23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

NUMERIC ANALYSIS OF AN ADIABATIC MODEL IN A STIRLING ENGINE

Ana Carolina Ávila Santos
Fábio Alfaia da Cunha
Universidade de Brasília - Faculdade Gama,
Área Especial de Indústria Projeção A Brasília
ZIP: 72.444-240
ana.avilasantos@gmail.com, fabioalfaia@yahoo.com.br

Augusto César de Mendonça Brasil
Universidade de Brasília - Faculdade Gama,
Área Especial de Indústria Projeção A Brasília
ZIP: 72.444-240
augbrasil@gmail.com

Abstract. *Concentrating Solar Power systems concentrate heat provided from the radiation into a point and can be connected to an external combustion engine, as in the case of a stirling engine. In order to improve the system's behavior, a precise analysis of the engine is needed so it is possible to define the best templates of temperatures, pressure and working fluid. For that, this paper uses mathematic tools in ideal thermodynamic hypotheses based in adiabatic and isothermal processes to provide a numeric solution method through the solution of ordinary differential equations by runge kutta method. The solution was applied in a MatLab code to predict the operation of a stirling engine provided by Genoastirling s.r.l company, it was acquired a 71,96% efficiency using 26,85ºC and 926,85ºC cooling and heating temperatures respectively and nitrogen as working fluid.*

Keywords: *Stirling Engine, Adiabatic Model.*

1. INTRODUCTION

A Stirling Engine is driven by difference of temperature between its chambers, which leads to pressure difference inside the machine due expansion and compression of the working fluid. This flow will move pistons that are connected to a rotator, generating mechanical energy which can be linked to an electric generator (MARTINI, 1983). It is possible to implement a parabolic mirror plate to concentrate solar radiation to heat up the engine working fluid, this type of system is an example of a Concentrated Solar Power (CSP) system (EPE, 2012) and it can lead the fluid to reach temperatures over 650 ºC (HINRICHS, 2010). The advantages of these systems are: applicability in remote areas and harsh conditions, no need of long transmission lines, renewable energy source and no costs with other natural resources. Qnergy company is building a 1,5 MW energy plant with this technology in Tooele, Utah (US) with total efficiency of 24% (QNERGY, 2014). There is already another one in Peorina, Arizona (US) also with 1,5 MW and 26% efficiency (NREL, 2014). These efficiencies are higher than the common photovoltaic technology, which is between 10 and 15% (HINRICHS, 2010).

In order to preview these efficiencies is indispensable to know and understand the components that will act in the engine's performance such as temperature difference, working fluid, dead volume and the regenerator. Therefore, it is important to study these factors. Qvale, in 1967, developed a second order adiabatic modeling that predicts a sinusoidal pressure, volume and mass flux behavior (apud CHEN, 1983). In 1977, Urieli developed a strict nodal analysis considering the kinetic energy effects in small control volumes in different shapes and sizes and the inert gas (apud CHEN, 1983). Later, in 1984, Urieli developed another mathematical model based on isothermal and adiabatic processes hypotheses (apud ZIABASHARHAGH, 2012). The purpose of this paper is to analyze the Urieli's adiabatic hypotheses as important concepts about thermodynamic cycles and efficiencies, stirling engine's operation and propose a numeric solution.

2. THEORETICAL BASIS

The thermodynamic processes understanding is very important as the running principles of a stirling engine are based on them. A thermodynamic process can be classified as a reversible or irreversible process. In fact, all nature processes are irreversible, according to the 2nd law of thermodynamic, all processes occur in a certain direction (that is related to energy quality, a higher temperature is better than a lower one as it can be converted, like heat in to work) searching of an equilibrium state. Therefore, after a certain process took place, beside the change in the system there is also a change in its surrounding, that makes impossible for the system to return to its initial conditions, if that did happen it would be a reversible process, which are idealizations to facilitate turbine and engine's studies. (ÇENGEL, 2006).

A thermal engine that is governed by a cycle composed only by reversible processes have higher efficiency, since there are no losses to the surrounding. They operate with two energy reservoirs that are considered large enough so they are not influenced by the system, a high temperature (T_h) source and a low temperature (T_k) sink. According to the Carnot Principle, as two thermal engines operate with same temperature reservoirs, they will present same efficiency, the Carnot Efficiency:

$$\eta_c = 1 - T_k/T_h \tag{1}$$

The stirling cycle has a carnot efficiency and it is defined by two isothermal and two isochoric process (figure 3) the work done by the cycle occurs only during the constant temperature processes, however, heat must be exchanged during all four processes (STINE, 2001).

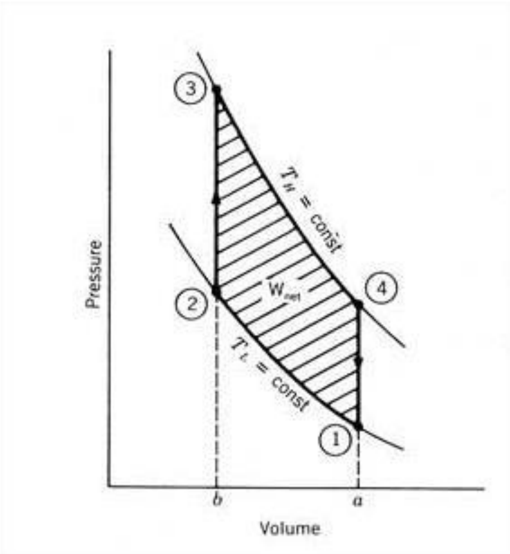

Figure 1 PV Diagram of a Stirling Cycle (STINE, 2001)

The stirling engine was invented by Robert Stirling in 1816 (PARLAK, 2008) and works through four steps: heat transfer from the source into the gas which is expanded and moves a piston, than heat transfer from the gas into the sink which leads to gas compression, bringing back the piston (STINE, 2001). In effect, these processes are not ideal, there are three important factors that will degrade the engine's operation, they are: piston's sinusoidal movement, imperfect regeneration and dead volume (STINE, 2001). The last one exercise the biggest influence on the losses, and it is due part of the gas remaining non active during operation, it remains steady inside the engine chamber's. Regeneration is a heat recycling process, which leads the heat lost in cooling back to the fluid during heating, so less energy is necessary. These factors end up rounding the PV Diagram which will look similar to figure 2.

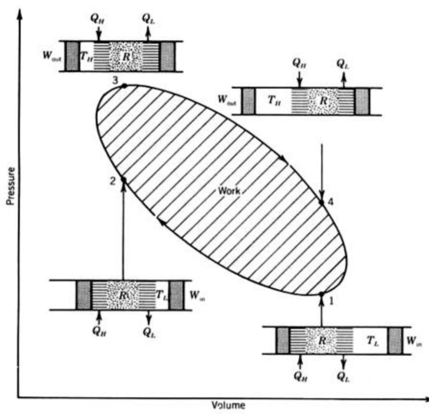

Figure 2 PV Diagram of a Stirling Engine (STINE, 2001)

3. ADIABATIC MATHEMATICAL MODELING

A stirling engine is divided into five chambers: expansion, heating, regeneration, cooling and compression spaces. The first and the last one are the working spaces due piston movement, so the walls are not built to suffer heat exchange but to endure the friction. Therefore, to make the analysis easier, adiabatic processes can be assumed, which named the modeling. The adiabatic model was first developed and worked by Urieli in 1984 (apud, ZIABASHARHAGA, 2012) and will be presented in following. Another good assumption is to consider the heat exchange process as isothermal. In addition, as the working fluid is gas, usually helium, air (nitrogen) or hydrogen, they can be treated as perfect gas, as the cycle occurs in high temperatures. The specific heats are considered constant. The engine runs with no pressure drop. The regeneration temperature changes linearly throw the chamber's geometry. As a cycle is defined, there are boundary conditions, such as: temperature in the connection between regeneration and cooling is the same one as the cooling space and the temperature between the heating and regeneration connection is the same as the heating space. To make the analysis even less complex, a medium effective temperature will be used inside the regeneration chamber (Tr), obtained through the mass equation:

$$m = \int \rho \, dV \tag{2}$$

Where,

$$dV = A \, dx \tag{3}$$

$$\rho = P \,/\, (RT) \tag{4}$$

As said, T = T(x) is the linear behavior of the regenerator's temperature dependent of x and in boundery conditions Tr(0) = Tk and Tr(L) = Th, where L is the length of the regenerator, so

$$Tr(x) = [(T_h - T_k) \,/\, L]\, x - T_k \tag{5}$$

So, if Eq. (5) and Eq. (2) are applied in Eq. (4)

$$Tr = (T_h - T_k) \,/\, \ln(T_h/T_k) \tag{6}$$

The engine's chambers are connected in the following order: Compression (c), Cooling (k), Regeneraton (r), Heating (h) and expansion (e), as can be seen in figure 3.

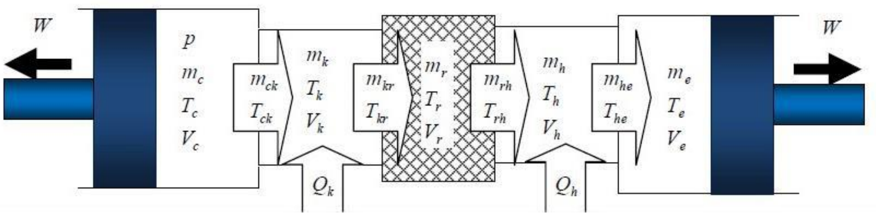

Figure 3 Stirling Engine's Chambers (ZIABASHARHAGH, 2012)

The transitory mass flow between the work chambers and heat transfer chambers will be at the temperature equals to the temperature in its origin chamber, therefore, if mck > 0, Tck = Tc and if mck < 0, Tck = Tk. The same analysis goes for the warm side of the engine, if mhe > 0, The = Th and if mhe < 0, than The = Te, as the convention for positive flow is from the colder to the warmer side.

Applying energy balance equation in each heat exchange chamber considering the isothermal condition, it is possible to isolate the heat transfer variation at each one:

$$dQ_k = [(V_k c_v)/\, R]\, dP - (c_p T_{ck} m_{ck} - c_p T_k m_k) \tag{7}$$

$$dQ_r = [(V_r c_v)/\, R]\, dP - (c_p T_k m_{kr} - c_p T_h m_{rh}) \tag{8}$$

$$dQ_h = [(h_r c_v)/\, R]\, dP - (c_p T_h m_{kr} - c_p T_{he} m_{rh}) \tag{9}$$

Now, as the energy balance equation is applied in the working spaces considering the adiabatic condition, it is possible to study the temperature variation in them:

$$dTc = Tc\ ({}^{dP}/_{P} + {}^{dVc}/_{Vc} - {}^{dmc}/_{mc}) \quad (10)$$

$$dTe = Te\ ({}^{dP}/_{P} + {}^{dVe}/_{Ve} - {}^{dme}/_{me}) \quad (11)$$

From the ideal gas equation, for the total mass inside the engine (M) the instantaneous pressure is:

$$P = MT/(\ Vc/Tc + Vk/Tk + Vh/Th + Ve/The) \quad (12)$$

The work equations in expansion and compression cells are:

$$dWe = PdVe \quad (13)$$

$$dWe = PdVe \quad (14)$$

At last, if the energy balance equation considers the entire engine as its volume control, it is possible to study the system's entropy variation:

$$\Delta S = \int_0^x {}^{1}/_{T}\, dQ \quad (15)$$

$$\Delta S = M\ c_v \ln({}^{Tx}/_{To}) - MR({}^{Vx}/_{Vo}) \quad (16)$$

4. NUMERIC SOLUTION

In order to solve the equations with differentials, it is important to stablish a parameter. Therefore, the set of equations is set in function of the crank angle (θ). The equation's set can become a dY vector with initial conditions Y(θ = 0). It is possible to integrate dY using the First Order Runge-Kutta method, where $Y(\theta_i)$ is expanded in a Taylor series as a θ_{i-1}, which results in $Y_i = Y_{i-1} + hdY_{i-1}$, where h is the size of the step taken (dθ) (RUGGIERO, 2000) . It is important to assure that the boundary conditions are respected, such as initial temperature of the cycle (in θ = 0) is the same as the final (when θ = 2π).

4.1 Study case

The explained models (mathematical and numeric) above will be applied to a gamma stirling engine, that means that the engine has a piston and a displacer in different cylinders (CHENG, 1992), from the Genoastirling s.r.l company.

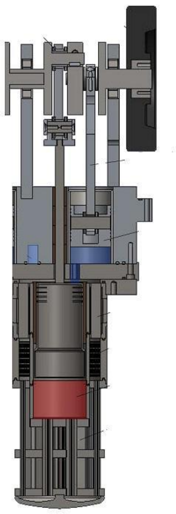

Figure 4 Genoastirling s.r.l. Gamma Stirling Engine

The numeric solution was coded and run in MatLab R2012b. The running parameters are: Th = 1200K, Tk= 300K, M = 0.005315475 kg of nitrogen gas in atmospheric pressure, maximum volume Vmax = 0.000618532 m^3, minimum volume Vmin = 0.00052624 m^3, medium volume Vmed = 0.000572078 m^3 and a 2π/100 step. The volumes behave in a sinusoidal way, as show in figure 5.

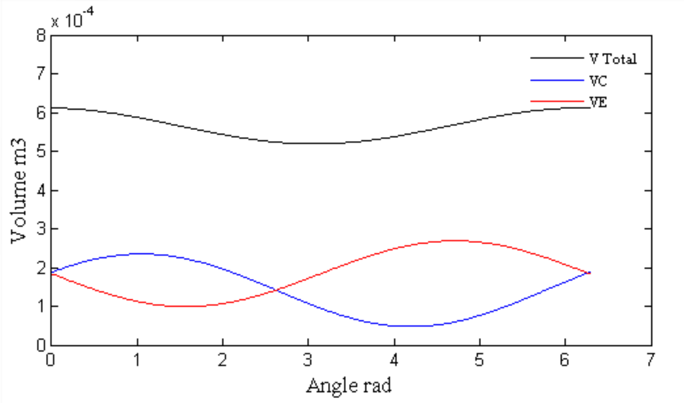

Figure 5 Volume vs Angle graph

The blue curve represents compression volume, the red curve expansion volume and the black curve is the total volume of the working fluid inside the engine.

The work produced by the cycle can be calculated by the sum of dW, and within the stablished parameters, the cycle provided W = 47.2428 J. Another way to confirm that value is to calculate the area of the PV diagram. That is the integration of Eq. (13) plus Eq. (14), the result is 47.2918 J, this slightly difference is due the different forms of calculation in MatLab, due sum and due the area of a curve, the curve can be seen in the black curve of figure 6.

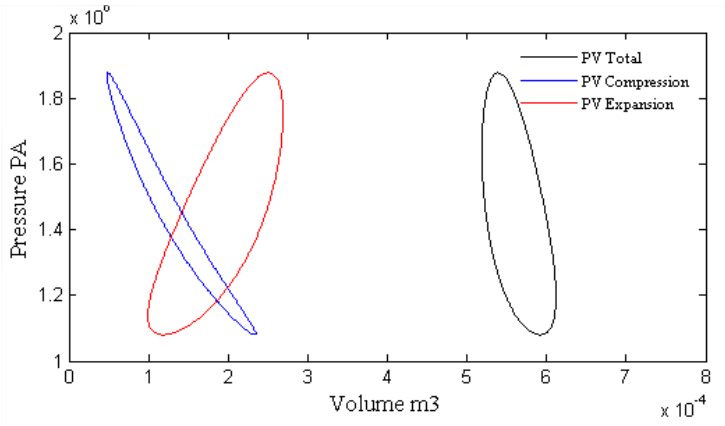

Figure 6 PV Diagrams

The expansion chamber, as it receives fluid in higher temperature, produces more work and its PV diagram's area equals to the heat provided by the source (Qh), which lead to work, while, the compression's PV diagram is equal to the amount of heat loss to the sink (Qk). From the graphs in figure 6, Qh = 66.3850 J and Qk = 19.0931 J. The net heat is the amount of energy that was converted into net work, so W = Qh – Qk = 47.2918 J, that shows how the expansion and compression's PV diagrams generate the total PV diagram.

The energetic cost to generate work inside the machine is paid in the compression and expansion cells. It occurs due the heating of the fluid through Qh in the heat exchangers and also, due its cooling through heat loss Qk to the sink. Part of that heat is recovered as Qr and goes to the heating process, so less Qh is needed, as the recovered heat never leaves the cycle, net Qr is equal to zero, characterizing an ideal regeneration process. The result of runge-kutta integration of equations 7 and 9 are Qh = 65.6503 J and Qk = - 18.7435 J.

The study of the needed heat to run an engine to produce a certain amount of work, in defined temperatures is very important to project a power plant. These parameters are used to decide which source of heat is the best choice, CSP, electrical heating or maybe a combustion chamber.

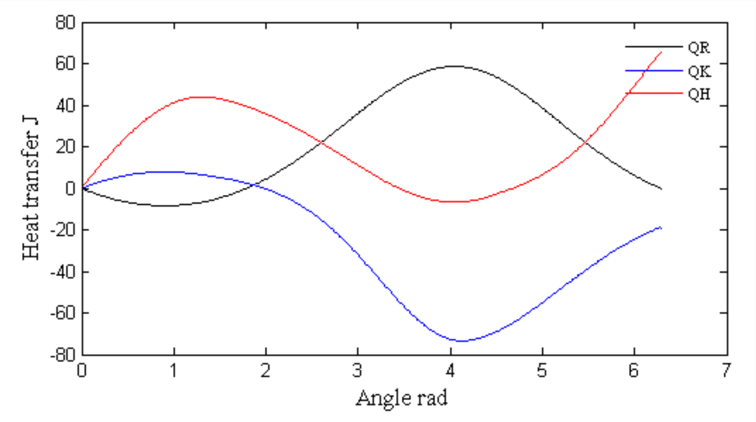

Figure 7 Heat exchange during cycle

Another interesting study aspect is the entropy variation during the cycle, which can lead into how heat is converted into work or internal energy. When a process path (0 to x) is defined and the equation 15 integral is inverted, net heat transfer is obtained associated to a reversible process. The Temperature vs Entropy graph's area provides the reversible work produced by the system (ÇENGEL, 2006). The result for the area of figure 8 is W = 47.2919 J.

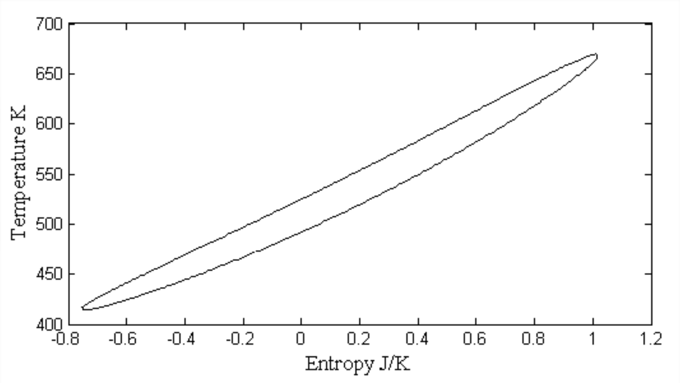

Figure 8 Temperature vs Entropy diagram

The engine's efficiency is determined by the ratio of net energy produced as work by the total of heat received.

$$\eta = W/Q_h \tag{17}$$

The simulation presented 71.96% efficiency. This value is high, but it is still lower than the carnot efficiency calculated using Eq. (1), which is 75% for the same temperature difference. This result was expected, as the engine's processes are not entirely reversible, which means that the model is, even if just a little, closer to reality.

5. CONCLUSION

The engine that was used in the model operates with a velocity around 5 hz. That means that it can generate around 473 W of mechanical power. Considering a 98% efficiency from the transformation into electric power, the system would provide 463.54. In order to make a power plant of 1,5 MW, over 3000 of these engines would be needed.

The CSP plants mentioned in introduction have an overall efficiency of 24 and 26%, when compared to the simulation result, even though the result does not consider the use of an electric generator, which will decrease the system's efficiency, the value of 71.96% is too high. It is recommended than, to make the models even more realistic, as heat loss should be considered in the work cells and imperfect regeneration should be assumed.

As consistency goes, the model is very educative, its application clearly demonstrates thermodynamics principles, its laws can be proven in different ways, for example the net work can be found trough PV diagram's or TS diagram's area, and the heat gain and loss difference.

6. REFERENCES

Hinrichs, R. A.; Kleinbach, M. and Reis, L. B. 2010. "Energia e Meio Ambiente". Cengage Learning. 4th Edition, São Paulo.

EPE, 2012, “Nota Técnica: Análise da Inserção da Geração Solar na Matriz Elétrica Brasileira”, Rio de Janeiro: MINISTÉRIO DE MINAS E ENERGIA.

Martini, W. R., 1983, “Stirling Engine Design Manual”, Second Edition. NASA, 1983.

Qnergy, “Solar Stirling Engine”. 1 oct. 2014 <http://www.qnergy.com/solar-stirling-engine->.

NREL, “Concentrating Solar Power Projects”. 1 oct. 2014: <http://www.nrel.gov/csp/solarpaces/project_detail.cfm/projectID=58>.

Chen, N. 1983, “A Review of Stirling Engine Mathematical Models”, U.S. Department of Energy. Oak Ridge National Laboratory, Oak Ridge, Tennessee, aug 1983.

Ziabasharhagh, M., 2012, “Numerical Solution of Beta-type Stirling Engine by Optimizing Heat Regenerator for Increasing Output Power and Efficiency”, Journal of Basic and Applied Scientific Research, 2012.

Çengel, Y., 2006, “Thermodynamics: An Engineering Approach”, McGraw Hill, 5th Edition.

Stine, W., 2001, “Power From The Sun”. 16 oct. 2014 <http://www.powerfromthesun.net/book.html>.

Ruggiero, M. A. G., 2000, “CÁLCULO NUMÉRICO – Aspectos Teóricos e Computacionais. Departamento de Matemática Aplicada – IMECC – UNICAMP”. Pearson Editor, São Paulo, 2nd Edition.