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KINESTATIC ANALYSIS OF SERIAL-PARALLEL COOPERATIVE ROBOTIC SYSTEMS

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Abstract. *This paper concerns the kinestatic analysis of a cooperative robotic system composed by a serial and a parallel manipulator, both planar. Cooperative robotic systems are mostly composed by serial manipulators and, in this scenario, tasks are successfully performed with two or more manipulators. Although parallel robots are gaining much attention in research in recent decades and its use in industrial applications is increasing, few is found in the literature considering the cooperation involving parallel robots. An advantage of such configurations is the precise positioning and stiffness given by a parallel manipulator and the dexterity provided by a serial robot. In this work a planar cooperative robotic system composed by a 3RPR platform and a serial RRR robot is addressed. Besides the kinematics, the forces (wrenches) at the common workpiece and the torques at the actuators level must be considered. The objective of this work is to obtain the kinestatic model of the system, presenting a detailed formulation based on graph and screw theories in a unifed and structured procedure. The formulation derives from previous works of Davies applied to general mechanisms. An advantage of the procedure is its applicability regardless the type of the robots (serial and/or parallel) since is based on graph analysis of the system.*

Keywords: *screw theory, cooperative robotic system, parallel robot, serial robot*

1. INTRODUCTION

A Cooperative Robotic System (CRS) is a system composed by two or more robot manipulators working simultaneously and together in the accomplishment of a common task. These systems present an increased workspace, greater load capacity and more dexterity than a single robot arm. Also, the execution of tasks not feasible to a single robot and an improvement in productivity can be obtained (Caccavale and Uchiyama, 2008; Huang and Lin, 2003). Possible applications are in handling of common objects or workpieces - with the possibility of flexible objects or objects with some degree of freedom -, the assembly of multiple parts without the usage of specific designed tools, packaging, palletizing, assembly and machining. In fact, cooperative manipulators can lead to new tasks to be executed – or executed in a new way –, all leading to an increased flexibility in production. Due to the flexibility of using two or more robots, it's possible to perform complex tasks and tasks that require greater load capacity or heavier-weight collaborative processes, for example: Knowing the wrench capability of the CRS, it's possible to evaluate some machining applications, like prototyping, drilling and milling.

On the other hand, cooperative robotic systems are more complex, both in modeling and in task execution. One main difficult if that the system is coupled, since the movement of one arm influences the other(s) and vice versa. Then, there is a natural need for modeling the system as a whole as some coordinated control. In many cases only the trajectories of the system are specified and the robots controlled under a position-control scheme, which is the usual case, without taking into account the forces involved: the internal stresses on the common workpiece and the interchanged forces. This may lead to increased contact forces due to position and/or geometric errors (Huang and Lin, 2003). Also, when the forces on the common object must be specified, the torques on the actuators must also be planned. So one of the general problems of such systems is how to specify and control the trajectory of the object or end-effector, the mechanical stresses on it and the forces and moments on the joints.

Different strategies have been proposed, mainly based on position specifications, like master-slave, error synchronization and kinematic modeling control and on force and motion, like impedance, hybrid force position or the symmetric formulation (Caccavale and Uchiyama, 2008; Siciliano *et al.*, 2009; Freitas *et al.*, 2010). Among them, it can be seen that are mainly applied to cooperative systems composed solely by *serial manipulators*.

The differential kinematics of cooperative robotic systems have been modeled using screw theory, but without taking into account forces and moments (Rocha *et al.*, 2012; Ribeiro *et al.*, 2007; Henriques *et al.*, 2005). Due to the necessity of considering the forces involved in the cooperation, the main contribution of this work is to model both the differential kinematics *and* the statics of a cooperative robotic system based on kinestatic modeling. Also, the application example is composed by a serial and a parallel platform (RRR and 3RRR, respectively). Kinestatics refers to the dualistic relations between differential kinematics and statics. In this work, this relation is derived using graphs – to describe the topology – and screw theory – to describe the geometry of the system. Both are related by Kirchhoff laws, as proposed by Davies (Davies, 2006) and applied to the mechanical system under study.

2. KINEMATIC CHAINS, GRAPHS AND SCREW THEORY

2.1 Graph representation of kinematic chains

A kinematic chain is composed by a finite set of n bodies connected by e joints. Each joint has a degree of freedom f_i permitting a relative motion between the pair of bodies it couples and the gross degree of freedom F_N of the kinematic chain is $F_N = \sum f_i$. The joints are also called couplings and, as the bodies of the kinematic chain, form another finite set[1]. Hence, the topology of any coupling network can be uniquely represented by a coupling graph G_C, with the nodes representing the bodies and the edges representing the joints. The graph is always connected and directed edges are used according to the convention (Tsai, 2000; Davies, 2006).

2.2 Screw theory, twists and wrenches

3. DAVIES' METHOD

The CRS under study are composed for one 3R planar manipulator and one 3<u>R</u>RR planar parallel manipulator, one called *Leader* (with the tool fixed at the end-effector) and another called *Follower* (with the workpiece fixed at the end-effector) as shown in Fig. 1. The leader manipulator (3R planar) have the actuated joints g, h and i. The end-effector of the leader is connected to the moving platform of the follower manipulator (3<u>R</u>RR planar) by the virtual F edge. The actuated joints of the follower manipulator are the P, Q and R joints. The other joints a,b,c,d,e and f are passive joints.

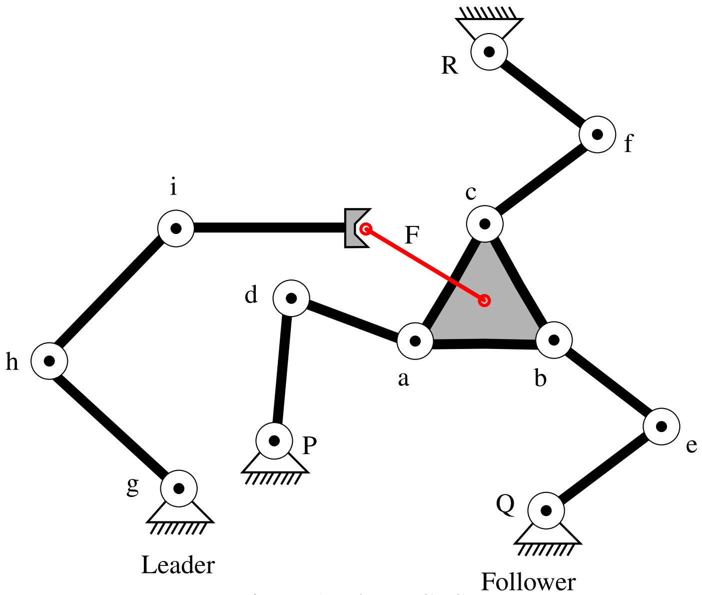
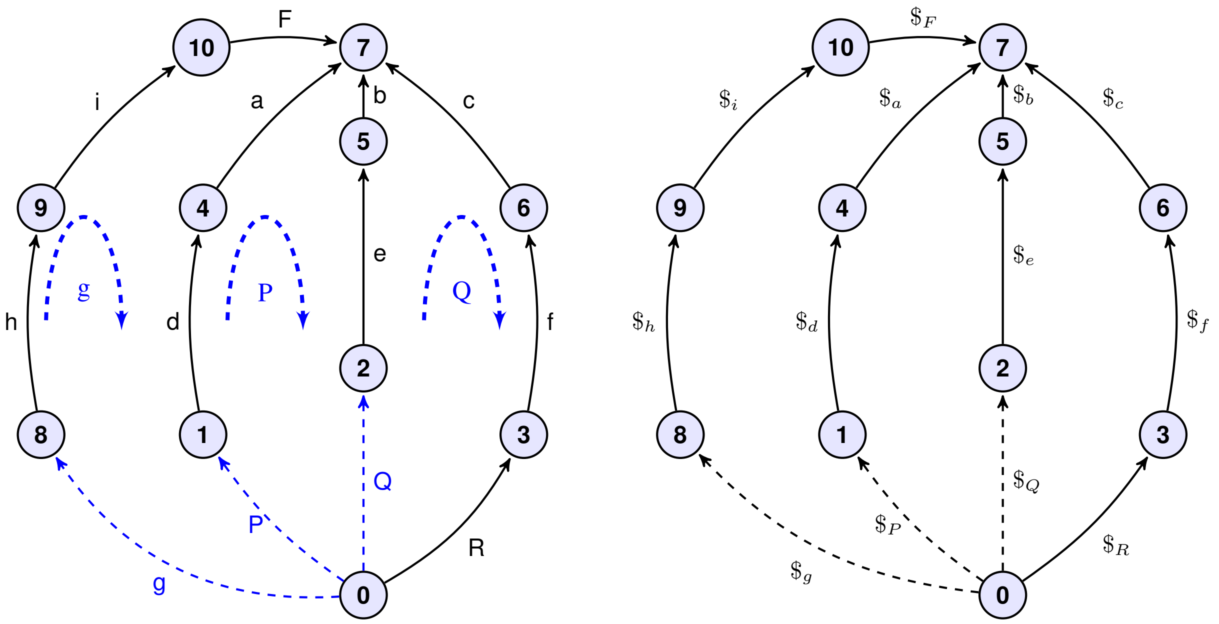


Figure 1: Planar CRS.

In the kinematic analysis of the CRS, the main goal is to determine the angular velocity ω and the translational velocity v_p requirements for the joints in relation to the twists applied at the end-effector.

Using graph theory and the circuit law (Davies, 2006), it is possible generate the motion graph G_M, as shown in Fig. 2a. In the motion graph it is possible prove that are $l = 3$ independent circuits, selecting the edges g, P and Q as chords.

[1]The group of both sets is often called *coupling network*.The term *kinematic chain* is used for coupling networks with $F_N > 0$.



(a) Motion graph of the planar CRS. (b) Action graph of the planar CRS.

Figure 2: Planar CRS and the respectively motion graph and action graph.

Now it is possible write the circuit matrix $[B]_{l \times e}$, where the rows are the circuits and the columns are the edges, as shown in Eq. **??**.

$$[B]_{3\times 13} = \begin{array}{c} \\ \text{g} \\ \text{P} \\ \text{Q} \end{array} \begin{array}{c} \begin{array}{ccccccccccccc} \text{g} & \text{P} & \text{Q} & \text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{h} & \text{i} & \text{F} & \text{R} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & -1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

Using screw theory the twist (angular velocity) applied by the end-effector can be represented by the ω_z. The angular velocities at the actuated joints in the leader and the follower manipulators are denoted as ω_g, ω_h, ω_i and ω_P, ω_Q, ω_R. Using the circuit law it's possible to prove that the gross network dof F is 13 and the nett dof F_N is 4.

Considering the primary variables as the twist acting in the end-effector and in the actuated joints of the leader manipulator, the kinematic solution can be written in the form:

$$\{\Psi_s\}_{9\times 1} = \left[-\hat{M}_{N_s}^{-1}\right]_{9\times 9} \left[\hat{M}_{N_p}\right]_{9\times 4} \begin{Bmatrix} \omega_g \\ \omega_h \\ \omega_i \\ \omega_F \end{Bmatrix} \tag{1}$$

where Ψ_s is the secondary variables magnitudes, $\hat{M}_{N_s}$ is the network unit motion matrix of the secondary variables and $\hat{M}_{N_p}$ is the network unit motion matrix of the primary variables, where all wrenches must be written in the same reference frame.

Equation (1) can be rewritten in its unfolded form as shown in Eq. (2). In this new representation, it's possible to select the secondary variables in the vector Ψ_s, selecting the angular velocities in the actuated joints of the followr manipulator. Multiplying the primary and secondary matrix it's possible to extract the twelve kinematic variables $a_1, \ldots, a_{12}$ (where the $a_1, \ldots, a_{12}$ terms represent kinematic expressions as a function of the manipulator joint positions).

$$\begin{Bmatrix} \tau_P \\ \tau_Q \\ \tau_R \end{Bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \cdot \begin{Bmatrix} \omega_g \\ \omega_h \\ \omega_i \\ \omega_F \end{Bmatrix} \tag{2}$$

In the statics analysis of the CRS, the main goal is to determine the force and moment requirements for the joints in relation to the wrenches applied at the end-effector. In this work the formalism presented by Davies (Davies, 2006) is used as the mathematical tool to analyse the static problem. The Davies method appears in many publications in the literature and further explanations can be found in (Davies, 2006; Weihmann *et al.*, 2011; Mejia *et al.*, 2015).

The Davies method establishes a systematic way to relate the forces and moments in closed kinematic chains (Weihmann *et al.*, 2011). This method is based in graph theory, screw theory and Kirchhoff-Davies cut-set law and circuit law and it can be used to obtain the static equations of a robot in a matricial form (Weihmann *et al.*, 2011).

Using screw theory the wrenches (force and moments) applied by the end-effector can be represented by the vector $\$_F = [f_x \, f_y \, m_z]^T$ where f_x and f_y denotes the force in the directions x and y respectively, and m_z denotes the moment around the z axis. The torques at the actuated joints in the leader and the follower manipulators are denoted as τ_g, τ_h, τ_i and τ_P, τ_Q, τ_R.

Using graph theory and the cut-set law (Davies, 2006), it is possible to prove that the gross of this manipulator is C=33 and, in the absence of singularities, the nett degree of constraint is $C_N = 3$. The nett degree of constraint $C_N = 3$ means that only three independent variables are needed in order to solve the statics of the CRS. The CRS action graph is shown in Fig. 2b, where the edges are the joints and the vertex are the manipulator links. The wrenches in each joint are represented by the symbol \$.

Applying the cutset law it's possible to write the cutset matrix $[Q]_{k\times C}$, where k represents the number of cuts and C the gross dof. The 3R+3$\underline{R}$RR CRS have 11 (eleven) bodies, for this the cutset law require that 10 (ten) cutset cuts are applied.

$$[Q]_{10\times 13} = \begin{array}{cc} & \begin{array}{ccccccccccccc} g & P & Q & a & b & c & d & e & f & h & i & F & R \end{array} \\ \begin{array}{c} h \\ i \\ F \\ d \\ a \\ e \\ b \\ R \\ f \\ c \end{array} & \begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Considering the primary variables as the wrenches acting in the end-effector of the leader manipulator, the static solution can be written in the form:

$$\{\Psi_s\}_{30\times 1} = \left[-\hat{A}_{N_s}^{-1}\right]_{30\times 30} \left[\hat{A}_{N_p}\right]_{30\times 3} \begin{Bmatrix} f_x \\ f_y \\ m_z \end{Bmatrix} \tag{3}$$

where Ψ_s is the secondary variables magnitudes, $\hat{A}_{N_s}$ is the unitary action matrix of the secondary variables and $\hat{A}_{N_p}$ is the secondary action matrix of the primary variables, where all wrenches must be written in the same reference frame.

Equation (3) can be rewritten in its unfolded form as shown in Eq. (4). In this new representation, it's possible to select the secondary variables in the vector Ψ_s, selecting the torques in the actuated joints. Multiplying the primary and secondary matrix it's possible to extract the eighteen kinematic variables $a_1, \ldots, a_{18}$ (where the $a_1, \ldots, a_{18}$ terms represent kinematic expressions as a function of the manipulator joint positions (Frantz, 2015)).

$$\begin{Bmatrix} \tau_g \\ \tau_h \\ \tau_i \\ \tau_P \\ \tau_Q \\ \tau_R \end{Bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \cdot \begin{Bmatrix} f_x \\ f_y \\ m_z \end{Bmatrix} \tag{4}$$

The solution obtained in Eq. (4) is similar to the classic static mapping obtained through virtual working method (Siciliano *et al.*, 2009). But the solution used in this work is not restricted to the absence of gravity.

3.1 Solving for secondary variables

Number and choice of primary and secondary variables; the net degrees. The solution equations.

4. A SERIAL-PARALLEL COOPERATIVE SYSTEM

- General characterists, number of loops, DoF, DoC, etc.

4.1 Planar 3R-3RRR system

- Example and considerations/simplifications. Why this example? What it represents?
- Derivation of the equations according to the example. A table with the twists and wrenches in symbolic (or numerical) form.
- Figures: system, coupling graph, action graph.

4.2 Results

The results of apply the Davie's method in a Cooperative Robotic System (CRS) is discussed in this section. The schematic representation of the studied CRS is shown in Fig. 3, this is a planar CRS with mobility equal to six (M = 6) and with nett dof equal to four ($F_N = 4$) and a nett degree of restriction equal to three ($C_N = 3$). The kinematic and the static of the CRS was solved using the formalism presented by Davies (2006) as was previously discussed in Section and the particular kinematic solution and the partuicular backward statics solution for the CRS looks like shown in Eq. (2) and Eq. (4), respectively.

The graphical results, were obtained using arbitrary topological information for the links: l_1=0.31 m, l_2=0.20 m, l_3=0.23 m for the leader manipulator and l_4=0.20 m for the follower, the end effector position (in meters) of the leader robot (x,y) = (0.273;0.144) and the follower robot (0.250;0.144), the joint angles (in rad) $(\theta_1, \theta_2, \theta_3) = (\pi/2, -\pi/2, -\pi/4)$, the angular velocities $\omega_g = \omega_h = \omega_i = 4.5$rad/s, $omega_F = 13.5$rad/s and the maximum torques in the joints of the leader $\tau_{g_{max}} = \tau_{h_{max}} = \tau_{i_{max}} = \pm 4.2$Nm and the follower $\tau_{P_{max}} = \tau_{Q_{max}} = \tau_{R_{max}} = \pm 8$Nm.

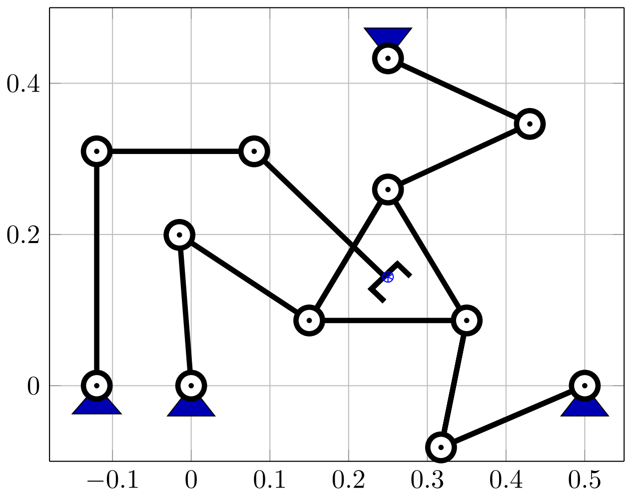


Figure 3: Planar CRS.

Variyng the joint angles of the leader manipulator by $[\pi : -\pi/360 : \theta_1]$, $[0 : -\pi/360 : \theta_2]$ and $[0 : -\pi/360 : \theta_3]$, it's possible obtain the angular velocities map of the follower manipulator, as shown in Figs. 5a, 5c and **??**.

5. CONCLUSIONS

This work discussed some problems etc of cooperative robotic systems. A solution for differential kinematics and statics of a system composed by a serial and a parallel robot, both planar, was formulated based on the analysis of Kircchhofian mechanical networks as proposed by Davies, as the usage of virtual kinematic chains where appropriated. Future works: formulation of a spatial case, force control.

6. ACKNOWLEDGEMENTS

This optional section must be placed before the list of references.

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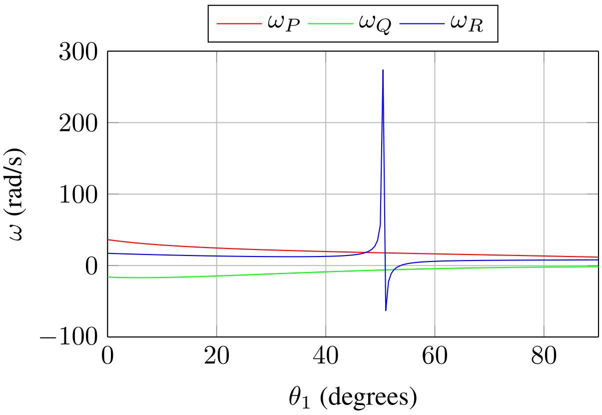
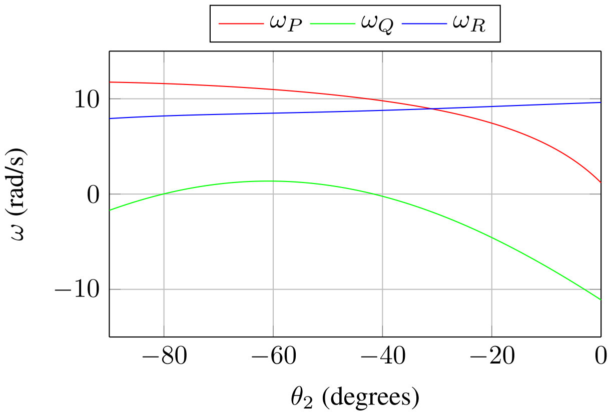
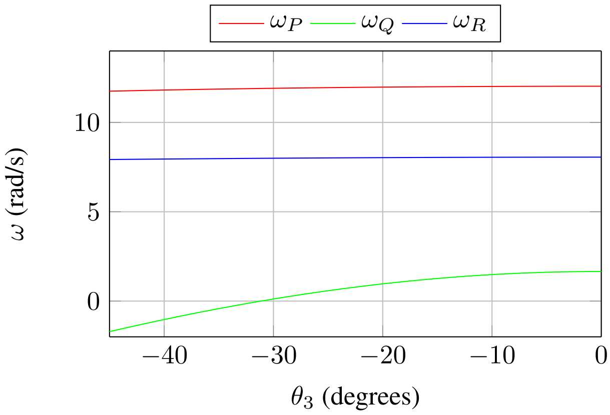


(a) Kinematic map of the follower varying θ_1.

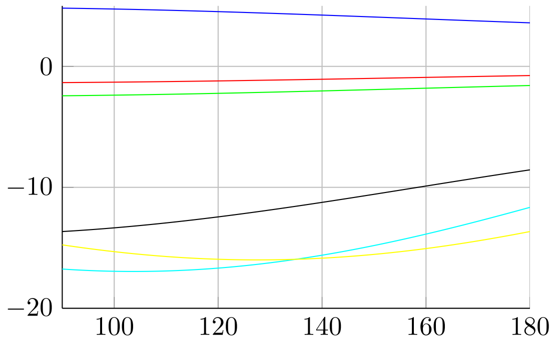


(b) Kinematic map of the follower varying θ_2.

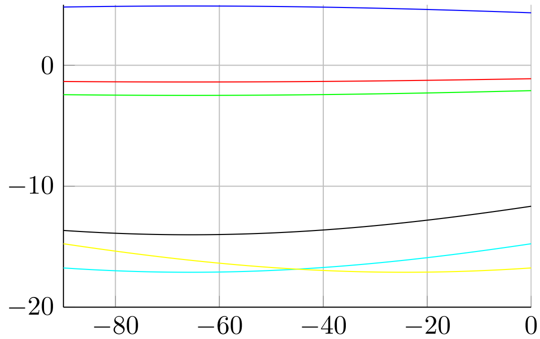


(c) Kinematic map of the follower varying θ_3.

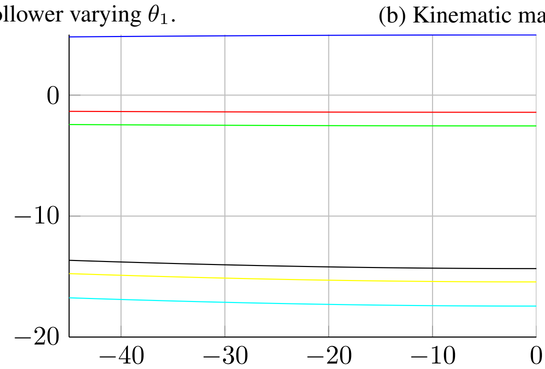
Figure 4: Kinematic map of the follower manipulator.



(a) Kinematic map of the follower varying θ_1.



(b) Kinematic map of the follower varying θ_2.



(c) Kinematic map of the follower varying θ_3.

Figure 5: Kinematic map of the follower manipulator.

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