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# Inflight Parameter Estimation based on Output Error Method

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***Abstract.*** *This paper aims at performing a feasibility analysis of an inflight aircraft parameter estimation method. By inflight is meant iterative estimation during the flight maneuvers execution, but without a real-time processing constraints. Based on the time domain Maximum Likelihood Estimation principle, the Output Error Method was selected for parameter estimation.*
*The proposed inflight parameter estimation method is based on a predefined model and searches for an optimal set of longitudinal motion parameters that describes an already known aircraft longitudinal dynamics. This is a necessary first step, since the estimated parameters can be compared to the true ones.*
*Inflight parameter estimation can bring significant savings in time and fuel consumption to flight test campaigns, providing quantitative results of the parameter estimation accuracies to the flight test engineer and pilots. Hence, quicker decision about when to terminate the flight tests can be made.*

***Keywords:*** *Inflight Estimation, Output Error Method, Flight Tests*

## 1. INTRODUCTION

System identification is a multidisciplinary area that investigates the physical system behavior, starting by a model postulation and then devising proper to enable the input-output measurements gathering. This data is used to find an optimized set of parameter values for the postulated model, in a way that best describes the system behavior. This work focus on a supplementary flight test decision method, using a longitudinal aircraft dynamics simulation and implementing an inflight parameter estimation technique based on Maximum Likelihood Estimation principle. The Output Error Method was selected due to its simplicity, fast computational processing and robustness, when compared to other time domain methods such as Filter Error Method and Equation Error Method.

A method is proposed and applied for each maneuver, for automatically deciding if the maneuvers are adequate to support the inflight parameter estimation, informing to the flight test engineers and pilots the quality of estimations and recommending the flight test duration. The main contribution with respect to Deiler (2012) is the use of standard deviation of parameter estimates as an additional decision method with minimal computational effort. The proposed method could be expanded in future works to an inflight parameter estimation tool to be applied to support real flight campaigns, following a procedure similar to Deiler (2012).

## 2. AIRCRAFT DYNAMIC MODEL

The aircraft motion can be described as the fixed wing rigid body 6-DOF EOM (Equations of Motion), influenced by forces and moments, mainly from gravity, aircraft thrust and aerodynamic effects. Conveniently, on the pursuit of reducing the computational costs to solve the set of EOM, they are simplified to uncoupled motion particular cases, such as longitudinal and lateral aircraft motion.
The aerodynamic forces and moments estimation are the main point of the aircraft system identification problem, due to observability difficulties and an intrinsic uncertainty related to the aerodynamic derivatives estimation. These aerodynamic derivatives are the model parameters to be estimated, representing the individual forces and moments contributions of each aircraft aerodynamic part, such as wings, flaps, stabilizer, and control surfaces influences, obtained from a first order linearization of the nonlinear aerodynamic model. The aerodynamic derivatives are very useful parameters to feed the flight dynamics models, improving the design of flight control system, loads assessment and flight test simulators, and are estimated by many ways throughout the aircraft development lifecycle, starting by Computational Fluid Dynamics (CFD) simulations, later validating a prototype on wind tunnel and finally verifying these values with flight test data analysis.

The proposed inflight parameter estimation will be based on a simplified longitudinal modeling approach, only considering the short period response dependant from the states of angle of attack ($\alpha$) and pitch rate ($q$), and the input of elevator deflection angle ($\delta_e$). The aircraft velocity, trajectory angle and altitude will be assumed constant during the entire simulation. Equations (1) and (2) describes the simplified longitudinal model:

$$\dot{\alpha} = Z_0 + Z_\alpha * \alpha + (1 + Z_q) * q + Z_{\delta e} * \delta e \tag{1}$$

$$\dot{q} = M_0 + M_\alpha * \alpha + M_q * q + M_{\delta e} * \delta e \tag{2}$$

where $Z_0, Z_\alpha, Z_q, Z_{\delta e}, M_0, M_\alpha, M_q$ and $M_{\delta e}$ are the nondimensional aerodynamic stability and control derivatives, and are denoted in Eq. (3) as the parameters vector $\boldsymbol{\theta}$.

$$\boldsymbol{\theta} = [Z_0, Z_\alpha, Z_q, Z_{\delta e}, M_0, M_\alpha, M_q, M_{\delta e}] \tag{3}$$

## 3. MAXIMUM LIKELIHOOD PARAMETER ESTIMATION

The parameter estimation is the search for an set of estimated parameters vector $\hat{\boldsymbol{\theta}}$ that gets sufficiently close to the unknown real physical system parameters vector $\boldsymbol{\theta}$ of an assumed model structure, based on noisy measurement obtained from real system. The Fischer's uncertainties model for the parameters and measurements will be adopted (Schweppe, 1973), where the $\boldsymbol{\theta}$ is a vector of unknown constant parameters and the $\boldsymbol{\nu}$ is a random vector with probability density $p(\boldsymbol{\nu})$, respectively. The mathematical model of a dynamic system, without taking into account the process noise, have the following nonlinear general represesentation by Eqs. (4), (5) and (6):

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\theta}], \quad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{4}$$

$$\boldsymbol{y}(t) = \boldsymbol{h}[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\theta}] \tag{5}$$

$$\boldsymbol{z} = \boldsymbol{y} + \boldsymbol{\nu} \tag{6}$$

where $\boldsymbol{x}$ is a vector of state variables with initial conditions $\boldsymbol{x}_0$ , $\boldsymbol{y}$ the aircraft output vector, $\boldsymbol{z}$ is the measurement output vector, $\boldsymbol{u}$ is the input vector, $\boldsymbol{f}$ and $\boldsymbol{h}$ are general nonlinear functions, and $\boldsymbol{\nu}$ is the measurement noise error, assumed as a gaussian distribution random variable $\mathbb{N}(0, \boldsymbol{Q})$, i.e., with zero mean and covariance matrix $\boldsymbol{Q}$.

### 3.1 Output Error Method

The Output Error Method (OEM) is adopted as the parameter estimation method, consisting in a batch maneuver processing based on the difference between measured flight data $\boldsymbol{z}$ and the estimated output $\hat{\boldsymbol{y}}$, as shown in Fig. 1. This method was chosen for exhibiting a good robustness, simplicity and reduced computational costs. The scalar cost function

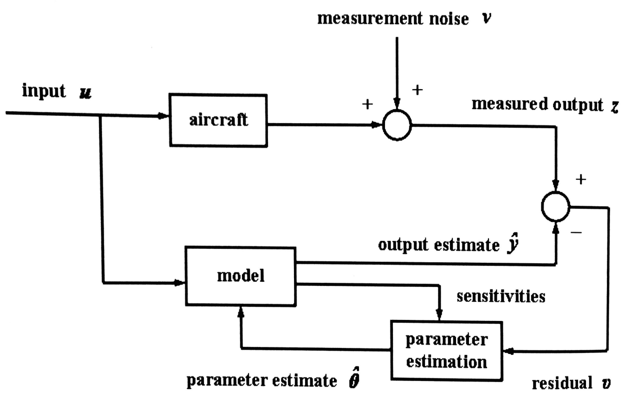


Figure 1. Output-Error Parameter Estimation (Klein and Morelli, 2006)

is derived from a Maximum Likelihood function simplification, based on the determinant of the residual error covariance matrix $\boldsymbol{R}$, and according to Deiler (2012) the determinant can be approximated by the product of the main diagonal terms $\sigma_i$ of the matrix $\boldsymbol{R}$, as in Eq. (7).

$$J(\hat{\boldsymbol{\theta}}) = det(\boldsymbol{R}) \approx \prod_{i=1}^{n} \sigma_i^2 \tag{7}$$

$$\boldsymbol{R} = E\{\hat{\boldsymbol{v}}^T \hat{\boldsymbol{v}}\} \tag{8}$$

$$\hat{\boldsymbol{v}} = \boldsymbol{z}(\boldsymbol{\theta}) - \hat{\boldsymbol{y}}(\hat{\boldsymbol{\theta}}) \tag{9}$$

### 3.2 Nonlinear Optimization Method

The parameter estimation block (see Fig. 1) is basically an optimization problem, adjusting the parameter estimate vector that generate the lowest value of cost function $J(\hat{\boldsymbol{\theta}})$. This optimization problem is a critical issue on inflight parameter estimation, due to the potentially high computational processing time cost and algorithm nonconvergence issues. For the aircraft parameter estimation problem, any multivariable nonlinear optimization method, constrained or unconstrained, can be applied.
There are many multivariable nonlinear optimization algorithms available in literature, including the most used Gauss-Newton and Levenberg-Marquardt methods (Jategaonkar, 2006). In this work the *fminsearch* was selected, a *Matlab*® unconstrained nonlinear optimization command based on Nelder Mead Simplex algorithm, due to its simplicity, the good convergence behavior, overall performance and stability.

## 4. INFLIGHT PARAMETER ESTIMATION

The inflight parameter estimation, also known as online parameter estimation, is a parameter estimation that occurs during the flight test iteratively at each maneuver execution. This method can bring interesting benefits as the reduction of repetitive redundant maneuvers, significantly shortening the flight test campaign duration. It can also can detect instrumentation problems or degraded aerodynamic performance, e.g. damage or icing formation to the lifting surfaces. This estimation approach is not intend to replace the flight data processing on ground, but act as a supporting tool on preliminary analysis.

Figure 2 shows the proposed inflight parameter estimation method, with the flight test validation based on the last three applied maneuvers.

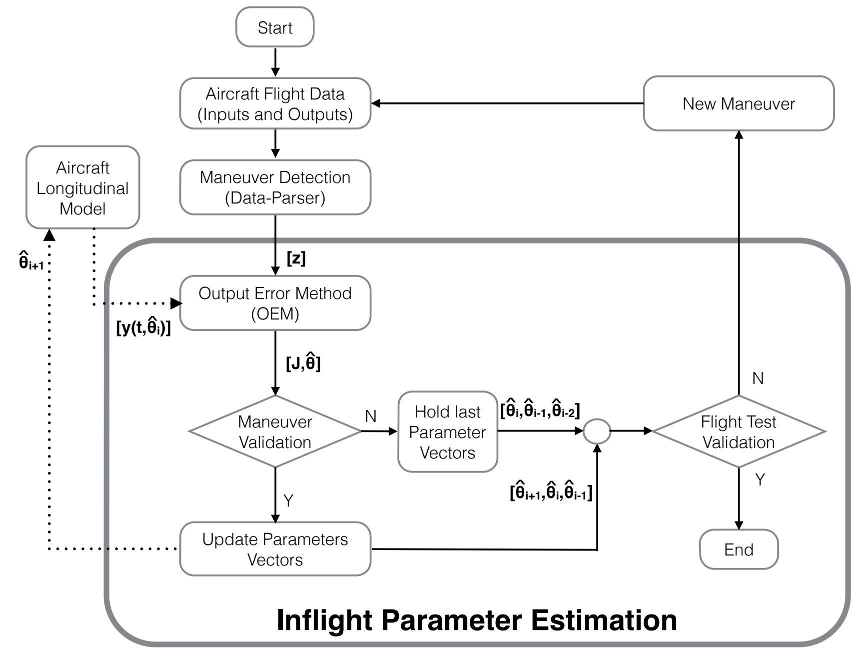


Figure 2. Inflight parameter estimation method

A multiple sequence of maneuvers feeds the inflight parameter estimation method, being preprocessed by a data-parser to automatically select the flight data segment associated to each maneuver (Deiler, 2014). For each maneuver, the OEM is executed, yielding the estimated parameters $\hat{\boldsymbol{\theta}}_i$ with their respective associated cost function $J(\hat{\boldsymbol{\theta}}_i)$, being validated by two decision methods, the maneuver validation and flight test validation, as shown in Fig. 2.

### 4.1 Maneuver Validation

The maneuver validation is a decision method to determine, which iterative parameter estimate should be incorporated, or in other words, when the parameter estimation should be updated or discarded, for each maneuver execution. The validation is essential for discarding an estimation at some iteration, when a poor maneuver is detected. This way, a poor parameter estimate does not hamper the parameter estimation procedure.

Equations (10) and (11) presents two distinct decision method criteria for maneuver validation (Deiler, 2012), the continuous improvement and separate computaional methods, respectively.

- Continuous Improvement

$$J(\hat{\boldsymbol{\theta}}_i) < 2 \cdot J(\hat{\boldsymbol{\theta}}_{i-1}) \tag{10}$$

- Separate Computation

$$J(\hat{\boldsymbol{\theta}}_i) < \text{median}(J(\hat{\boldsymbol{\theta}}_1) \ldots J(\hat{\boldsymbol{\theta}}_{i-1})) \tag{11}$$

### 4.2 Flight Test Validation

The flight test validation is the main contribution of this paper, suggesting a secondary decision method to decide if the last executed group of maneuvers are contributing to the overall estimation, defining when is needed to apply new maneuvers or assume that the parameter inflight estimation is good enough to stop the campaign.

The timeframe of the last executed groups of maneuvers selection is represented by the gray boxes in Fig. 3, where *n* is a predefined size of the last executed maneuvers group and *i* is the total number of executed maneuvers.

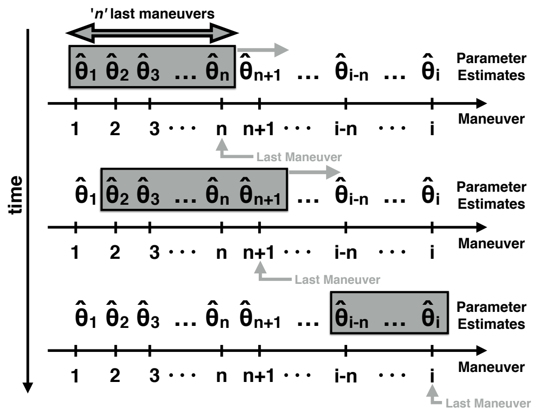


Figure 3. Last executed group of maneuvers timeframe

This decision method predefine a flight test stop criteria value based on the standard deviation of the estimated parameters obtained from the *n* latest accepted maneuvers execution divided by the average of the parameter estimates, as shown in Eq. (12) and Eq. (13).

$$\text{norm}\left(\frac{5 \cdot \boldsymbol{\sigma}(\hat{\boldsymbol{\theta}}_{i-n} \ldots \hat{\boldsymbol{\theta}}_i)}{avg(\hat{\boldsymbol{\theta}}_{i-n} \ldots \hat{\boldsymbol{\theta}}_i)}\right) \cdot 100\% < 25\% \tag{12}$$

$$\left\| \frac{5 \cdot \sum\limits_{k=i-n}^{i} \left(\hat{\boldsymbol{\theta}}_{\boldsymbol{k}} - \overline{\hat{\boldsymbol{\theta}}}\right)^2}{n \cdot \overline{\hat{\boldsymbol{\theta}}}} \right\| \cdot 100\% < 25\% \tag{13}$$

where *i* is the total number of executed maneuvers and the parameters average term $\overline{\hat{\boldsymbol{\theta}}}$ is given by Eq. (14).

$$\overline{\hat{\boldsymbol{\theta}}} = \sum_{k=i-n}^{i} \frac{\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}}{n} \tag{14}$$

For a defined length *n* of the maneuvers block, shown in Fig. 3, there is a quick decay in the standard variation, as the maneuvers are executed. At each maneuver execution, the block of maneuvers is displaced to become the new block, and as the maneuvers blocks are processed, a more regular behaviour is expected for the standard variation. Thus, the choice of *n*, the block size, is important for the proposed decision method. For instance, small values of *n* can prematurely ends the flight test or big values might unnecessarily prolong the flight test. In this work, empirical studies shows that three maneuvers is an adequate value for the block length.

## 5. SIMULATED RESULTS

The Inflight Parameter Estimation, shown in Fig. 2, is now illustrated, for the continuous improvement and separate computation maneuver validation decision methods, as in Eq. (10) and Eq. (11), respectively. For the flight test validation, the standard deviation of recent maneuver is considered for the case of the last 3 maneuvers, based on Eq. (12).

It should be stressed that highly exciting maneuvers significantly improves the parameter estimation, but might be associated with a higher cost function value and prone to be rejected by the decision method. Also, a steady flight data with poor dynamic response do not stimulate properly the aircraft can produce lower cost function values. Therefore, it is recommended the execution of only rich dynamic maneuvers.

In this first test, the data parser is replaced by simulated data in Fig. 4. As already discussed in the last section, for better illustrating how the decision methods work, the maneuver sequences are a composition of modified 3-2-1-1 maneuver and reverse 3-2-1-1 maneuver, as shown in Fig. 4. The choice for modifying the 3-2-1-1 maneuver is a common practice to equalize the positive and negative command signal areas, thereby avoiding the aircraft returns to an equilibrium point different from the initial trimmed one.

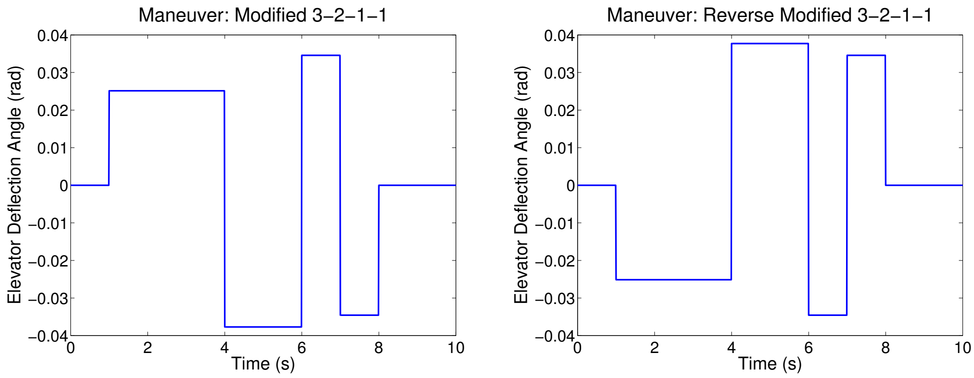


Figure 4. Elevator deflection angle ($\delta_e$) maneuvers data segments

The same maneuvers sequence were provided for each simulation case, composed by seven elevator deflection angle ($\delta_e$) data segments occurrences at the following order: 3-2-1-1 maneuver, two consecutives reverse 3-2-1-1 maneuver, 3-2-1-1 maneuver, reverse 3-2-1-1 maneuver, 3-2-1-1 maneuver, reverse 3-2-1-1 maneuver.

The simulation results are divided in two parts, where the first one shows the convergence behavior of standard deviation estimates, defined in Eq. (12), along the maneuvers execution for each decision method from Eq. (10) and (11), and applied to three different measurement noise levels. The results are shown in Fig. 5. The second part is presented in Table 1, with the nominal errors and the required number of maneuvers for the proposed method to define the end of flight test campaign.

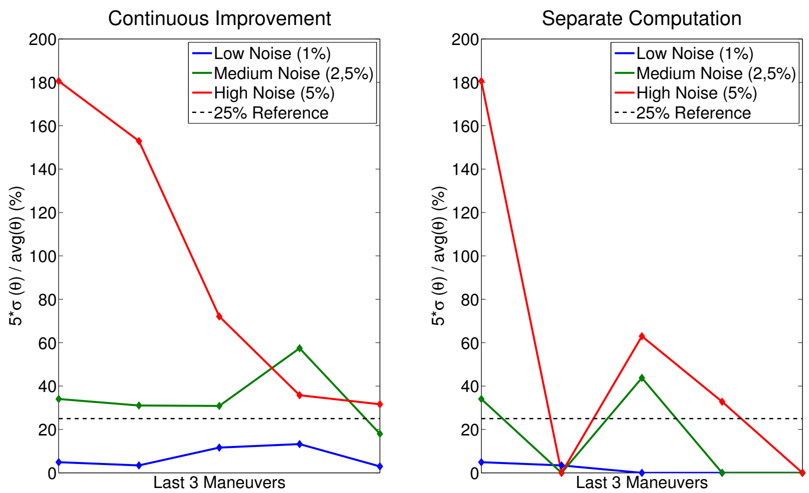


Figure 5. Simulation 1, flight test validation results for the last three maneuvers

Analysing the results in Fig. 5, the fast decreasing behavior on standard deviation is observed on both maneuver validation methods, as expected in the section 4.2, evidencing the estimation stablishment on the measured data along the

Table 1. Nominal error and required number of flight test maneuvers

| Validation Method | Noise Intensity | Nominal Error (%) | | | | | | | | Required number of maneuvers |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $Z_0$ | $Z_\alpha$ | $Z_q$ | $Z_{\delta e}$ | $M_0$ | $M_\alpha$ | $M_q$ | $M_{\delta e}$ | |
| Continuous Improvement | Low (1%) | 0.12 | 0.28 | 0.54 | 0.44 | 0.32 | 0.15 | 0.56 | 0.07 | 3 |
| | Med (2,5%) | 0.87 | 0.74 | 6.38 | 3.35 | 0.73 | 0.10 | 1.69 | 1.21 | 7 |
| | High (5%) | 0.07 | 5.50 | 16.37 | 0.89 | 2.18 | 2.35 | 1.28 | 0.70 | - |
| Separate Computation | Low (1%) | 0.12 | 0.28 | 0.54 | 0.44 | 0.32 | 0.14 | 0.56 | 0.07 | 3 |
| | Med (2,5%) | 0.29 | 2.79 | 3.59 | 0.70 | 1.19 | 0.60 | 1.82 | 0.88 | 4 |
| | High (5%) | 5.43 | 4.86 | 30.85 | 9.41 | 1.26 | 0.56 | 3.46 | 1.20 | 4 |

seven proposed maneuvers.

Table 1 shows the percentual nominal error for each parameter estimate component and the number of maneuvers required to pass the flight test validation. The results show the distinct behavior of the decision methods, requiring different quantities of executed maneuvers.

At the particular case of continuous improvement method with high noise, presented in Fig. 5 and in the third line of Tab. 1, the standard deviation do not assume values lower than the reference value. Therefore, the flight validation method not end the flight test, and possibly request for more maneuvers to be executed or to abort the flight test without reaching the reference value goal.

## 6. CONCLUSIONS

A flight test validation method was presented to automatically decide the end of flight test campaign. The results obtained from simulations show, for different measurement noise intensities and maneuver validation methods, that the proposed method can be a good terminating criteria for inflight parameter estimation.

For future work, real flight data can be used and the method be adapted to a supporting analysis tool for inflight parameter estimation, augmenting the situational awareness and improving the decision capability of pilots and flight test engineers.

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