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THERMAL ANALISYS AND GEOMETRIC OPTIMIZATION OF A TUBULAR ARRAY USING CONSTRUCTAL DESIGN

Vagner Andrade Pedrotti
Jeferson Avila Souza
vagnerpedrotti@hotmail.com
jasouza@furg.br

Elizaldo Domingues dos Santos
Elizaldosantos@furg.br

Liércio André Isoldi
liercioisoldi@furg.br
Universidade Federal do Rio Grande – FURG, Av Itália, Km 8, s/n, Cep 96203-900, Rio Grande, RS, Brasil.

Abstract. *This paper discussed a study with the objective of maximize energy efficiency in a tubular array containing four tubes, subjected to a transverse flow. Taking into account that a large variety of tubular arrangements can exist, in present work it was developed a tubular array assemble which does not uses any kind of preformed arrangement (aligned or staggered) for positioning the cylinders. Using the principle of constraints and objectives of Constructal Design and a control function dependent on the speed and temperature fields, it was calculated, in a deterministic way, the location where the cylinders should be positioned. Proposed tubular array thermal performance was than compared with standard configurations suggested in the literature. The distance between tubes (p) is also considered one of the metrics constraints of the study. It was studied four variations: p = 1D, p = 1.25D, p = 1.5D, p = 2D, where D is the diameter of the cylinders. Fluid flow is set as laminar, permanent, two- dimensional, incompressible, with forced convection and Prandtl number equal to 0.71. It was evaluated three values for the Reynolds number: Re_D= 100, Re_D= 50 and Re_D=10. Mass, momentum and energy equations were solved by the Finite Volumes Method (FVM) using FLUENT software. Geometry creation and mesh generation were performed with GMSH software while VISIT software was used for the post processing. Results show that the Constructal assemble (optimized) was more efficient, creating geometric formations that allows greater heat exchange between the fluid and the cylinders.*

Keywords: *Constructal Design, Geometric optimization, Tubular array*

1. INTRODUCTION

Investigations on external flow are an important issue because several engineering equipment make use of i. Examples are: towers, bridges, air conditioning, vehicles, etc.

Due to the major technological advances, it is possible to create favorable environments to perform experimental and numerical studies and investigate complex phenomena.

For a single cylinder, Catalano, et al., (2003) has studied the application of the wall model boundary condition and concluded that this formulation can be used in simulations with Reynolds number up to $1x10^6$. Researching the occurrence of vortex formation when the cylinder approach to the wall, Sarkar and Sarkar., (2010) noted that the stagnation point moves towards the wall according to cylinder approaches to the wall, generating a force that deflects the boundary layer.

For the two cylinders case Sumner, (2010), presented a good review of studies that consider a large range of Reynolds number and various assembles with different distances between tubes.

In a study involving tree cylinders, using a triangular assemble Bao, et al., (2010) concluded that when the cylinders are close to the flow is become similar to a single body. However, with intermediate distances there is a vortex shedding behind downstream cylinders. With large spacing vortex shedding occurs in all cylinders.

Using seven cylinders Darócky, et al., (2014) developed a multi-objective study, aimed maximize the rate of heat exchange while obtaining a minimum pressure loss. Using the GA (genetic algorithms) they evaluated more than 140000 configurations.

Presenting result for the experimental and numerical assembles with 12, 14, 16 and 23 tubes subjected to forced convection (Stanescu., et al, 1996) searched for the optimal distance among tubes for the maximization of the thermal conductivity. Tube occupation area was kept constant and runs were performed for Reynolds number varying from 40 to 2000. Four degrees of freedom (distance between tubes, *S*, over tube diameter, *D*) were analyzed: *S/D* = 0.25, 0.5, 0.75 and 1. Results Showed that optimized *S/D* values decreases with the increase of Reynolds number.

Some works using *Constructal Design* have also developed studies aiming to find better thermal efficiency in tubular arrangements. Analyzing cases where cylinders of different sizes are aligned and in rotation and subjected to forced convection, Bello-Ochente, et al., (2011) observed that the rotation is beneficial in terms of increasing heat transfer when cylinders are aligned on the same axis. For natural convection and also using aligned cylinders with different diameters Bello-Ochente and Bejan, (2005) concluded that the heat transfer rate increases as the optimized structure becomes more complex. Investigations using two cylinders aiming the optimization of heat transfer and drag reduction Rocha., et al, (2013) indicated that best performance is obtained when tubes are aligned with an angle varying between 45 ° and 60 ° to the main fluid flow stream. Analyzing a triangular arrangement of cylinders in order to maximize heat transfer and minimize drag Galarça, et al, (2015) used the *Constructal Design* to assess the optimal ratio between the step of transverse distance by the diameter (*ST/D*). The found optimal ratio for multi-objective study was *ST/D ~2*.

Even with the large number of studies involving external flow passing through tubular arrangements it has not yet been developed any studies to deterministic indicate the cylinder position that maximize system efficient. Considering that exists a large number of possible tubular arrangements our goal was to develop a standard formation without establishing any initial arrangement. Using the principle of constraints and objectives of *Constructal Design* and an objective function dependent on the velocity and temperature fields, the position where the cylinders should be positioned was deterministic obtained. In seeking to increase the efficiency of heat transfer in tubular arrays, the heat exchange between cylinders and fluid stream of the optimized assemble (*constructal*) is compared with the heat exchanged obtained for the aligned and staggered assembled. The influence of Reynolds number in the pattern of formation in the *Constructal* assemble (placement of the cylinders) will be investigated.

2. NUMERICAL AND MATHEMATICAL FORMULATION

In this paper the computational domain was created and discretized using the open source GMSH software (Geuzaine and Remacle, 2009). ANSYS FLUENT 14.0 (2011) software was then used to solve the problem using the FVM (Finite Volume Method).

The methodology used to build the optimized array is the Constructal Design. A theory that describes deterministic as flow systems evolve over time. According to this principal everything changes all the time and the process of designing is not static but a dynamic phenomenon. The Constructal Theory becomes a powerful methodology to explain the tendency of all things to flow through pathways that generate less “resistance”. It is a physical principle that unite the “animate” and “inanimate” (Bejan, 2000). For being a general theory, it can be applied in all areas where there exists some type of evolution. Thus, it has been utilized to solve biological, physical, social organization, technology developments, sustainability and engineering problems Bejan and Zane, (2012).

In the present paper the flow is considered two-dimensional, laminar, incompressible, permanent in forced convection and with constant physical properties. Continuity, momentum and energy equations, already with the model simplifications, are formulated as:

$$\nabla.(\vec{V}) = 0 \tag{1}$$

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \mu \nabla^2 \vec{v} \tag{2}$$

$$\rho \, c_p \frac{DT}{Dt} = \nabla.(k\nabla T) \tag{3}$$

where $\vec{v}$ is the velocity vector [m/s], ρ is the density [kg/m^3], μ is the absolute viscosity [Pa s], c_p is the specific heat at constant pressure [J/kg K] and T temperature [K].

The system formed by Eqs. (1)-(3) is solved with FLUENT software. In current solution, pressure-velocity coupling is solved with SIMPLE algorithm and second order upwind was used for the advective terms discretization. Residues of 10^{-6} were used as convergence criterion for mass and momentum equations while 10^{-8} was used for energy equation. Even though only the steady state solution was of interest, in order to facilitate convergence, it was used a transient numerical solution. Thus, it was employed in all simulations a time step of 10^{-3} s. The steady-state condition was identified when the convection heat transfer coefficient, *h*, calculated on tubes for each integration time step, stops varying with time.

Grid independence test has also been based on the convective heat transfer coefficient. The grid was considered independent when the *error* is smaller than 1%. Equation (4) shows the way that the deviation (*error*) was calculated.

$$error = 100\left|\frac{h^{j}_{médio} - h^{j+1}_{médio}}{h^{j}_{médio}}\right| \quad (4)$$

with *j* for a certain mesh and *j +1* a more refined mesh. The test was performed with a grid built with just one tube. The independence simulations are summarized in Tab 1. Grid M3 was considered independent used in all simulations.

Table 1. Grid independence test.

Grid	N° of volumes	h	Error
M1	9347	60.302	6,55%
M2	46821	56.351	1,55%
M3	225507	55.473	0,19%
M4	495191	55.364	--

3. PROBLEM DESCRIPTION

In the present work the computational domain has a length of 69D (were D is diameter of the cylinder) and a height of 7D. The region were cylinders can be placed is represented by the white area in Fig. 1. This area is square and has length and height of 5D. The area of occupation (white) and computational domain are shown in Fig. 1.

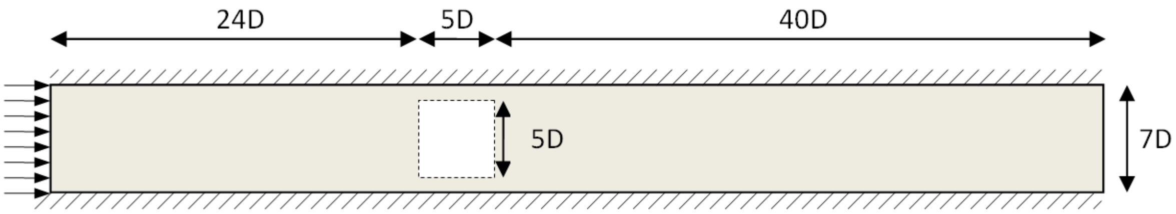


Figure 1. Computational domain

Four different cylinder distance constraints have been used in current work. They are:

- *p = 1D*
- *p = 1.25D*
- *p = 1.5D*
- *p = 2D*

The air is forced to flow between two parallel plates with prescribed velocity U_∞ and temperature T_∞. The temperature at the cylinders surface is constant and defied as T_t. The walls are considered isolated with non-slip flow boundary condition. At the right wall, parabolic boundary condition is specified. The Fig. 2 shows these boundary conditions in a staggered tube array.

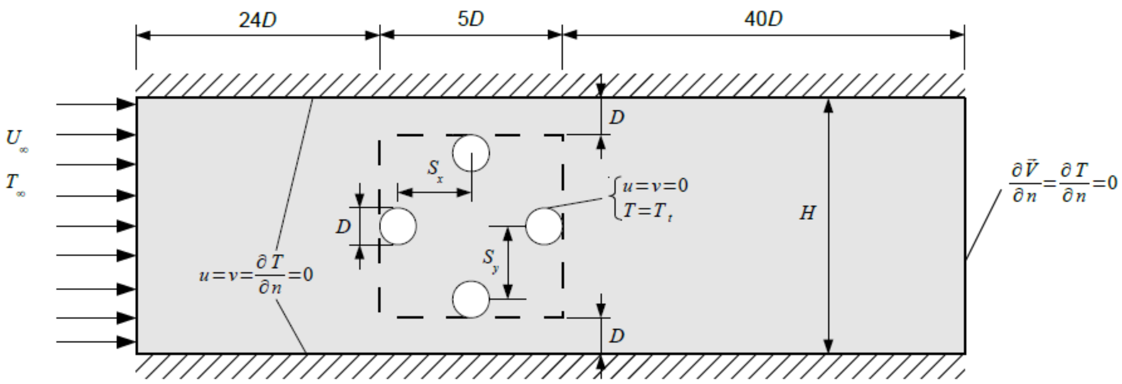


Figure 2. Boundary conditions (not in scale)

4. CONTROL FUNCTION AND SOLUTION ALGORITHM

In current solution it is necessary to define a control function that will be used to calculate where in the computational domain each cylinder should be place. Considering that the heat flux between the cylinders and the flow stream may be estimated by:

$$q = hA\Delta T \tag{5}$$

where q is heat transfer rate [W], h the convective heat transfer coefficient [W/m^2 K] and A the external superficial cylinder area[m^2].

The control function was then formulated based in Eq. (5). Being h proportional to flow velocity, as larger as the velocity, larger will be the heat transfer between cylinders and fluid. In the same manner, as larger as the temperature difference, larger will be the heat transfer. Thus, it is possible to build and dimensionless expression such as:

$$\varphi = \frac{U}{U_\infty}\left(\frac{T_t - T}{T_t - T_\infty}\right) \tag{6}$$

In Eq. (6) it is possible to observe that the temperature term will vary from zero (when $T = T_t$) to one (when $T = T_\infty$) and the velocity term may assume values larger than one in regions where the velocity is larger than the main stream velocity U_∞.

With Eq. (6) it is possible to determine to flow regions where the heat transfer between cylinders and fluid is potentially more efficient. Consequently it is possible to calculate where a new cylinder should be positioned.

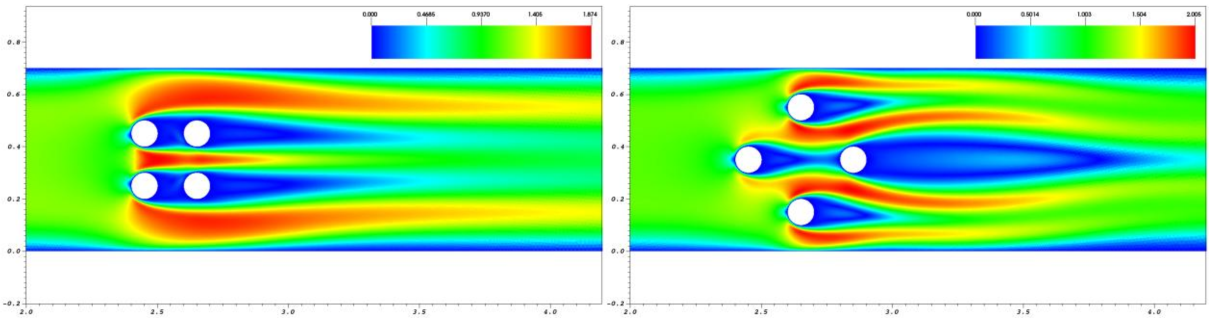
Array construction will be performed with the following algorithm:

1) define position of the first cylinder;
2) solve fluid flow and the thermal problem (Eqs (1)-(3));
3) calculate φ with Eq (6);
4) calculate φ_{max} and the coordinate of the center of the new cylinder (x_c, y_c);
5) create the new geometry with a new cylinder;
6) redo steps 2 to 5 until all cylinders are positioned;

5. RESULTS

Initially the traditional aligned and staggered assembles (Çengel e Cimbala, 2011, Incropera, et al., 2010) have been simulated with the goal of analyze the heat transfer rate that occurs between the fluid and the cylinders. This efficiency was evaluated by computing the heat flux balance (Δq) between the inlet (q_{in}) and outlet (q_{out}) sections. These arrangements were evaluated for three Reynolds number: $Re_D = 100$, $Re_D = 50$ and $Re_D = 10$.

In Fig. (3) it is possible to visualize the positioning of the cylinders in the aligned and staggered assembles. It is also shown in Fig. (3) the scalar field for the control function.



(a) (b)

Figure 3. Control function: aligned (a) and staggered (b) assembles for the $Re_D = 100$

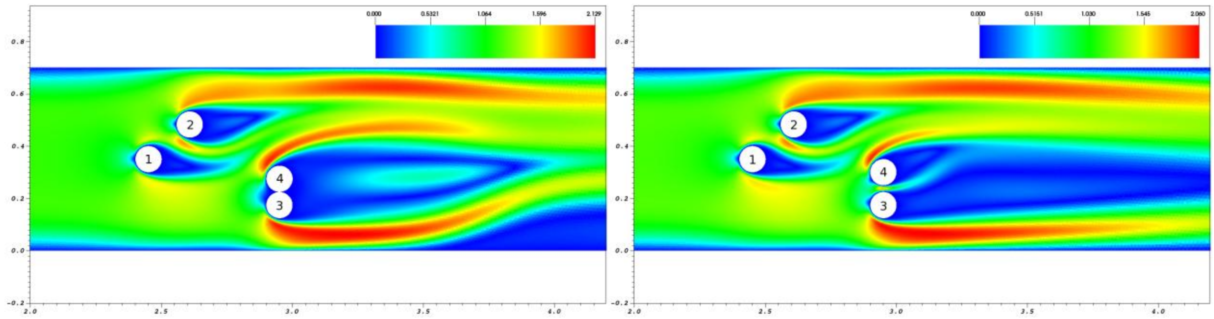
Heat flux balance ($\Delta q = q_{out} - q_{in}$) calculate for the $Re_D = 100$, $Re_D = 50$ and $Re_D = 10$. Are shown in Tab. 2 for the aligned and staggered assemble.

Table 2. Heat transfer for aligned and staggered assemble.

Re_D	q_{in} [W]	q_{out} aligned[W]	q_{out} staggered[W]	Δq aligned[W]	Δq staggered[W]
100	919.44	2015.58	2335.23	1096.13	1415.78
50	459.72	1289.60	1510.91	829.88	1051.18
10	91.94	490.93	632.41	398.98	540.64

The *Constructal* assemble was obtained by initially solving the problem for the geometry with only one cylinder. This cylinder was positioned at the middle section between upper and lower walls. With calculated velocity and temperature fields, the control function is evaluated and the calculation of its maximum value coordinates will determine the second cylinder center. After that, flow and thermal problems are again solved and this procedure is repeated till the last cylinder is positioned inside the computational domain. It is important to highlight that the number of simulations needed to build the *Constructal* assemble is always equal to the number of the cylinders. This procedure was repeated for the four constraints ($p = 1D$, $p = 1.25D$, $p = 1.5D$, $p = 2D$) are three Reynolds number ($Re_D = 100$, $Re_D = 50$ and $Re_D = 10$).

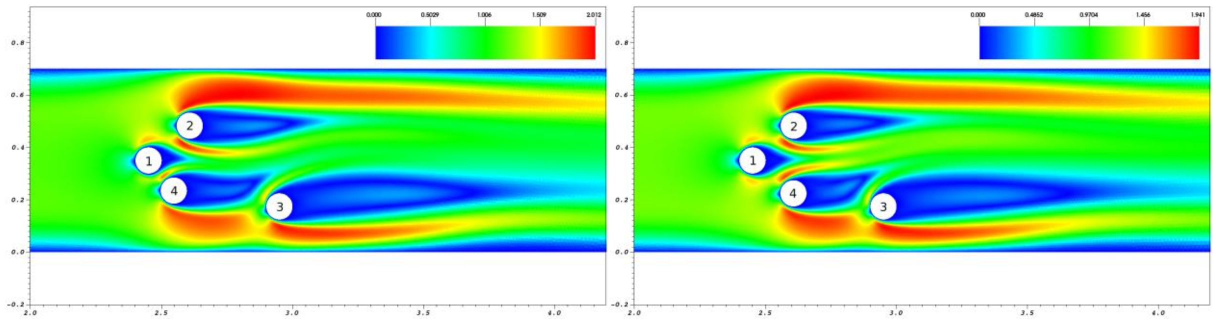
First will be shown the arrangements for $Re_D = 100$. In these arrangements the position of the first three cylinders were identical for all distance restriction. In Fig. (4) is shown the *Constructal* assemble for the case with $p = 1D$ and $p = 1.25D$. The number inside the cylinders indicates how (in which order) the cylinders were positioned.



(a) (b)

Figure 4. Control function, *Constructal* assemble $p = 1D$ (a) and $p = 1.25D$ (b) for the $Re_D = 100$

Figure 5 shows the scale fields of the control function φ for the case with $p = 1.5D$ and $p = 2D$. For this restrictions, the fourth cylinder was positioned on the left side of the occupancy area. For the case with $p = 2D$ the fourth cylinder is positioned in alignment with the second cylinder.



(a) (b)

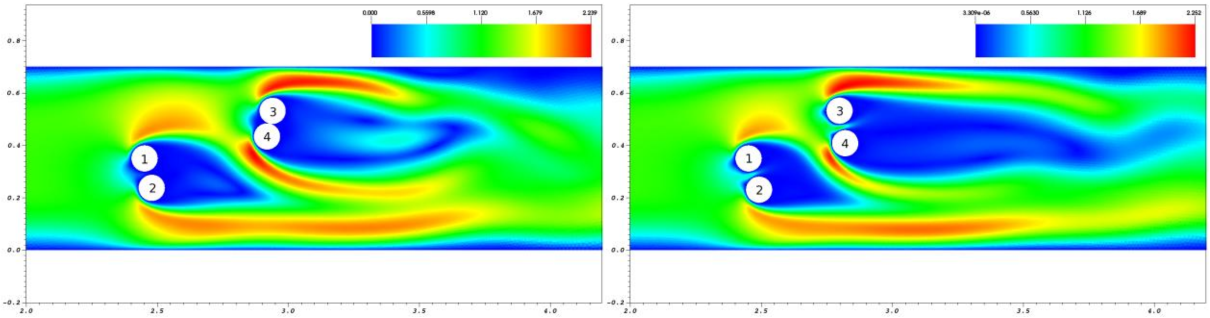
Figure 5. Control function, *Constructal* assemble $p = 1.5D$ (a) and $p = 2D$ (b) for the $Re_D = 100$

To view the thermal difference of each arrangement, in Tab. 3 is shown results for heat flux exchange for the $Re_D = 100$.

Table 3. Heat transfer for *Constructal* assemble for the $Re_D = 100$.

restriction	q_{in} [W]	q_{out} [W]	Δq[W]
1D	919.45	2129.12	1209.67
1.25D	919.45	2429.99	1510.54
1.5D	919.45	2442.39	1522.94
2D	919.45	2409.34	1489.89

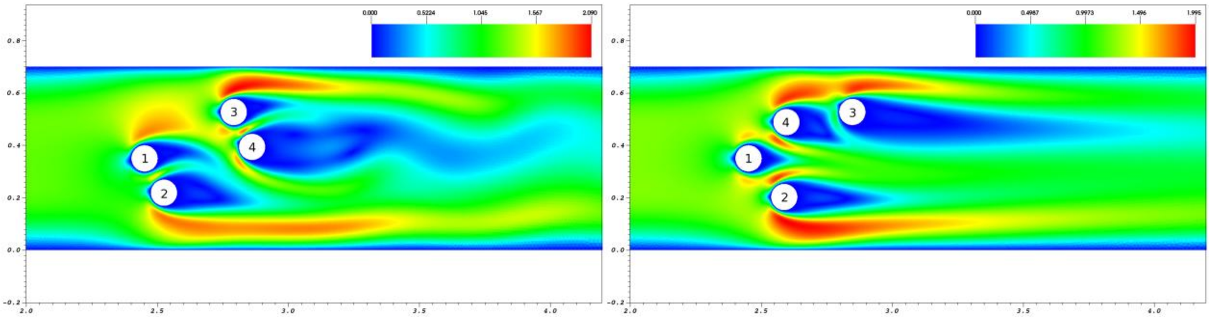
Next results for $Re_D = 50$ are presented in Fig. (6) for the *Constructal* assemble scalar fields of the control function φ for $p = 1D$ and $p = 1.25D$ constrains are shown in Fig. (6).



(a) (b)

Figure 6. Control function, *Constructal* assemble $p = 1D$ (a) and $p = 1.25D$ for the $Re_D = 50$

Figure 7 shows the scalar fields for the control function φ for the cases with $p = 1.5D$ and $p = 2D$.



(a) (b)

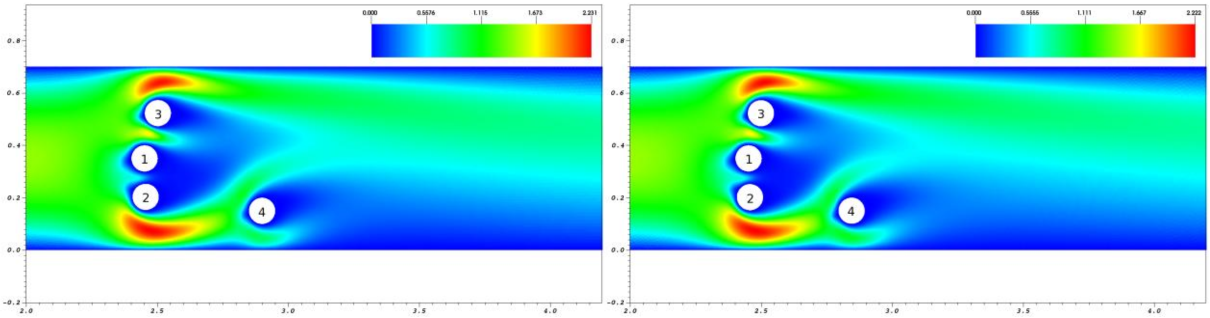
Figure7. Control function, *Constructal* assemble $p = 1.5D$ (a) and $p = 2D$ (b) for the $Re_D = 50$

In tab. 4 is shown the results for heat flux exchange for the $Re_D = 50$.

Table 4. Heat transfer for *Constructal* assemble for the $Re_D = 50$.

restriction	q_{in} [W]	q_{out} [W]	Δq[W]
1D	459.72	1196.33	736.61
1.25D	459.72	1477.19	1017.47
1.5D	459.72	1587.08	1127.35
2D	459.72	1539.2	1079.48

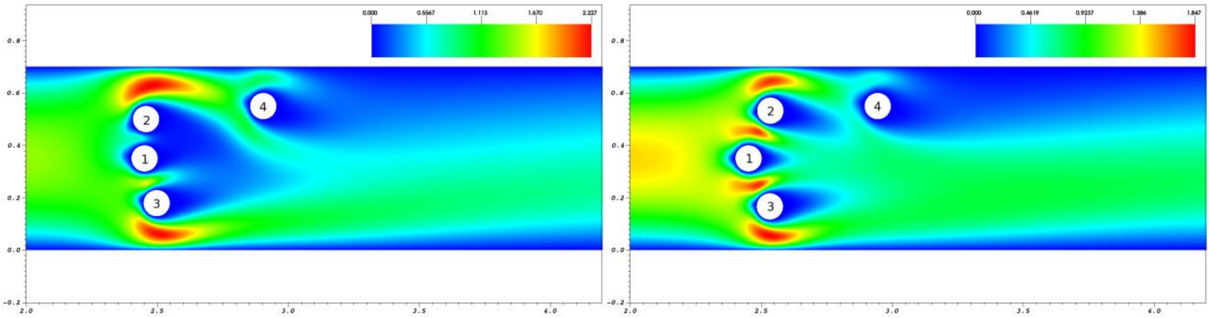
Next, results for $Re_D = 10$ are presented in Fig. (8) for the *Constructal* assemble scale field of the control function φ for $p = 1D$ and $p = 1.25D$ constraints are shown.



(a) (b)

Figure 8. Control function, *Constructal* assemble $p = 1D$ and $p = 1.25D$ for the $Re_D = 10$

Figure 9 shows the scalar field of the control function φ for the cases with $p = 1.5D$ and $p = 2D$.



(a) (b)

Figure 9. Control function, *Constructal* assemble $p = 1.5D$ and $p = 2D$ for the $Re_D = 10$

In table 6 is shown the results for heat flux exchange for the $Re_D = 10$.

Table 5. Heat transfer for *Constructal* assemble for the $Re_D = 10$.

restriction	q_{in} [W]	q_{out} [W]	Δq[W]
1D	91.94	609.67	517.75
1.25D	91.94	614.42	522.48
1.5D	91.94	614.40	522.45
2D	91.94	640.08	548.13

For results found using $Re_D = 100$ it was observed that *Constructal* assemble $p = 1.5D$ was shown to be the most efficient, with 39% and 8% more efficient than the aligned and staggered assembles, respectively.

Using $Re_D = 50$ it was observed that *Constructal* assemble $p = 1.5D$ was shown to be the most efficient, with 36% and 8% more efficient than the aligned and staggered assembles, respectively.

Using $Re_D = 10$ it was observed that *Constructal* assemble $p = 2D$ was shown to be the most efficient, with 37% and 2% more efficient than the aligned and staggered assembles, respectively.

6. CONCLUSIONS

Results for the occupation of the cylinders show that the distance constraint has direct influence on the formation of the assembles and thus on the heat transfer rate. The exception was for $Re_D = 100$ with arrangement of up to three cylinders. In this case, the distance restriction did not influence cylinders positioning and the construction of Constructal assemble generated the same formation for the first three cylinders.

Analyzing results shown in this article it is possible to observe that *Constructal Design* method, ruled by the control function defined in Eq. (6), leads to an assemble that, depending on the distance constrain, may be as efficient as the traditional aligned and staggered assembles.

7. ACKNOWLEDGEMENTS

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