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General Approach to Modeling Quadrotor Dynamics Using Newton-Euler Techniques

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Abstract. *A quadrotor is an Unmanned Aerial vehicle (UAV), which has an X-shaped structure, which the ends have an assembly of engine (or electric motor) and propeller blade. The rotation of this rotor set create lift force and angular momentum. The combination of lift and angular momentum, of the four rotors, could be used to change the attitude of quadrotor, changing pitch, yaw, roll and tridimensional position, due to conservation of momentum and angular momentum. The technological development allowed an advance of the aerodynamic techniques that shows up both the development of these vehicles, but also has its common use in many ordinary applications. This raise a need to develop an UAV, which presents better stability than today UAV models. In turn, made possible through development of dynamic models that describe permanent and transient response of the dynamical system. However, the study of Dynamical Systems is a complex area of many Engineering branches like Control, Mechanical and Mechatronics Engineering. Furthermore, although there are models available in the literature, they are much particularized to a specific variant of the possible Quadrotors types. Thus, there are difficulties in carrying out dynamic modeling quadrotors considering these different variants, using known methods. This is due to poor documentation of mathematical procedures used to obtain the existing models. These factors are responsible for the lack of references that expose the methods used for modeling and became predominant choice for the paper theme. Thus, in order to formulate generalized models, it is proposed to obtain the equations of motion of a quadrotor by means of the Newton-Euler method, whereas, a local coordinate system and other inertial or global. Thus, the relationship of equations of motion with the Euler angles are established. Subsequently, the Taylor series for linearization of the equations of motion and therefore analysis was realized in state space mathematical description. Therefore, this paper presents a guide to obtain dynamic models for the many variants of quadrotors.*

Keywords: *Dynamical Modeling, Quadrotor, Newton-Euler method Space State Mathematical*

1. INTRODUCTION

The quadrotor is an unmanned aerial vehicle known as multicopter, that features low weight and the possibility of flying low rotation speeds. In virtue of these characteristics, it has great maneuverability and accuracy, can hover, land and perform vertical takeoff. It shows up efficient on replacing many man activities, as high risk operations, surveillance, remote monitoring and detailed inspections. In recent years, the quadrotors are becoming research's attractive theme, especially on studies focused on dynamic modeling and control methods development automatic to flight stabilization and regulation due to control difficulty (SA, 2012; PEREIRA, 2008).
The design and analysis of these multicopters can be made through the classical techniques as Newton-Euler and Euler-Lagrange (BOUABDALLAH and Siegwart, 2005). The modeling of these systems using these formalisms is used to obtain the motion equations and consequently linearization of a Differential Equations System. The modeling focus is obtain equations to simulate or design control systems. In this work the focus is obtain motion equation to stability analysis of Nonlinear Systems. Despite the focus displayed the six second order nonlinear differential equations was linearized by Taylor Series and after that simulated. The results shown compatibility with model found on literature and result are similar.

2. MOTION EQUATION FOR QUADRIROTOR

The quadrotor motion arises from the rotation of four thrusters rotors. Thus, the lifting force is the sum of the contributions of the speeds and torques of the four rotors, which allows the realization of an aerodynamic analysis quadrotor (McKerrow, 2004). To achieve the study is conducted analysis of all the forces acting on the aircraft, on the move, including his weight force.

The appearance of the major forces and moments are due exclusively the variation in speed of the rotors, it is necessary to balance the system, which in pairs, these rotate in opposite directions (and NAIDOO STOPFORTH , 2011). Figure 2.1 shows a quadrotor X-quad type, in which the rotors 1 and 3 rotate in the counterclockwise direction and 2, 4 rotors rotate clockwise, so as to balance the system.

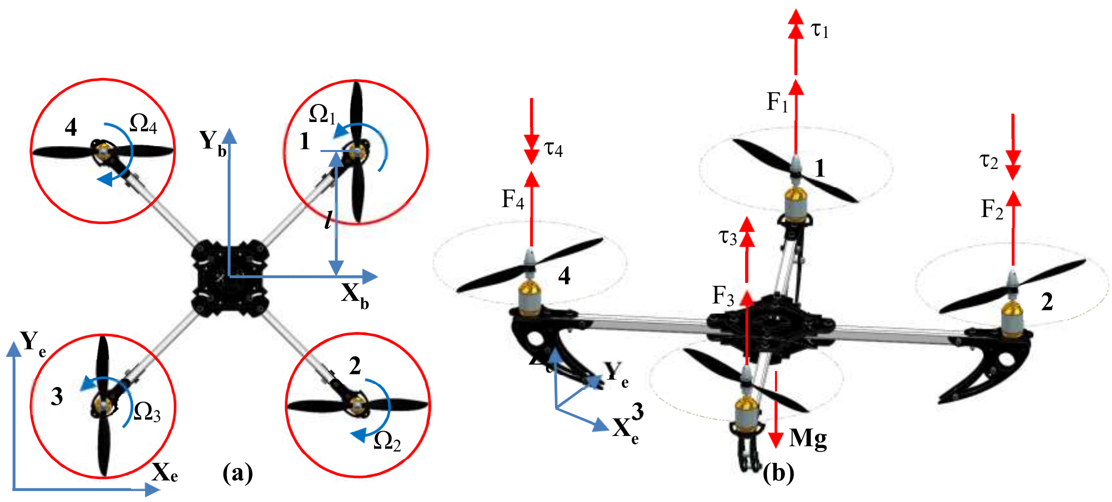

Figure 1: Quadrotor coordinate systems (a) and free body diagram (b) **Source:** MOURA (2014)

For analysis let $X_eY_eZ_e$ be an Absolute or Inertial Coordinate System (ACS) and $X_bY_bZ_b$ the local or body coordinate systems (BCS). A combination of four lift forces the quadrotor contrary to gravitational action and four drag torques due air resistance to the motion. The system of forces and torques which must be considered is shown on Figure 1. Besides, it is important to consider the quadrotor as a rigid body.

The motion of the quadrotor can be categorized into roll, pitch and yaw, described according to the local coordinate system (Bresciani, 2008), shown in Figure 1. These motions correspond to the rotation of quadrotor system for each axe, respectively, $X_bY_bZ_b$.

Symbol	Definition	Symbol	Definition
X_e, Y_e, Z_e	Position in ACS	L	Lift Force
X_b, Y_b, Z_b	Position in BCS	Ω	Rotor angular velocity
$I_{X_b}, I_{Y_b}, I_{Z_b}$	Body inertia	$\mathbf{\Omega}$	Quadrotor angular velocity vector
$\ddot{X}_e, \ddot{Y}_e, \ddot{Z}_e$	Acceleration in ACS	l	Lever Arm
$F_{X_e}, F_{Y_e}, F_{Z_e}$	Forces in the ACS	M	Quadrotor Mass
$F_{X_b}, F_{Y_b}, F_{Z_b}$	Lift forces on the rotors on BCS	g	Gravitational acceleration
ϕ, θ, ψ	Roll, Pitch, Yaw angular position	J_r	Polar moment of inertia of the rotors
$\dot{\phi}, \dot{\theta}, \dot{\psi}$	Roll, Pitch, Yaw angular velocity	$\mathbf{g}(\mathbf{x})$	Gradient Operator
$\ddot{\phi}, \ddot{\theta}, \ddot{\psi}$	Roll, Pitch, Yaw angular acceleration	$\mathbf{H}_e$	Quadrotor Angular Momentum
$\tau_{X_e}, \tau_{Y_e}, \tau_{Z_e}$	Torque System in ACS	b	Thrust factor
$\tau_{X_b}, \tau_{Y_b}, \tau_{Z_b}$	Torque System in BCS	d	Drag factor
D	Drag Force		

The roll motion is rotation around the axe X_b, this is possible by the combination of rotors speeds, causing an unbalance and the appearance of the rotation around the direction X_b. Theoretically, increasing up of angular velocity of the rotors 1 and 4 and reducing up rotors 2 and 3, generating a positive torque, or in another case, generate a negative torque in the opposite direction to the rotation increasing up speed of rotors 2 and 3 and reducing up 1 and 4. These speed changes cause variations in relation to the angle ϕ.

Similarly occurs pitch motion, being identified as the rotation around the axis $Y_{b,}$, further characterized by the variation θ which arises from the rotor unbalance. A positive torque in relation $Y_{b,}$ is generated by the speed increasing of the rotors 1 and 2 and reducing speed of rotors 3 and 4. Otherwise it have been a negative torque. The yaw motion, which refers to rotation around Z_b, it is necessary to unbalance between the four rotors being determined by the variation ψ. Thus, if the rotors 1 and 3 intensify the rotation, 2 and 4 reduce, proportionately, or the reverse, producing a yaw motion around Z_b (Guimarães, 2012). To vertically upward movement, the rotation of all rotors is intensified simultaneously (BOUABDALLAH, and MURRIERI SIEDWART, 2004).

The performances of drag and lift forces define the torques. These forces are influenced by parameters of the profiles used in propellers, which have special features in addition to the angular velocity of the rotors, pointed at Naidoo and Stopforth (2011). So for the specific type of propeller uses a relation to the lift (L) and one for drag (D).

Considering a profile already defined, these forces have a direct relationship with the angular velocity of the rotor and constants b and d, which represent lift and drag, respectively, which are the type of wing functions. Considering them, get the equations (1) and (2):

$$D = d\Omega^2 \tag{1}$$

$$L = b\Omega^2 \tag{2}$$

Knowing the constant b and d are attributes of the propeller, defined by propeller design, the only variable is the angular velocity of the rotors. Thus, the lifting forces acting on the rotors are set acting in the positive direction Z_b, as shown in Equation (3):

$$\begin{bmatrix} F_{X_b} \\ F_{Y_b} \\ F_{Z_b} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ b\left[\Omega_1{}^2 + \Omega_2{}^2 + \Omega_3{}^2 + \Omega_4{}^2\right] \end{bmatrix} \tag{3}$$

Considering l the distance between the rotation shaft and the rotor and knowing the forces as equations (1) and (2), it is possible to determine the torques of each of rotation axes as shown by equations (4):

$$\begin{bmatrix} \tau_{X_b} \\ \tau_{Y_b} \\ \tau_{Z_b} \end{bmatrix} = \begin{bmatrix} lb\left(\Omega_1{}^2 + \Omega_2{}^2 - \Omega_3{}^2 - \Omega_4{}^2\right) \\ lb\left(-\Omega_1{}^2 + \Omega_2{}^2 + \Omega_3{}^2 - \Omega_4{}^2\right) \\ d\left(-\Omega_1{}^2 + \Omega_2{}^2 - \Omega_3{}^2 + \Omega_4{}^2\right) \end{bmatrix} \tag{4}$$

To obtain the equations of motion, it is held coordinate transformation in relation to the inertial system $X_e Y_e Z_e$. Thus, knowing the forces on local coordinate system are acting only on Z_b, considering it parallel to Z_e, then the weight force only present in installments Z_e. Thus, it is presented according to equation (5):

$$\begin{bmatrix} F_{X_e} \\ F_{Y_e} \\ F_{Z_e} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ F_{Z_b} \end{bmatrix} \rightarrow \begin{bmatrix} F_{X_e} \\ F_{Y_e} \\ F_{Z_e} \end{bmatrix} = \begin{bmatrix} F_{Z_b}\alpha_{13} \\ F_{Z_b}\alpha_{23} \\ F_{Z_b}\alpha_{33} \end{bmatrix} \tag{5}$$

Where the columns of $\left[\alpha_{ij}\right]$ are direction cosines of local coordinate system. The $\left[\alpha_{ij}\right]$ is obtained by rotation by three successives rotations in relation to local coordinate system, respectively $R_{Z_b} R_{Y_b} R_{X_b}$. That configuration was choose because generate simple components for $\left[\alpha_{i3}\right]^T$. Thus, the equations can be handled with greater simplicity. Considering the gravitational field action, the acceleration equations are shown in system equations (6):

$$\begin{bmatrix} \ddot{X}_e \\ \ddot{Y}_e \\ \ddot{Z}_e \end{bmatrix} = \begin{bmatrix} F_{Z_b} \sin\phi \sin\theta / M \\ F_{Z_b} \cos\phi \sin\theta / M \\ F_{Z_b} \cos\theta / M + g \end{bmatrix} \tag{6}$$

Subsequently, the torque equations are obtained deriving angular momentum of system in relation to the local coordinate system (Lemos,2007):

$$\dot{\mathbf{H}}_e = \left(\dot{\mathbf{H}}_e\right)_b + \mathbf{\Omega} \times \mathbf{H}_e \rightarrow \boldsymbol{\tau}_e = \boldsymbol{\tau}_b + \mathbf{\Omega} \times \mathbf{H}_e \tag{7}$$

Where $\boldsymbol{\tau}_b$ is torque generated by lifting and drag forces on rotors on local coordinate system, $\mathbf{\Omega}$ is angular velocity of the local coordinate system and $\mathbf{H}_e$ angular momentum of quadrotor. Thus realizing the cross product, knowing that $\mathbf{\Omega} = \begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T$ and writing angular momentum for each direction, beginning to X_e direction in Equation (8) have been:

$$\mathbf{\Omega} \times \mathbf{H}_e = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dot{\phi} & \dot{\theta} & \dot{\psi} \\ \mathbf{I}_P\mathbf{\Omega} & 0 & 0 \end{vmatrix} = \dot{\psi}\mathbf{I}_P\left(\mathbf{\Omega}^T\mathbf{j}\right) - \dot{\theta}\mathbf{I}_P\left(\mathbf{\Omega}^T\mathbf{k}\right) = \left(I_{Y_b} - I_{Z_b}\right)\dot{\theta}\dot{\psi} \tag{8}$$

Where $\mathbf{I}_p$ is the inertia matrix of system, represented by principal moments of inertia and $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors in the directions, respectively, X_e, Y_e, Z_e.

Applying the result obtained at equations (4) and equation (8) in equation (7), resulting in the torque component in X_e direction equation (9).

$$\tau_{X_e} = lb\left[\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2\right] + \left(I_{Y_b} - I_{Z_b}\right)\dot{\theta}\dot{\psi} \tag{9}$$

Similarly, the torque components in the Y_e and Z_e can be obtained. In the directions X_e and Y_e can be add the gyroscopic effect due rotors. By these considerations, we can obtain the system equations (10):

$$\begin{bmatrix} \tau_{X_e} \\ \tau_{Y_e} \\ \tau_{Z_e} \end{bmatrix} = \begin{bmatrix} lb\left(\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2\right) + \left(I_{Y_b} - I_{Z_b}\right)\dot{\theta}\dot{\psi} - J_r\dot{\theta}\omega \\ lb\left(-\Omega_1^2 + \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right) + \left(I_{Z_b} - I_{X_b}\right)\dot{\phi}\dot{\psi} + J_r\dot{\phi}\omega \\ d\left(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right) + \left(I_{X_b} - I_{Y_b}\right)\dot{\phi}\dot{\theta} \end{bmatrix} \tag{10}$$

Therefore, the components of angular acceleration equations for the quadrotor motion are expressed by equations (11):

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dfrac{U_2}{I_{X_b}} + \left(\dfrac{I_{Y_b} - I_{Z_b}}{I_{X_b}}\right)\dot{\theta}\dot{\psi} + \dfrac{J_r}{I_{X_b}}\dot{\theta}\omega \\ \dfrac{U_3}{I_{Y_b}} + \left(\dfrac{I_{Z_b} - I_{X_b}}{I_{Y_b}}\right)\dot{\phi}\dot{\psi} + \dfrac{J_r}{I_{Y_b}}\dot{\phi}\omega \\ \dfrac{U_4}{I_{Z_b}} + \left(\dfrac{I_{X_b} - I_{Y_b}}{I_{Z_b}}\right)\dot{\phi}\dot{\theta} \end{bmatrix} \tag{11}$$

Where $U_1 = b\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right)$, $U_2 = lb\left(\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2\right)$, $U_3 = lb\left(-\Omega_1^2 + \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right)$, $U_4 = d\left(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right)$ are input of the systems and finally, $\omega = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4$ is a disturbance.
Therefore, the dynamical model can be presented as system of nonlinear ordinary differential equations, equations (12). The model obtained is similar to the model proposed Bouabdallah and Siegwart (2004).

$$\begin{bmatrix} \ddot{X}_e \\ \ddot{Y}_e \\ \ddot{Z}_e \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} bU_1 \sin\phi \sin\theta / M \\ bU_1 \cos\phi \sin\theta / M \\ bU_1 \cos\theta / M + g \\ \frac{lb}{I_{X_b}} U_2 + \left(\frac{I_{Y_b} - I_{Z_b}}{I_{X_b}} \right) \dot{\theta}\dot{\psi} + \frac{J_r}{I_{X_b}} \dot{\theta}\omega \\ \frac{lb}{I_{Y_b}} U_3 + \left(\frac{I_{Z_b} - I_{X_b}}{I_{Y_b}} \right) \dot{\phi}\dot{\psi} + \frac{J_r}{I_{Y_b}} \dot{\phi}\omega \\ \frac{lb}{I_{Z_b}} U_4 + \left(\frac{I_{X_b} - I_{Y_b}}{I_{Z_b}} \right) \dot{\phi}\dot{\theta} \end{bmatrix} \tag{12}$$

3. LINEARIZATION OF MOTION EQUATION FOR QUADROTOR

To obtain the control system of a quadrotor, it is necessary linearization of the equations obtained in mathematical modeling, which represent state variables of a dynamic system. The linearization process follow the procedure proposed by Duke, Antoniewicz and Krambeer (1988). For simplification, it is shown here only the linearization of the equation of linear acceleration in the X_e direction, as for other equations, the procedure is analogous to obtain.

$$\ddot{X}_e = \frac{b \sin\phi \sin\theta}{M} U_1 \tag{13}$$

From equation (13) is applied to the Taylor series linearization, according to equation (14).

$$\ddot{X}_e = \ddot{X}_{e0} + \mathbf{g}(\mathbf{x})_{\mathbf{x}=\mathbf{x_0}} \mathbf{\Delta} \tag{14}$$

In the equation (14), $\ddot{X}_{e0}$ is acceleration component in the direction X_e in the initial instant, $\mathbf{g}(\mathbf{x})_{\mathbf{x}=\mathbf{x_0}}$ is gradient operator in the initial instant. Since $\mathbf{g}(\mathbf{x})_{\mathbf{x}=\mathbf{x_0}}$ is function of $\mathbf{x}$ and, it is a vector dependent of the time. Knowing that $\ddot{X}_e$ is function ϕ, θ and U_1, then $\mathbf{x} = [\phi \quad \theta \quad U_1]^T$. Thus, derivation were carried out with respect to $\mathbf{x}$, as shown in the equations (15).

$$\begin{aligned} \mathbf{g}(\mathbf{x})_{\mathbf{x}=\mathbf{x_0}} &= \frac{d}{d\mathbf{x}} \left(\ddot{X}_e \right) \Big|_{\mathbf{x}=\mathbf{x_0}} \\ \mathbf{g}(\mathbf{x})_{\mathbf{x}=\mathbf{x_0}} &= \left[\frac{b}{M} \cos\phi_0 \sin\theta_0 U_{10} \quad \frac{b}{M} \sin\phi_0 \cos\theta_0 U_{10} \quad \frac{b}{M} \sin\phi_0 \sin\theta_0 \right] \end{aligned} \tag{15}$$

Assuming that $\mathbf{\Delta}$, is the variation of $\mathbf{x}$ expressed in equation (16):

$$\mathbf{\Delta} = [\mathbf{x} - \mathbf{x}_0] = \begin{bmatrix} \phi - \phi_0 \\ \theta - \theta_0 \\ U_1 - U_{10} \end{bmatrix} \tag{16}$$

It is assumed that $\ddot{X}_{e0}$ the acceleration in the direction x at the initial time, as shown in equation (17):

$$X_{e0} = \frac{b}{M} \sin\phi_0 \sin\theta_0 U_{10} \tag{17}$$

Consequently, the equation (18) results in equation 22:

$$\ddot{X}_e = \frac{b}{M} \sin\phi_0 \sin\theta_0 U_{10} + \left[\frac{b}{M} \cos\phi_0 \sin\theta_0 U_{10} \quad \frac{b}{M} \sin\phi_0 \cos\theta_0 U_{10} \quad \frac{b}{M} \sin\phi_0 \sin\theta_0 \right] \begin{bmatrix} \phi - \phi_0 \\ \theta - \theta_0 \\ U_1 - U_{10} \end{bmatrix} \tag{18}$$

As a consequence, considering the same sequence of calculations, the equations for the other linear and angular accelerations were obtained. Further, a change of variable, shown in equation (19), results in equation (20):

$$\mathbf{x} = [x_i] = \left[x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}\right]^T, i = 1..12 \tag{19}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ \left[\frac{b}{M}\cos\phi_0 \sin\theta_0 U_{10}\right]x_7 + \left[\frac{b}{M}\sin\phi_0 \cos\theta_0 U_{10}\right]x_9 + \left[\frac{b}{M}\sin\phi_0 \sin\theta_0 U_{10}\right]U_1 + R_1 \\ x_4 \\ \left[-\frac{b}{M}\sin\phi_0 \sin\theta_0 U_{10}\right]x_7 + \left[\frac{b}{M}\cos\phi_0 \cos\theta_0 U_{10}\right]x_9 + \left[\frac{b}{M}\cos\varphi_0 \sin\theta_0\right]U_1 + R_4 \\ x_6 \\ \left[\frac{b}{M}\cos\theta\right]U_1 - \left[\frac{b}{M}\sin\phi_0 U_{10}\right]x_9 + \left[\frac{b}{M}\sin\theta_0 U_{10}\theta_0\right] + g + R_6 \\ x_8 \\ \left[\frac{lbU_{20}}{I_x} + \frac{I_y - I_z}{I_x}\psi_0 + \frac{J_r}{I_x}\omega_0\right]x_{10} + \left[\frac{lbU_{20}}{I_x} + \frac{I_y - I_z}{I_x}\dot{\theta}_0 + \frac{J_r}{I_x}\dot{\theta}_0\omega_0\right]x_{12} + \\ \left[\frac{lbU_{20}}{I_{X_b}} + \frac{I_{Y_b} - I_{Z_b}}{I_{X_b}}\dot{\psi}_0\dot{\theta}_0 + \frac{J_r}{I_{X_b}}\dot{\theta}_0\right]\omega + \left[\frac{lb}{I_{X_b}} + \frac{I_{Y_b} - I_{Z_b}}{I_{X_b}}\dot{\psi}_0\dot{\theta}_0 + \frac{J_r}{I_{X_b}}\dot{\theta}_0\omega_0\right]U_2 + R_8 \\ x_{10} \\ \left[\left(\frac{lbU_{3_0}}{I_y}\right) + \left(\frac{I_z - I_x}{I_y}\right)\dot{\phi}_0\dot{\psi}_0 + \frac{J_r}{I_y}\omega_0\right]x_8 + \left[\left(\frac{lb}{I_y}\right) + \left(\frac{I_z - I_x}{I_y}\right)\dot{\phi}_0\dot{\psi}_0 + \frac{J_r}{I_y}\dot{\phi}_0\omega_0\right]U_3 + \\ \left[\left(\frac{lbU_{3_0}}{I_y}\right) + \left(\frac{I_z - I_x}{I_y}\right)\dot{\phi}_0\dot{\psi}_0 + \frac{J_r}{I_y}\dot{\phi}_0\right]\omega + R_{10} \\ x_{12} \\ \left[\left(\frac{I_x - I_y}{I_z}\right)\dot{\psi}_0\right]x_8 + \left[\left(\frac{I_x - I_y}{I_z}\right)\dot{\phi}_0\right]x_{12} + \left[\frac{J_r}{I_x}\omega_0\right]x_{10} + \left[\frac{J_r}{I_x}\dot{\theta}_0\right]\omega + \left(\frac{lb}{I_x}\right)U_3 + R_{12} \end{bmatrix} \tag{20}$$

The values represented by $(R_1, R_4, R_6, R_8, R_{10}, R_{12})$ represent the residual from the linearization process of the Taylor Series Method for differentiable functions. According (Kreyszig, 2005) for high orders differentiable functions, these values are infinitesimal, it can be ignored. In this case, these variables have their despicable values and wasn´t considered for analysis, for not intervening significantly on final results.

4. RESULTS

It was used the Matlab software, student version to Solve the equations system and obtain curves which describe the quadrotor behavior. It was decided to hold a quadrotor behavior analysis when the displacements and angular velocities were modified. The system equation (20) was rearranged as state space following Cook recommendation (2007), Equation (21):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{21}$$

In this case $\mathbf{A}$ represent a 12×12 matrix, formed by the coefficients accompanying state variables $\mathbf{x}$ and $\mathbf{A}$ a matrix 6×6, formed by the coefficients accompanying system input $\mathbf{u}$, to allow better handling and presentation of results. The constants in the system equations can be chosen to carry out an analysis it is required to perform. The values used for the simulation are shown in the table below(Table 1). In this case we aimed to evaluate the sensitivity of the model to initial conditions and thus to prove that the model is not stable by nature. Consequently, is necessary a control system to stabilize the quadrotor.

Table 1: Model parameter used for simulation

Symbol	Value	Unit	Symbol	Value	Unit
ϕ_0	0	rad	I_{X_b}	0.034	Kg/mm²
θ_0	0	rad	I_{Y_b}	0.035	Kg/mm²
ψ_0	0	rad	I_{Z_b}	3.086	Kg/mm²
l	209.275	mm	J_r	1.62E-4	Kg/mm²
b	3.788	Ns²	$\Omega_{i=1,2,3,4}$	809.80	Rad/s
M	1.0137	Kg			

The figure shows up the response to the initial condition of the model, posterior angular position and velocity.

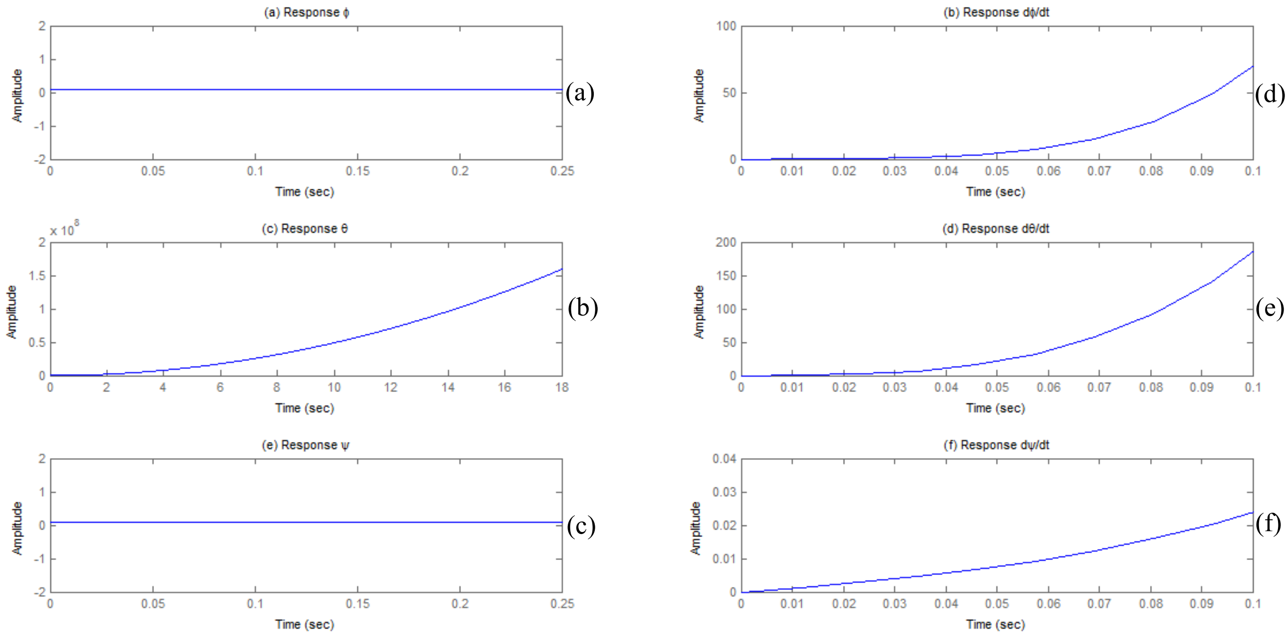

Figure 2: System Response to Initial Conditions. **Source:** Authors, 2015.

In figure 2 is shown the variation of the amplitude with respect to time when the angular velocity and displacement suffers some disturbance. In the figure 2-a the angular displacement, ϕ, around X_b direction, suffers a disturbance 0.1 rad. Note that the amplitude remains constant, which was expected, since an angular change from the X_b axis would not cause imbalance to quadrotor. But when it comes to the angular velocity $\dot{\phi}$ (figure 2-d), undergoes a change of 0.1 rad / s, it is clear that amplitude increases exponentially over time, showing that this change in angular velocity causes an imbalance to quadrotor, It is necessary to the performance of a control system. In (figure 2-b), it is noted that when the angular displacement θ, around Y_b direction, undergoes a change of 0.1 rad, the amplitude grows to very high values in a very short time interval. Thus it is clear that the quadrotor is not stable, requiring a control system control. Similarly when the angular velocity, $\dot{\theta}$ around Y_b subject to perturbation of 0.1 rad / s, the quadrotor destabilizes. When the angular displacement ψ toward Z_b undergoes a disturbance 0.1 rad, the quadrotor remains stable, which was expected, since a change in angular position Z_b, would not cause imbalance to quadrotor. In relation to angular velocity $\dot{\psi}$, in the Z_b direction when undergoes a perturbation of 0.1 rad / s, it is noted that the amplitude increases rapidly in a short time, showing an instability in quadrotor. Thus, it is clear that a control system is necessary to stabilize the quadrotor.

The model will be used in stability studies of non-linear systems. Will be found the roots of equations and, on neighborhood, these roots will be evaluated system properties concerning the dynamic stability.

5. ACKNOWLEDGEMENTS

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