always present and not accounted for by the classical HDD equation, the porous medium entrance and exit pressure-drop effect for an incompressible, Newtonian, saturated porous medium flow of uniform fluid properties have been only recently yielded a modification to the HDD equation, which is of the form

$$\frac{\Delta P}{e} = \frac{\Delta P_c + \Delta P_r}{e} = \frac{\mu}{K_m}\bar{u} + \rho C_m \bar{u}^2, \tag{2}$$

where ΔP_r is the inlet and exit (restriction) pressure-drop contributions, in Pa, ΔP_c is the porous medium core pressure-drop contribution—now ΔP is unambiguously the overall flow pressure drop, due to the presence of the porous medium, measured by pressure taps placed outside the porous medium, and e is unambiguously the porous medium length (no longer the distance between pressure taps)—K_m is the modified (overall) permeability, in m^2, and C_m is the modified (overall) form coefficient, in m^{-1}, given by

$$\frac{1}{K_m} = \frac{1}{K} + \frac{\kappa}{ed}, \qquad \text{and} \qquad C_m = C + \frac{\chi}{e}, \tag{3}$$

where ed/κ is an inlet and exit (restriction) permeability, κ being the dimensionless restriction viscous-drag coefficient, and d being the average porous medium pore size, in m; and χ/e is an inlet and exit (restriction) form coefficient, χ being the dimensionless restriction form-drag coefficient.

From Eqs. (2) and (3), one has:

$$\Delta P = \Delta P_c + \Delta P_r = e\left(\frac{\mu}{K}\bar{u} + \rho C \bar{u}^2\right) + \left(\frac{\mu\kappa}{d}\bar{u} + \rho\chi\bar{u}^2\right), \tag{4}$$

which explicitly gives the total pressure drop, ΔP, as the sum of core porous medium contributions, ΔP_c, which are linearly related to porous medium length e in a classical, HDD manner, and inlet/exit contributions, ΔP_r, which bears no relation to porous medium length—note the absence of e in the second parenthesis term—while keeping HDD-like features, such as the linear and quadratic dependence on average flow velocity, each velocity term defining its own constitutive parameter.

In spite of the consistency with the original HDD equation, which can be recovered from the modified equation by setting $\kappa = \chi = 0$, and the alignment with the theoretical expectations, in the sense that the pressure-drop is split between a component that does depend on the porous medium length in an HDD fashion (the core one), and on another component that does not depend on the porous medium length (the inlet/exit one), the model was only tested in the original works of Naaktgeboren *et al.* (2005, 2012) (i) numerically, (ii) for wall-bounded, laminar, two-dimensional and steady flow, and (iii) on a very simplified porous medium geometry, obtained by not only collapsing the porous medium in the flow direction, but also by grouping together all the pores into a single, larger center pore, yielding thus a slotted or perforated infinitely thin plate geometry with equivalent void surface fraction as a given porous medium. The proposed modified HDD model has thus never been validated by experiments.

Owing the lack of experimental validation, this work aims at validating the modified HDD pressure-drop model equation proposed by Naaktgeboren *et al.* (2005, 2012), with experimental data of Baril *et al.* (2008).

2. EXPERIMENTAL RESULTS FROM LITERATURE

Baril *et al.* (2008) have performed pressure-drop measurement experiments with metallic Ni-Cr open cell foams (`NC 2733`, and `NCX1116`, RECEMAT International), whose geometrical characteristics and test parameters are listed on Tab. 1. Figures 1 and 2 are Baril's test results for the `NC 2733` and `NCX1116` foam specimens, respectively.

Table 1. Geometrical characteristics and test parameters for the tested Ni-Cr foams. Compiled from Baril *et al.* (2008).

Grade	d (mm)	ϕ (%)	e range (mm)	$\bar{u}$ range (m/s)
`NC 2733`	0.6	89	2–56	2–20
`NCX1116`	1.4	89	2–63	2–20

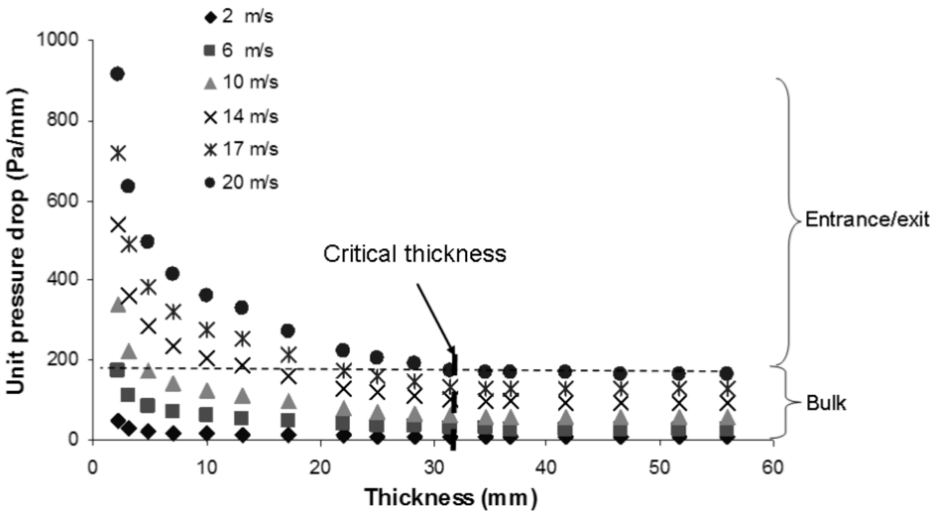

Figure 1. Pressure-drop per unit of porous medium length for the `NC 2733` foams. Source: Baril *et al.* (2008).

Figure 1 shows $\Delta P/e$ versus e for pressure taps placed outside each porous medium tested for several values of mean flow velocity $\bar{u}$ evaluated by volumetric flow rate and clear flow passage area. According to the classical HDD equation, one is expected to observe no variation of $\Delta P/e$ with increasing e; however, the experimental data shows that this is not so, especially for lower values of e, but this seems to be the case for higher values of e, indicated in the figures by the so-called critical thickness. Since the pressure taps were placed outside each porous medium, the excess pressure drop per unit of porous medium length must come from entrance and exit effects, since those, not being related to the porous medium length, must scale as $1/e$.

3. MODIFIED HDD MODEL VALIDATION

Numerical values of $\Delta P/e$ and e were recovered from Figs. 1 and 2 by recording each marker's centroid location in pixel coordinates and applying a linear transformation based on the horizontal and vertical axis scales of each figure. No uncertainty information was taken from the experimental data owing to the lack of error bars in Figs. 1 and 2.

The modified HDD model for thin porous media, Eq. (4), clearly states that both porous medium core and inlet/exit pressure-drop contributions have components that scale linearly and quadratically with the average flow velocity $\bar{u}$. Since pressure taps were placed outside each porous medium tested, one expects inlet/exit pressure-drop effects to be present in the measurements; therefore, the $\bar{u}$ pressure-drop dependence is verified by regrouping the data by average velocity while keeping the thickness constant within each group. In the bounds of the data points of such groups, one is expected to see only the HDD relation between $\Delta P/e$ and $\bar{u}$, since porous medium thickness is kept constant.

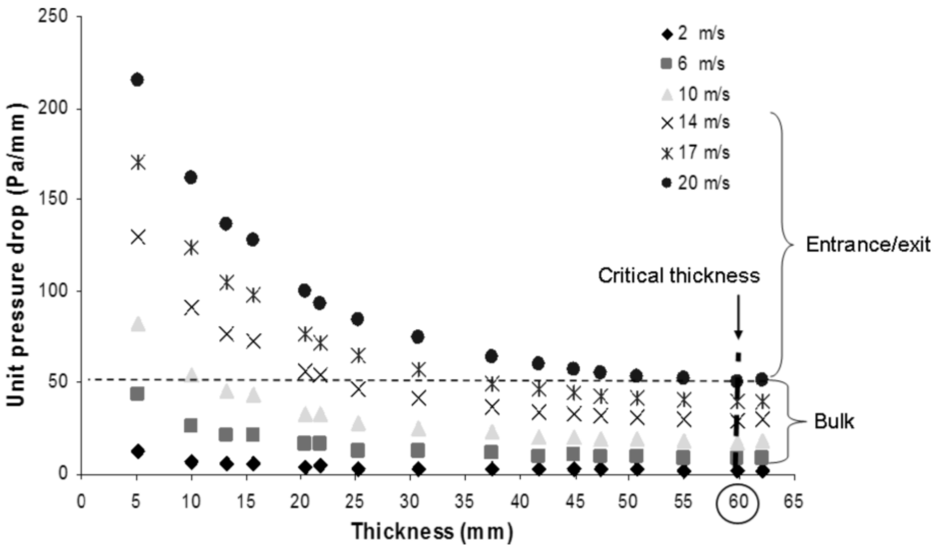

Figure 2. Pressure-drop per unit of porous medium length for the `NCX1116` foams. Source: Baril *et al.* (2008).

The linear and quadratic coefficients can be found by least squares by modeling each thickness data set i with

$$g_i(\bar{u}) = b_0^i \bar{u} + b_1^i \bar{u}^2. \tag{5}$$

Sample Eq. (5) curve-fit results are depicted on Fig. 3 for the `NC 2733` and `NCX1116` foams. A very good agreement between the model and experimental data is observed for every thickness data set i. Since inlet/exit pressure-drop effects are present in the underlying experimental data, the agreement *validates* the $\bar{u}$ dependence of the entrance/exit pressure-drop effects set forth by the modified HDD model equation.

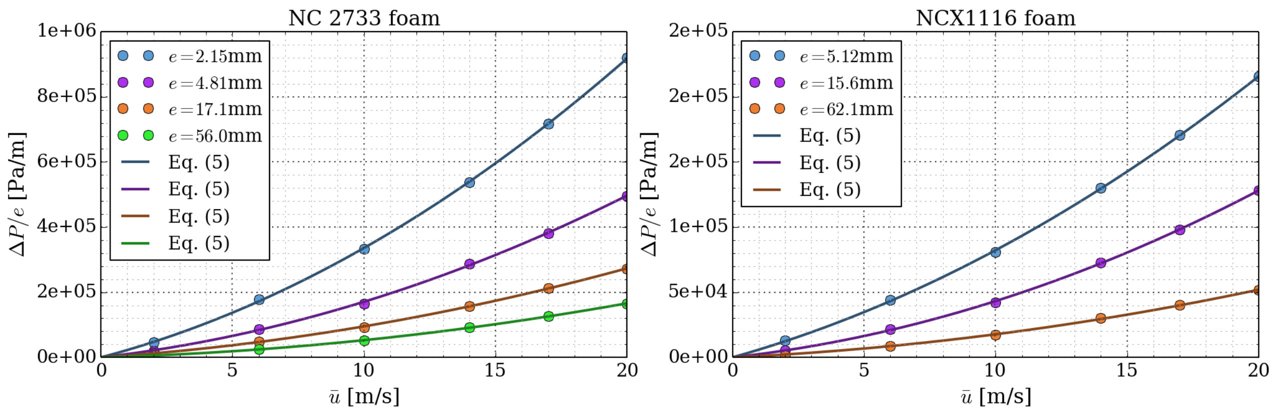

Figure 3. Experimental (circles) and curve-fit (solid lines) results of $\Delta P/e$ versus $\bar{u}$ for the `NC 2733` foam (left) and the `NCX1116` foam (right) for selected porous medium length e. The modeled linear and quadratic $\bar{u}$ dependence correlates very well with the data, even with significant inlet/exit pressure-drop effects (shorter e results).

In one hand, values of b_0^i and b_1^i obtained by modeling the data regrouped by velocities correspond to the *constant* μ/K and ρC terms of the HDD model equation, respectively. On the other hand, when using the HDD model modified for thin porous media, values of b_0^i and b_1^i correspond to the groups

$$b_0^i = \frac{\mu}{K_m} = \frac{\mu}{K} + \frac{\mu\kappa}{ed}, \qquad \text{and} \qquad b_1^i = \rho C_m = \rho C + \frac{\rho\chi}{e}, \tag{6}$$

which have an explicit e dependence, scaled as $1/e$, that is set forth by Eq. (6).

Table 2 presents the obtained b_0^i and b_1^i values by fitting the experimental pressure-drop data with Eq. (5), which correspond to the μ/K and ρC values one would obtain when applying the HDD equation for the tested porous media without the knowledge of entrance/exit pressure-drop effects and placing the pressure taps outside the porous medium.

It is worth noting that b_0^i values for the `NC 2733` foams cover a full order of magnitude, since this foam was tested for very thin configurations. This realization underlines the importance of properly accounting for porous media entrance/exit

Table 2. Curve-fitted $b_{0,1}^i$ values for both Ni-Cr foams.

NC 2733 foam			NCX1116 foam		
e^i (mm)	b_0^i $(\mathrm{Pa \cdot s/m^2})$	b_1^i $(\mathrm{Pa \cdot s^2/m^3})$	e^i (mm)	b_0^i $(\mathrm{Pa \cdot s/m^2})$	b_1^i $(\mathrm{Pa \cdot s^2/m^3})$
2.15	21280	1231	5.12	5665	256.2
3.11	12260	973.3	10.0	2689	271.7
4.81	9312	776.8	13.2	2144	235.9
7.06	6937	697.9	15.6	2236	208.6
10.0	6219	589.8	20.4	1592	171.3
13.1	5105	572.8	21.7	1872	139.5
17.1	5363	415.7	25.3	1151	155.8
22.1	3960	362.1	30.7	1196	127.5
25.1	3063	366.0	37.5	1334	93.9
28.4	2967	333.3	41.8	947.0	104.2
31.6	3113	281.1	44.9	1076	91.3
34.7	2449	302.0	47.2	1060	86.1
36.8	2237	314.0	50.8	1093	79.9
41.7	1979	325.3	55.0	888.7	87.6
46.6	2266	302.0	59.9	817.1	86.7
51.7	2266	302.0	62.1	949.2	82.4
56.0	2266	302.0	—	—	—

pressure-drop effects in the determination of bulk porous medium hydraulic parameters, while placing pressure taps outside the porous sample.

Equation (6) suggest the $b_{0,1}^i$ data to be modeled by the following function basis:

$$b_0(e) = a_0 + \frac{a_1}{e}, \qquad \text{and} \qquad b_1(e) = a_2 + \frac{a_3}{e}. \tag{7}$$

Upon inspection of Eqs. (6) and (7), one immediately recognizes:

$$a_0 = \frac{\mu}{K}, \qquad \text{and} \qquad a_1 = \frac{\mu\kappa}{d}, \qquad \text{and} \qquad a_2 = \rho C, \qquad \text{and} \qquad a_3 = \rho\chi. \tag{8}$$

Moreover, if Eq. (7) is able of well representing the experimental data, then the modified HDD equation, Eq. (2), is valid for the underlying porous medium types (metallic foam). This can be verified by the Fig. 4 for the `NC 2733` foam and by the Fig. 5 for the `NCX1116` foam.

4. MODEL VALIDATION

Figures 4 and 5 show good agreements between the modified HDD model equation and the experimentally obtained b_0^i and b_1^i data for the entire range of tested e, which suggests that the modified HDD model should be *valid* also for metallic foam porous media types.

The validity of the modified HDD model, Eq. (2), entails interesting consequences, the first being the possibility of determining the bulk porous medium hydraulic parameters K and C, *even when only thin porous medium samples are available for pressure-drop experimentation*, by a similar curve-fitting procedure described in this work.

Table 3 lists curve-fitted values of a_0, a_1, a_2, and a_3 and the corresponding porous medium hydraulic parameters K, C, κ, and χ values obtained by plugging in the fluid properties in the a_i definitions.

5. DISCUSSION

Porous metallic foam pressure-drop measurements with pressure taps located outside the porous samples due to Baril *et al.* (2008) *validate* the modified HDD equation proposed by Naaktgeboren *et al.* (2005, 2012).

The validation began by the confirmation of the $\bar{u}$ dependence on the ΔP data, for porous foam samples of increasing flow direction lengths e; hence, in the presence of various entrance and exit to porous medium core pressure-drop effect ratios. If ΔP_r was not, in fact, the sum of linear and quadratic average flow velocity pressure-drop contributions, as modeled by Eq. (2), then one wouldn't be able to represent the experimental data as closely by the modified HDD model as what is depicted in Fig. 3.

It can be said that pressure-drop resulting from flow adjustments upon entering and leaving either a perforated slot or plate, (Naaktgeboren *et al.*, 2012), or metallic foam types of porous medium very positively follow a $c_0\bar{u} + c_1\bar{u}^2$

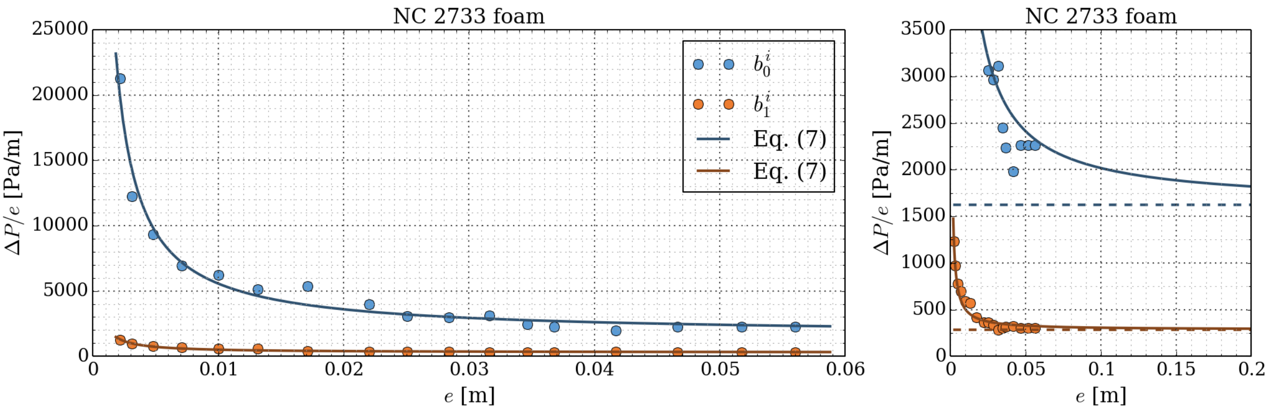

Figure 4. Plots of $b_0^i(e)$ and $b_1^i(e)$ for the `NC 2733` foam (circles) so that $b_0^i\bar{u} + b_1^i\bar{u}^2$ fits the i-th experimental $\Delta P/e_i$ data. Lines correspond to the modified HDD model. The left plot is scaled to the entire data set. The right plot zooms in on higher e values, showing model extrapolation and the $\Delta P/e$ value asymptotic in e with dashed lines.

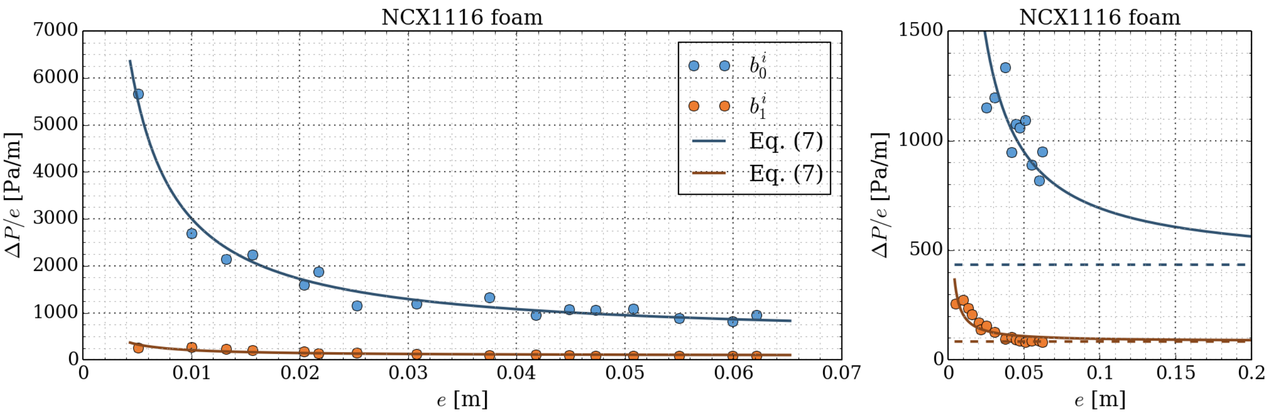

Figure 5. Plots of $b_0^i(e)$ and $b_1^i(e)$ for the `NCX1116` foam (circles) so that $b_0^i\bar{u} + b_1^i\bar{u}^2$ fits the i-th experimental $\Delta P/e_i$ data. Lines correspond to the modified HDD model. The left plot is scaled to the entire data set. The right plot zooms in on higher e values, showing model extrapolation and the $\Delta P/e$ value asymptotic in e with dashed lines.

Table 3. Modified HDD model hydraulic parameter values for the `NC 2733` and `NCX1116` foams.

Parameter	`NC 2733`	`NCX1116`
$a_0 = \mu/K$	1623.5	433.89
$a_1 = \mu\kappa/d$	39.415	25.798
$a_2 = \rho C$	284.20	83.501
$a_3 = \rho\chi$	2.1776	1.2302
K (m^2)	1.1456×10^{-8}	4.2868×10^{-8}
C (m^{-1})	243.11	71.430
κ	1271.5	1941.8
χ	1.8628	1.0524

relationship, as laid out by the modified HDD model equation. Since these two porous medium geometries are broadly different from each other, it is less doubtful that this relationship might be valid porous medium geometries whose core configurations obey the HDD law.

Since entrance and exit pressure-drop follow a relationship with respect to flow velocity of the same mathematical kind as the porous medium core does, they silently modify K and C values whenever the HDD equation is applied in association with pressure taps placed outside porous media. This feature helps explaining why it took so long for them to be noticed.

The modified HDD model validation proceeded by the verification of the functional forms of $1/K_m$ and C_m with respect to porous medium length e, which the modified HDD equation establishes as being $c_0 + c_1/e$, c_0 and c_1 being constants, according to Eq. (3), or, conversely, as Eq. (7).

Figs. 4 and 5 shows that the modified HDD model captures the data trend, closely following the experimentally fitted $b_0 = \mu/K_m$ and $b_1 = \rho C_m$ coefficient data for the entire range of tested porous medium lengths, e. This assertion fully validates the modified HDD model of Naaktgeboren *et al.* (2012) for porous metallic foams, which in turn was obtained numerically from slotted and perforated infinitesimally thin plates. Since these porous medium geometries are vastly different from each other, it is less doubtful that this relationship might be valid porous medium geometries whose core configurations obey the HDD law.

5.1 The concept of "critical thickness"

Is there a particular porous medium length (or thickness) that causes pressure taps placed outside the porous medium *not* to measure entrance and exit pressure-drop effects?

The modified HDD model makes no specific prediction of a critical porous medium thickness, beyond which the inlet/exit pressure-drop effects would cease to exist, according to Eqs. (2) and (3). Therefore, in this sense, there is no such critical porous medium thickness.

Conversely, it would be of practical interest to have a simple critical thickness rule, in terms of a given number of pore diameters worth of porous medium length, that would guarantee that porous medium K and C values could be accurately determined by the HDD equation associated with pressure-drop measurements made with pressure taps placed outside the porous medium—such as the 50 pore rule given by Baril *et al.* (2008).

This sort of concept is inherently problematic, since different accuracy requirements would lead to vastly different values of the so called critical thickness, as reducing the accuracy requirement by a factor of 10, would necessarily lead to a new critical thickness that is 10 times larger than one based on the original accuracy requirement.

Another problem with embracing a critical thickness concept is that it risks being understood in such a way as to make entrance and exit effects completely nonexistent for sufficiently long porous media, which is in contradiction with placing pressure taps outside the porous medium—recall from earlier discussion that these pressure-drop contributions are always present.

Figures 1 and 2 precisely depict this misconception. In them, the authors divide the $\Delta P/e$ ordinates in two regions, namely, "Entrance/exit," and "Bulk," thus indicating that a *null* entrance/exit pressure-drop regime has been reached within the short, finite porous medium thickness tested.

Even under the assumptions of (a) a consensus in the determination of the K and C accuracy factor and (b) the establishment of an adequate conceptual understanding of what a critical thickness would mean; the validation of the modified HDD model poses additional problems to the determination of a critical thickness that is proportional to the pore size. Let $\mathbb{A} \ll 1$ be the dimensionless accuracy factor, such that $\Delta P_c = \Delta P_r/\mathbb{A}$ for the critical thickness $e_{cr} = h_{cr} d$, h_{cr} being the number of average pores with size d—for Baril *et al.* (2008), $h_{cr} \approx 50$.

From Eq. (4), making $\Delta P_c = \Delta P_r/\mathbb{A}$ reveals that e_{cr}, and also h_{cr}, are at least functions of the average flow velocity $\bar{u}$, and might be functions of pore size d:

$$e_{cr}(\bar{u}, d) = \frac{1}{\mathbb{A}} \cdot \frac{\mu\kappa/d + \rho\chi\bar{u}}{\mu/K + \rho C\bar{u}} = \frac{1}{\mathbb{A}} \cdot \frac{a_1 + a_3\bar{u}}{a_0 + a_2\bar{u}}. \quad \text{and} \quad h_{cr}(\bar{u}, d) = \frac{1}{\mathbb{A}} \cdot \frac{\mu\kappa + d\rho\chi\bar{u}}{d^2(\mu/K + \rho C\bar{u})} = \frac{1}{\mathbb{A}} \cdot \frac{a_1 + a_3\bar{u}}{d(a_0 + a_2\bar{u})}. \tag{9}$$

The $\bar{u}$ dependence is sure, for very low flow average velocities, $\mathbb{A}e_{cr}$ approaches a_1/a_0—the linear porous medium bulk and inlet/exit pressure-drop terms dominate the quadratic ones—while for high average flow velocities, $\mathbb{A}e_{c,min}$ approaches a_3/a_2—the quadratic bulk and boundary pressure-drop terms dominate the linear ones. In general, one expects $a_1/a_0 \neq a_3/a_2$, making e_{cr} and h_{cr} at least a function of average flow velocity in general.

Further studies need to me made as to determine whether $h_{cr}(\bar{u}, d)$ collapses into $h_{cr}(\bar{u})$ only, say, by inlet and exit pressure-drop effects being linearly dependent upon pore size.

Even with the possibility of h_{cr} not being a function of pore size, the concept of a critical porous medium thickness beyond which entrance and exit pressure-drop effects become negligible (a) inherits the arbitrariness hidden in the "negligible" term, also shown by the $\mathbb{A}$ term on Eq. (9), (b) risks causing misconceptions about entrance and exit effects on longer porous media, and (c) is at least flow velocity dependent, thus hindering attempts to find a general critical thickness value for porous media.

Therefore, the modified HDD model for pressure taps placed outside the porous medium is a stronger, more consistent approach to porous medium characterization than attempting defining a so called critical thickness.

5.2 Are entrance and exit pressure-drop effects easily negligible?

Equation (9) along with the values of Tab. 3, allow for the determination of the accuracy factor used by Baril *et al.* (2008), yielding $\mathbb{A} = 0.3548$ for the `NC 2733` foam and $\mathbb{A} = 0.3993$ for the `NCX1116` foam, which are inconsistent and also seem to be very inadequate as $\mathbb{A}$ must be $\ll 1$. Moreover, adopting $\mathbb{A} = 0.01$, which will make core pressure-drop 100 times the entrance and exit ones, yield $e_{cr} = 1.135$ m ($h_{cr} = 1892$ pore sizes) for the `NC 2733` foam and $e_{cr} = 2.396$ m ($h_{cr} = 1711$ pore sizes) for the `NCX1116` foam, both for $\bar{u} = 20$ m/s. If $\bar{u} = 2$ m/s, then figures change significantly to $e_{cr} = 1.997$ m for the `NC 2733` foam and $e_{cr} = 4.702$ m for the `NCX1116` foam.

The quite moderate values of $\mathbb{A}$ (indirectly) adopted by Baril *et al.* (2008) might have stemmed from the apparent stabilization of the experimental pressure-drop data, encouraged by the illusion caused by the scale used in Figs. 1 and 2, the inherent presence of uncertainties in the measurements, that could have obscured a continuing descending trend with increasing porous medium length of the data, and lack of tests done with radically longer foam samples or with pressure taps located within the porous foams, which could have read a pressure-drop per unit of porous medium length value that the modified HDD model now predicts to be much lower than the ones observed for the lengthiest tested foams.

In fact, the modified HDD model states that the *relative* importance of the inlet/exit effects compared with the core ones decays proportionally as $1/e$. The right plots of Figs. 4 and 5 illustrate that a $1/e$ type of decay can actually be *very slow*. In fact, it is a well known mathematical fact that the harmonic series, $\sum_{n=1}^{\infty} 1/n$, is a sum of terms that decay slowly enough to cause the series to actually *diverge*.

The right subplots on Figs. 5 and 4 allow for no visual asymptote on the b_0 data, which are confirmed by the position of the dashed lines, which are the mathematical model asymptote. Thus, Figs. 5 and 4 show that the modeled asymptote can be quite different than a value obtained by visual inspection.

The porous medium bulk parameters are related to the asymptotic values $a_0 = \mu/K$, and $a_2 = \rho C$, as laid out by Eq. (8). Table 3, for instance, lists $a_0 = 1623.5$, and $a_2 = 284.20$ for the `NC 2733` foam, but $\Delta P/e$ versus $\bar{u}$ fits for e greater than the indicated critical thickness—the b_0 and b_1 data of Tab. 2 for higher e_i values—yield $\mu/K = 2266$, and $\rho C = 302.0$, therefore underestimating K by 28.4% and overestimating C by 6.3%. A similar calculation for the `NCX1116` foam finds that, using the indicated critical thickness $\Delta P/e$ versus $\bar{u}$ data underestimates K by 46.9% and overestimates C by 3.8%. This is consistent with the quite moderate values found for $\mathbb{A}$, and lead to very different values of K and C with respect to the ones given by the modified HDD model.

Finally, not only the bulk K and C parameters can be determined by the procedure outlined in this work, but also κ and χ, thus completing the modified HDD model. The determination of the four porous medium hydraulic parameters, K, C, κ, and χ, allows for total porous medium pressure-drop characterization—the total pressure drop introduced by having the porous medium placed in a flow circuit, as a function of the porous medium length.

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