23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

# PERFORMANCE ANALYSIS OF ATTITUDE CONTROL USING EULER ANGLES AND QUATERNION TECHNIQUES APPLIED TO A REAL-TIME WORK BENCH

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***Abstract.*** *This work presents an analysis of attitude control using Euler angles and Quaternions techniques applied to a real-time platform, especially mounted for control system validation. Nonlinear controllers with dead zone are implemented to perform attitude control using position and velocity feedback, firstly with Euler angles and after using Quaternions parameters as input to the controller. Gyros are used to measure body angular rates, real-time system for data processing and digital control actuation and a cold gas system to generate torque control. Tests are performed to compare attitude time history obtained from the two techniques as well as time performance and specific impulse estimating. Some details are discussed concerning control strategy, digital controller implementation and singularities that occur using the techniques.*

***Keywords:*** *Attitude control, Euler angles, Quaternions*

## 1. INTRODUCTION

The attitude control can be performed using Euler angles estimated from angular velocities measured by gyros and in other approach using Quaternion technique using parameter estimating. The following control system is implemented in a NI CompactRIO system, accessing AD and DA converters. The feedback consists of angular displacement and angular velocity signals, performing a typical servo position and velocity feedback. The Fig 1 shows the basic control system.

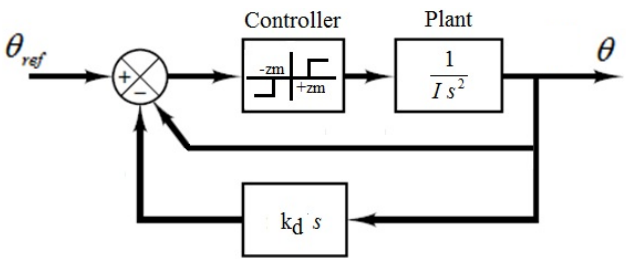


Figure 1. Feedback nonlinear control system.

The attitude control work bench is show in Figure 2, where an air bearing, a pneumatic cylinders and an aluminum disk are presented. Body angular velocities are measured by gyros sensors and analog filters for noise suppression.

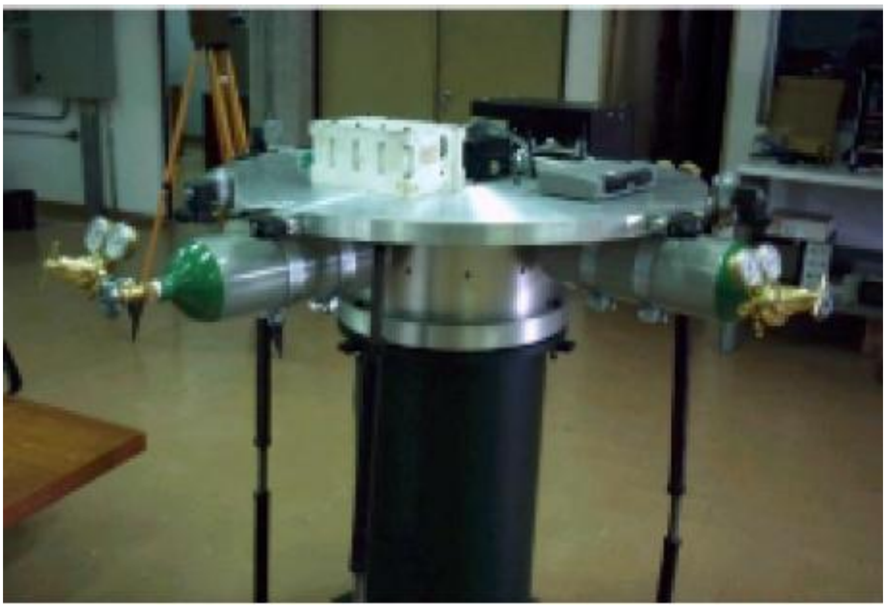

Figure 2. Attitude control work bench.

The Figure 3 shows the on-off actuator that performs 2 Newtons (Thrust) and so a torque to roll control of 4 N.m. The roll control is performed with 4 actuators (4 N.m), pitch with 2 actuators (2 N.m) and yaw with 2 actuators (2 N.m).

Figure 3. On-off cold gas actuators.

The Figure 4 presents the block diagram for attitude control system using Euler angles. The CE is a conversion matrix to obtain attitude, while the controller law has a nonlinear on-off action with 4 degrees of dead zone. The velocity feedback has a gain $k_d = 4$.

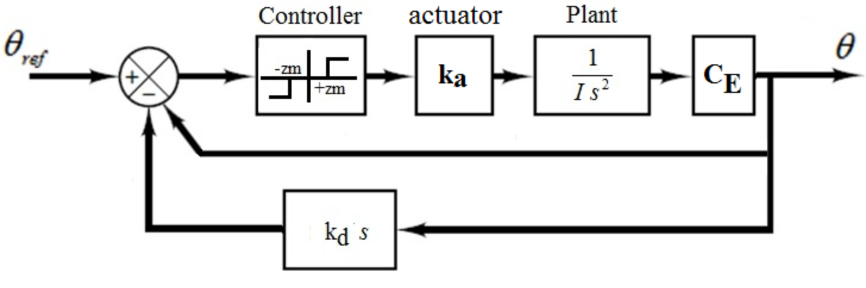


Figure 4. Attitude control system using Euler angles.

The error is calculated according to Eq. (1).

$$e = \phi_{ref} - (\phi + k_d \dot{\phi}) \tag{1}$$

The Figure 5 presents the block diagram for attitude control system using Quaternions technique. The CEQ and CQE are conversion Matrices to obtain attitude Quaternion and Euler respectively, while the control law has a nonlinear on-off action with 4 degrees of dead zone. The velocity feedback has a gain $k_d = 4$.

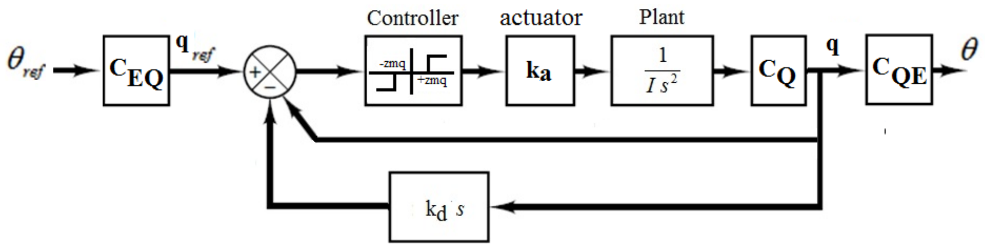


Figure 5. Attitude control system using Quaternion technique.

The error is calculated according to Eq. (2).

$$e = q_{ref} - (q + k_d \dot{q}) \tag{2}$$

The plant is represented by the well-known Equation of Motion, Eq. (3) as follows.

$$\begin{aligned} I_{xx}\dot{p} &= -p\dot{I}_{xx} + qr(I_{yy} - I_{zz}) + 4E_r D \\ I_{yy}\dot{q} &= -q\dot{I}_{yy} + pr(I_{zz} - I_{xx}) + 2E_p D \\ I_{zz}\dot{r} &= -r\dot{I}_{zz} + pq(I_{xx} - I_{yy}) + 2E_y D \end{aligned} \tag{3}$$

where:

p = body roll angular velocity, [rad/s]
q = body pitch angular velocity, [rad/s]
r = body yaw angular velocity, [rad/s]
I = Inertia [kg.m$^2$]

### 1.1 Euler angles approach

Several techniques are used to transform one frame into another, according to the dynamic system to the plant, e. g,. launcher vehicles, airplanes and missiles. Euler angles, Quaternions or a rotation-matrix can be used but Euler approach is the most popular. The Euler angles are obtained by processing body angular rates measured by gyros. The velocity transformation matrix is presented as follows in the Eq. (4), resulting in temporal Euler angles. A simple integration process is implemented to finally obtain the attitude. The rotation matrix $C_{Euler}$ is given by Eq. (4), as follows.

$$C_{Euler} = \begin{bmatrix} \cos\theta\cos\psi & \sin\psi & -\cos\psi\sin\theta \\ -\sin\psi\cos\theta\cos\phi + \sin\theta\sin\phi & \cos\psi\cos\phi & \sin\psi\sin\theta\cos\phi + \cos\theta\sin\psi \\ \sin\psi\cos\theta\cos\phi + \sin\theta\cos\phi & -\cos\psi\sin\phi & -\sin\psi\sin\theta\sin\phi + \cos\theta\cos\phi \end{bmatrix} \tag{4}$$

and the velocity transformation matrix is given by Eq. (5) as follows.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & -tg\,\psi\cos\phi & tg\,\psi\sin\phi \\ 0 & \cos\phi/\cos\psi & -\sin\phi/\cos\psi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{5}$$

### 1.2 Quaternion approach

The Quaternion approach consists of four quaternions and their relation to the Euler angles is given by Eq. (6) as follows.

$$\begin{aligned} q_0 &= \cos(\frac{\psi}{2})\cos(\frac{\theta}{2})\cos(\frac{\phi}{2}) + \sin(\frac{\psi}{2})\sin(\frac{\theta}{2})\sin(\frac{\phi}{2}) \\ q_1 &= \cos(\frac{\psi}{2})\cos(\frac{\theta}{2})\sin(\frac{\phi}{2}) - \sin(\frac{\psi}{2})\sin(\frac{\theta}{2})\cos(\frac{\phi}{2}) \\ q_2 &= \cos(\frac{\psi}{2})\sin(\frac{\theta}{2})\cos(\frac{\phi}{2}) + \sin(\frac{\psi}{2})\cos(\frac{\theta}{2})\sin(\frac{\phi}{2}) \\ q_3 &= -\cos(\frac{\psi}{2})\sin(\frac{\theta}{2})\sin(\frac{\phi}{2}) + \sin(\frac{\psi}{2})\cos(\frac{\theta}{2})\cos(\frac{\phi}{2}) \end{aligned} \tag{6}$$

And evaluating the four parameters we have the following kinematic equation presented in the Eq. (7).

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_4 & -q_3 & q_2 & q_1 \\ q_3 & q_4 & -q_1 & q_2 \\ -q_2 & q_1 & q_4 & q_3 \\ -q_1 & -q_2 & -q_3 & q_4 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \\ 0 \end{bmatrix} \tag{7}$$

The relation between Quaternions and the rotation matrix is obtained via Euler's formula, and so results in the following conversion matrix C.

$$C = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 - q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_1q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \tag{8}$$

## 2. REAL-TIME CONTROLLER

The real-time controller is implemented using a CompactRIO and LabVIEW environment. The Fig 6 shows the front panel used as Grafic User Interface (GUI).

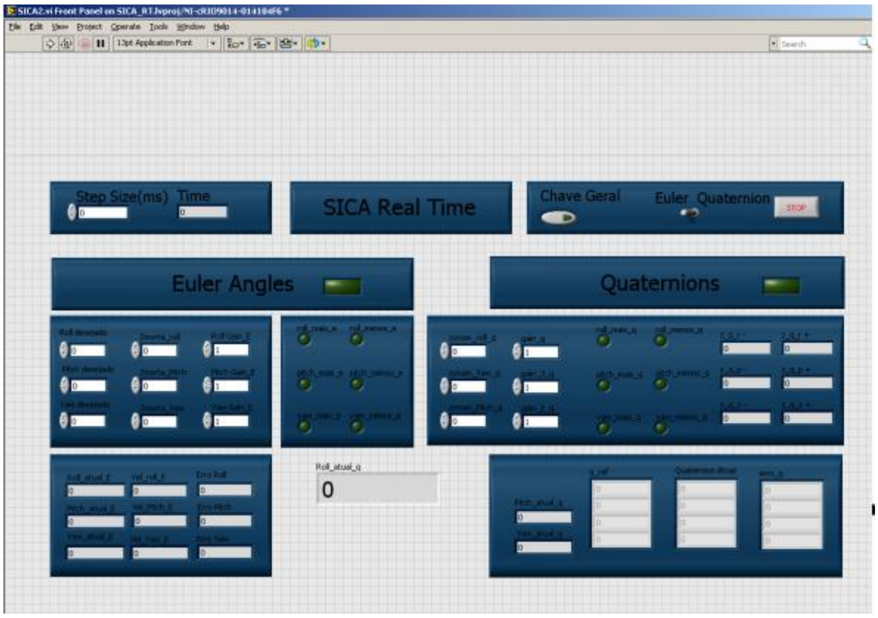


Figure 6. CompactRIO Control Panel.

The Quaternion is evaluated using matrices calculations to obtain the four parameters, as shown in Fig. 7. The body angular velocities are measured from gyros and then used to internal calculations to obtain the Euler parameters. This calculation considers the C-conversion matrix evaluation. The controller inputs are the three Quaternion parameters according to attitude of reference.

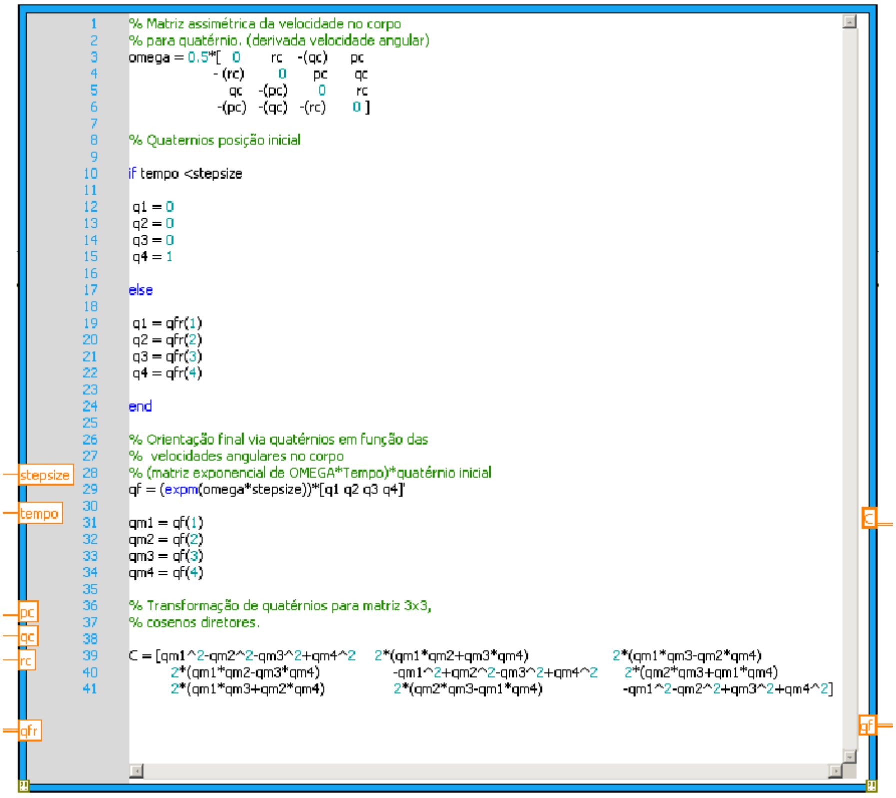


Figure 7. Quaternion approach.

## 3. RESULTS

The results obtained from tests using Euler angles are shown in the Figures 8, 9 and 10. The Fig. 8 presents the data history to roll angular attitude, angular velocity p, attitude of reference and the internal error to the digital controller. The roll reference is constant defined with value of 45 degrees.

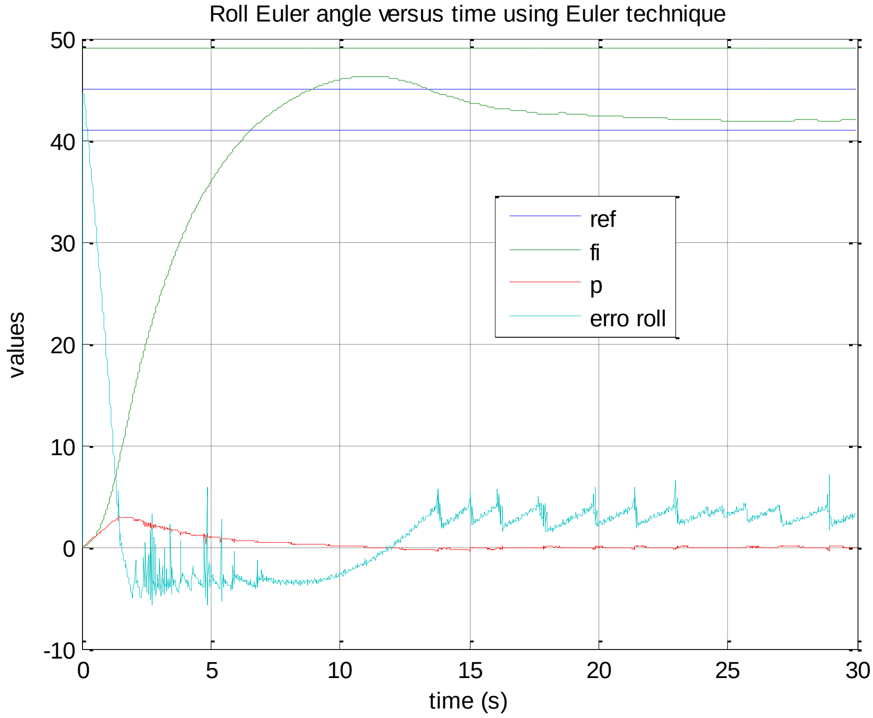


Figure 8. Results using Euler angle approach.

The results obtained from tests using Quaternion technique are shown in the Figure 9. It shows the data history to roll angular attitude, angular velocity p, attitude of reference and the internal error to the digital controller. The roll reference is constant defined with value of 45 degrees.

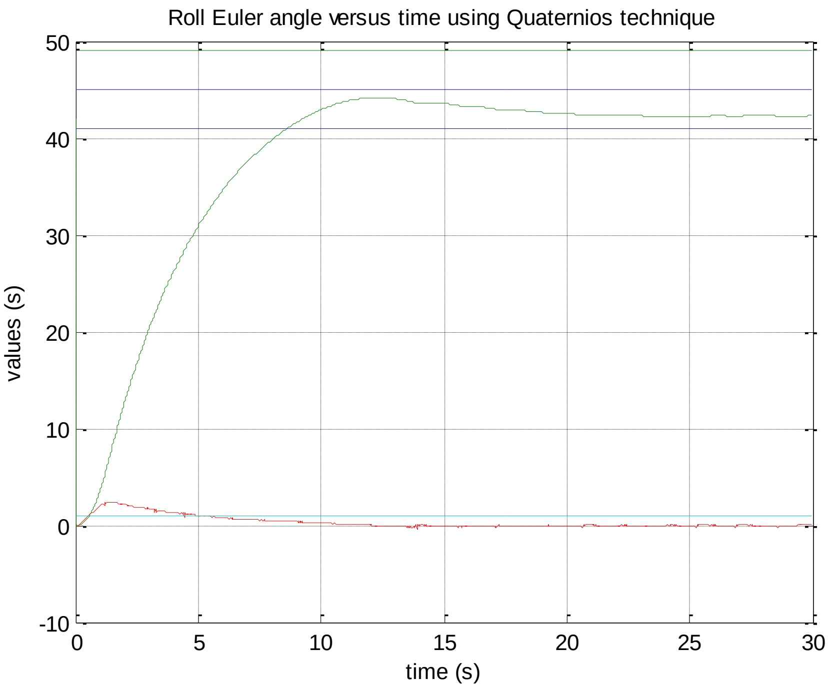


Figure 9. Results using Quaternion approach.

The Figure 10 shows the data history to roll angular attitude, angular velocity p, attitude of reference and the internal error to the digital controller, for both Euler and Quaternion techniques.

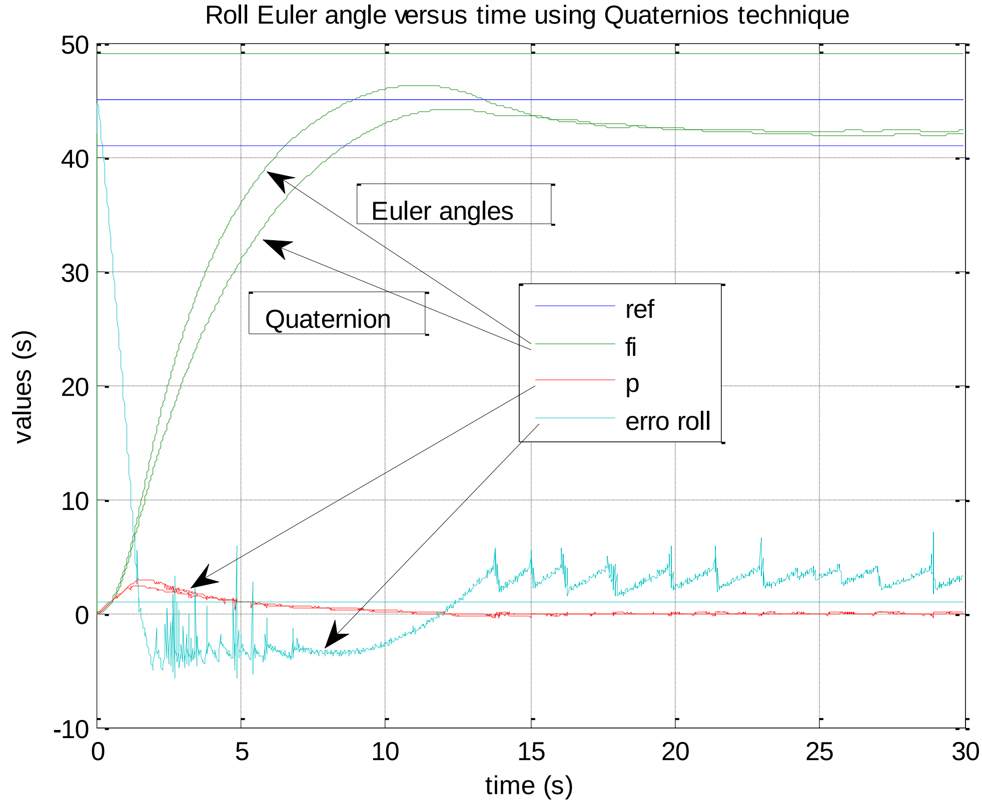


Figure 10. Comparison - Results using Euler and Quaternion approach.

## 4. CONCLUSIONS

The attitude control classically is performed using Euler angles and this work presents results from implementation of Quaternion technique. Both approaches showed similar attitude data histories. The Controller using q-parameters is performed with no restrictions and the tools, for control design, can be used the same way as using Euler angles. The unit feedback is related to angular displacement (estimated from temporal Euler angles integration) and the velocity feedback is related to body angular displacements measured from gyros. The gain associated to velocity feedback can be easily obtained from phase plane analysis using $\dot{\phi}$ versus $\phi$ diagram. The results using Euler angles approach presented an overshoot of 5% against 0% for Quaternions, but with a better rise time. The settling time is the same for both approaches. This work bench permits validation of control strategies as presented in this work using Quaternion technique, as well as others control requirements can be verified, e. g. influences of digital filtering, sampling frequencies, sensors bandwidth, converters resolution and mainly an analysis on attitude configuration leading on singularities due to Euler and Quaternion approaches.

## 5. ACKNOWLEDGEMENTS

The authors acknowledge the contribution from Carlos Henrique da Silva for LabVIEW program development.

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