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EXPERIMENTAL AND NUMERICAL ESTIMATION THE LOCATION AND INTENSITY OF HEAT GENERATION USING CORRELATION VIA THE SURFACE TEMPERATURE

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Abstract. *Breast cancer is the second most frequent cancer in the world. It is the most frequent among women. The most of equipment used for diagnosis of this disease are expensive, such as mammography, computerized tomography, ultrasound and magnetic resonance imaging. The aim of this work is the analysis of the heat transfer process in living tissues. The development of a new method to assist in the detection of breast tumors using thermographic images is presented. The properties and parameters for the numerical simulation of the breast were defined based on literature, and for the experimental was used o PVC. Then the direct problem to obtain the surface temperature of the breast was solved using both a commercial software COMSOL and analytical method. These simulated surface temperatures were used as input parameters for the solution of the inverse problem with the correlation method. The technique of the correlation method estimates the location and intensity of the tumor present in the breast (by testing all locations). The numerical results show that in a body with tumor, the precision in the estimation of the intensity and the location of the inclusion in the breast was determined accurately. The experimental results showed positive estimates, where the maximum error found was 7,8%. Thus this technique has potential for applications in vitro experimental cases and with the possibility of application also in vivo since the analysis considers regions of low sensitivity, as in real cases.*

Keywords: *breast cancer, correlation method, localization and intensity of tumor, surface temperature, numerical and experimental*

1. INTRODUCTION

The first quantitative relationship that described the energy transport in living tissues which included the effects of blood flow on the tissue temperature (on an ongoing basis), was presented by Harry H. Pennes in 1948. The equation derived from this study, originally designed to predict the temperature field in the human forearm, is the most common representation of the spatial and temporal distribution of temperature in biological systems and is called " bioheat equation " or "Pennes equation" Pennes (1948). After the study of Pennes and its use in many biological systems, new models have been proposed to describe more realistically the bioheat transfer process Nakayama and Kuwahara (2008), Mitchell and Myers (1968), Keller and Seiler (1971), Wulff (1974), Klinger (1978), Chen and Holmes (1980), Weinbaum and Jiji (1985), Roetzel and Xuan (1998).

According to the National Cancer Institute - INCA, the increasing number of new cancer cases will cause shortage of resources to meet the needs of diagnosis, treatment and monitoring. Breast cancer is the second most frequent cancer in the world and the most frequent among women. The average survival rate after five years is 61% INCA (2014).

Thus the importance of the development and improvement of the techniques used in the prevention and treatment of this disease, constituting a public health issue.

2. BREAST TUMOR

Figure 1 shows the anatomy of a breast which is basically comprised of three types of tissues: glandular, fibrous and adipose tissues. The glandular tissue (mammary lobes), responsible for the production of milk, is composed of units called secretory acini which are interconnected by a set of branching tubules (milk ducts) - draining the milk produced in the acini to the nipple. The fibrous tissue has the function of supporting the internal structures. The adipose tissue (fat), in addition to lining the entire gland, fills the spaces between the acini, ducts and the fibrous tissues, contributing significantly to the shape, volume and contour of the breasts Winnikow (2014).

As with other organs of the human body, the breasts also have blood vessels, which serves to irrigate it with blood and lymph vessels where lymph circulation occur. Lymph is a clear liquid and has function quite similar to blood of carrying nutrients to various areas of the human body and collecting the substances which are considered undesirable to the body Winnikow (2014).

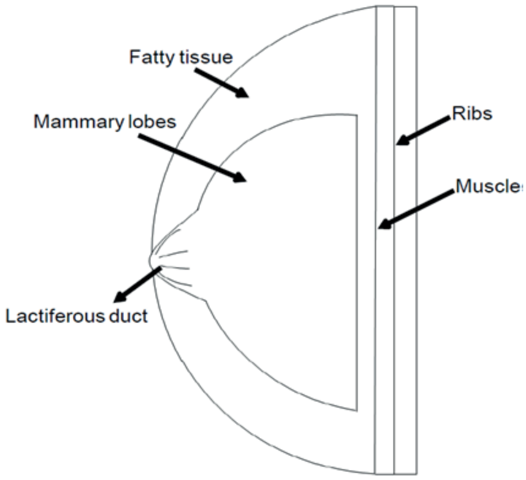

Figure 1. Schematic diagram of the anatomy of a breast

Breast cancer consists of a malignant tumor which develops from cells of the breast. Generally, it originates in the epithelial cells lining the innermost layer of the breast duct. Rarely breast cancer can start in other tissues, such as adipose and fibrous tissues de Mastologia (2014).

The change detection in the breast can be performed by self-examination, clinical examination (medical) or images. The indication that the alteration in the breast is cancer is only through a biopsy performed at the suspected region de Mastologia (2014).

Examinations based on images to aid diagnosis of breast diseases can be classified as structural and functional. Structural exams allow the visualization of internal structures of the breast, the main ones are mammography, ultrasound and MRI. The functional as exams allow the visualization of the functioning of the organs and the flow of liquids, such as ultrasound, magnetic resonance imaging and thermography Bezerra *et al.* (2013).

Thermography is characterized by the use of thermographic images acquired by an infrared camera capable of measuring temperature simultaneously at several points by detecting the infrared radiation emitted by bodies. It is more economically viable than traditional methods such as mammography, ultrasound and MRI.

This work aims at early detection of breast cancer through the use of temperatures on the surface of the breast (simulated thermographic images). To reach this objective is necessary to understand the heat transfer phenomenon in living tissues through the bioheat equation.

3. BIOHEAT EQUATION

Equation 1, which characterizes the heat transfer in living organisms, also known as Pennes equation Pennes (1948) can be written as:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + Q_p + Q_m + Q_e = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

where Qp is the source of heat due to blood perfusion, Qm is the volumetric metabolic heat generation and Qe is related to an external source of heat.

The source of heat due to blood perfusion (Q_p)is characterized by convective heat transfer effected by the blood through the capillary vascularization present in living tissue, which is proportional to the temperature difference of arterial blood entering the tissue and venous blood exiting the tissue Charny (1992). This term is given by

$$Q_p = w\rho_s c_s (T_a - T) \tag{2}$$

where w is blood flow rate is blood specific weight, is specific heat of blood, is arterial blood temperature and T is temperature of the tissue.

3.1 1D Analytical Solution

Figure 2 shows the one dimensional model used for obtaining the solution of the equation of Pennes. The biothermic one-dimensional problem to be analyzed should represent the heat transfer in a human tissue, simulating a body composed of layers of normal (healthy) tissues and a layer with a tumor. 1D model can provide initial information about the heat transfer in the tissue and enables faster the efficiency of the technique developed for the inverse problem. In this model, one surface is exposed to a convective medium while the other maintains a constant temperature prescribed by the internal body temperature.

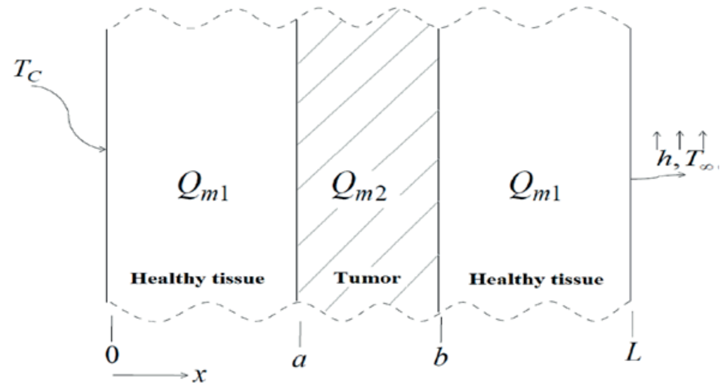

Figure 2. One-dimensional model of three layers of tissue exposed to a convective medium

Thus, the problem shown in Fig. 2 can be described by the Pennes equation as

$$\frac{\partial^2 T}{\partial x^2} + w\rho_s c_s (T_a - T) + Q_m = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

and subjected to the boundary conditions, at $x = 0$

$$T(0,t) = T_c \tag{4}$$

and at x = L

$$-k\frac{\partial T}{\partial x}\bigg|_{x=L} = h(T - T_\infty) \tag{5}$$

and the initial condition

$$T(x,0) = T_a \tag{6}$$

In Fig. 2 there are two different terms of metabolic heat generation, Qm_1 and Qm_2, simulating the normal metabolism and the tumor metabolism respectively. For analytical solution by the Green's functions method, first the variable θ is defined as

$$\theta(x,t) = T(x,t) - T_\infty \tag{7}$$

And a change of variable W of an auxiliary problem as

$$W(x,t) = \theta(x,t)e^{m^2\alpha t} \tag{8}$$

where $m^2 = \frac{w\rho_s c_s T_a}{k}$. Thus, Eq. 3 can be written as

$$\frac{\partial^2 W}{\partial x^2} + \frac{1}{k}Q' e^{m^2\alpha t} = \frac{1}{\alpha}\frac{\partial W}{\partial t} \tag{9}$$

where $Q' = Q_m + w\rho_s c_s (T_a - T_\infty)$

The solution of Eq. 9 can be obtained using the Green's function method Beck *et al.* (1992), or

$$\begin{aligned} W(x,t) = &\int_{x'=0}^{L} G_{X13}(x,t|x',\tau)|_{\tau=0} W(x',0)dx' + \\ &\frac{\alpha}{k}\int_{\tau=0}^{t}\int_{x'=0}^{L} G_{X13}(x,t|x',\tau)Q'e^{m^2\alpha\tau}dx'd\tau + \\ &\alpha\int_{\tau=0}^{t} W(0,t)\frac{\partial G_{X13}(x,t|x',\tau)}{\partial x'}|_{x'=0}d\tau \end{aligned} \tag{10}$$

where the first term refers to the initial temperature, the second refers to metabolic heat generation and the last refers to boundary condition at $x = 0$ of prescribed temperature. The Green function of the problem is given by

$$G_{X13}(x,t|x',\tau) = \frac{2}{L}\sum_{n=1}^{\infty} e^{\frac{-\beta_n^2\alpha(t-\tau)}{L^2}} \frac{(\beta_n^2 + B^2)\sin\left(\frac{\beta_n x}{L}\right)\sin\left(\frac{\beta_n x'}{L}\right)}{\beta_n^2 + B^2 + B} \tag{11}$$

where $\beta_n \cot \beta_n = -B$ and $B = \frac{hL}{k}$, β_n are the eigenvalues of the problem.
Rewriting the first term of Eq. 10 as

$$W_1(x,t) = \int_{x'=0}^{L} (T_0 - T_\infty) G_{X13}(x,t|x',t-\tau)|_{\tau=0} dx' \tag{12}$$

Considering the three different regions involving heat generation in Fig. 2 as

$$W_{2a}(x,t) = \frac{\alpha}{k} \int_{\tau=0}^{t} \int_{x'=0}^{a} G_{X13}(x,t|x',t-\tau) Q'_a e^{m^2\alpha\tau} dx' d\tau \tag{13a}$$

$$W_{2b}(x,t) = \frac{\alpha}{k} \int_{\tau=0}^{t} \int_{x'=a}^{b} G_{X13}(x,t|x',t-\tau) Q'_b e^{m^2\alpha\tau} dx' d\tau \tag{13b}$$

$$W_{2c}(x,t) = \frac{\alpha}{k} \int_{\tau=0}^{t} \int_{x'=b}^{L} G_{X13}(x,t|x',t-\tau) Q'_c e^{m^2\alpha\tau} dx' d\tau \tag{13c}$$

where Q'_a, Q'_b e Q'_c respectively represent the domains $[0:a]$, $[a:b]$ and $[b:L]$.
The third term of Eq. 10 may be written as

$$W_3(x,t) = \alpha \int_{\tau=0}^{t} (T_c - T_\infty) e^{m^2\alpha\tau} \frac{\partial G_{X13}(x,t|x',\tau)}{\partial x'}|_{x'=0} d\tau \tag{14}$$

Thus, a Eq. 10 may be rewritten as

$$W(x,t) = W_1 + W_{2a} + W_{2b} + W_{2c} + W_3 \tag{15}$$

Returning to the original variable , the temperature distribution is obtained as

$$T(x,t) = \left(W_1(x,t) + W_{2a}(x,t) + W_{2b}(x,t) + W_{2c}(x,t) + W_3(x,t)\right) e^{-m^2\alpha t} + T_\infty \tag{16}$$

or

$$\begin{aligned} T(x,t) = \Bigg\{ & -2(T_0 - T_\infty) \sum_{n=1}^{\infty} e^{\frac{-\beta_n^2 \alpha t}{L^2}} \frac{(\beta_n^2 + B^2)\sin\left(\frac{\beta_n x}{L}\right)[\cos(\beta_n) - 1]}{\beta_n(\beta_n^2 + B^2 + B)} \\ & \frac{-2L^2 Q'_a}{k} \sum_{n=1}^{\infty} (e^{m^2\alpha t} - e^{\frac{-\beta_n^2 \alpha t}{L^2}}) \frac{(\beta_n^2 + B^2)\sin\left(\frac{\beta_n x}{L}\right)[\cos\left(\frac{\beta_n a}{L}\right) - 1]}{\beta_n(\beta_n^2 + B^2 + B)(\beta_n^2 + m^2L^2)} \\ & \frac{-2L^2 Q'_b}{k} \sum_{n=1}^{\infty} (e^{m^2\alpha t} - e^{\frac{-\beta_n^2 \alpha t}{L^2}}) \frac{(\beta_n^2 + B^2)\sin\left(\frac{\beta_n x}{L}\right)[\cos\left(\frac{\beta_n b}{L}\right) - \cos\left(\frac{\beta_n a}{L}\right)]}{\beta_n(\beta_n^2 + B^2 + B)(\beta_n^2 + m^2L^2)} \\ & \frac{-2L^2 Q'_c}{k} \sum_{n=1}^{\infty} (e^{m^2\alpha t} - e^{\frac{-\beta_n^2 \alpha t}{L^2}}) \frac{(\beta_n^2 + B^2)\sin\left(\frac{\beta_n x}{L}\right)[\cos(\beta_n) - \cos\left(\frac{\beta_n b}{L}\right)]}{\beta_n(\beta_n^2 + B^2 + B)(\beta_n^2 + m^2L^2)} \\ & +2(T_c - T_\infty) \sum_{n=1}^{\infty} (e^{m^2\alpha t} - e^{\frac{-\beta_n^2 \alpha t}{L^2}}) \frac{\beta_n(\beta_n^2 + B^2)\sin\left(\frac{\beta_n x}{L}\right)}{(\beta_n^2 + B^2 + B)(\beta_n^2 + m^2L^2)} \Bigg\} e^{-m^2\alpha t} + T_\infty \end{aligned} \tag{17}$$

4. Correlation method

The estimate of the localization and intensity of the tumor for a 1D model will be performed using the Pearson correlation coefficient of the experimental temperatures with the unknown part of the analytical solution Gibbons and Chakraborti (2003).

Considering that for the model shown in Fig. 2 is known all information about healthy tissue, ie the terms W_1, W_{2a}, W_{2c} and W_3 are known in Eq. 15. Thus, the unique term to determine is the W_{2b} which is related to the intensity of information and localization of tumor.

The correlation will be performed between Y^* (experimental temperature Y subtracting the known terms of analytical solution) and the term W_{2b}^*. Both expressions will be dimensionless in relation the heat generation in the tumor Q'_b.

$$Y^*(x,t) = \frac{(Y(x,t) - T_\infty)e^{m^2\alpha t} - (W_1 + W_{2a} + W_{2c})}{\frac{2L^2 Q'_b}{k}} \tag{18}$$

$$W_{2b}^{*} = \frac{\frac{-2L^2Q_b'}{k}\sum_{n=1}^{\infty}(e^{m^2\alpha t} - e^{\frac{-\beta_n^2\alpha t}{L^2}})\frac{(\beta_n^2+B^2)\sin(\frac{\beta_n x}{L})[\cos(\frac{\beta_n b}{L})-\cos(\frac{\beta_n a}{L})]}{\beta_n(\beta_n^2+B^2+B)(\beta_n^2+m^2L^2)}}{\frac{2L^2Q_b'}{k}} \tag{19}$$

The correlation between the Eqs. 18 e 19 was performed using the function *corr* in *MATLAB*. Where the results are analyzed as follows:

- corr = 1: Perfect positive correlation between the two variables.
- corr = -1: Perfect negative correlation between the two variables. That is, if one increases, the other decreases each.
- corr = 0: This means that the two variables do not depend linearly to one another.

5. RESULTS AND DISCUSSIONS

The correlation method was used for estimating the location and intensity of metabolic heat generation of the tumor for a one-dimensional case. The properties used to solve the inverse problem are from the experimental study of Gautherie (1980), where thermal properties of breast cells are shown in Table1.

Table 1. Thermal properties for breast Gautherie (1980)

Properties	Symbols	Healthy tissue	Tumor
Thermal conductivity (W/mK)	k	0,42	0,42
Blood perfusion s^{-1}	w	0,00018	0,009
Specific weight (kg/m^3)	ρ	920	920
Specific heat (J/kgK)	c	3000	3000
Volumetric metabolic heat generation (W/m^3)	Q_m	450	29000

The great difficulty of solving an inverse problem of multiple variables is due to the presence of local minimums. In this study, the location and the metabolic tumor heat generation were predetermined. The tumor of *1 mm* is in a body of length *L = 51 mm*, as shown in Fig. 3.

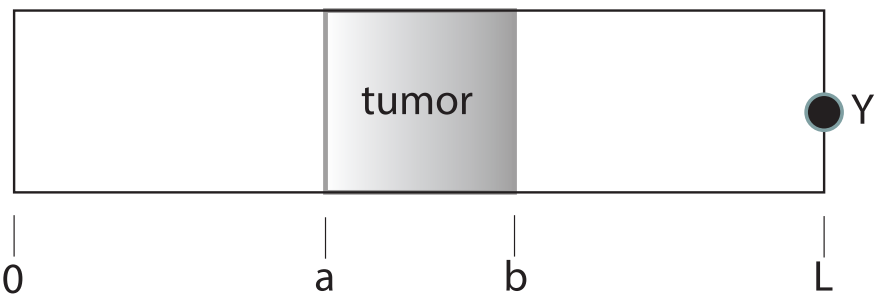

Figure 3. Schematic of 1D model used in the correlation method (outer surface at x=L)

The estimation of the profile of the tumor metabolic heat generation using the correlation method consists of the following steps:

1. Define the properties and the direct problem parameters, including the generation term (size, location and magnitude).
2. Resolve the direct problem: Numerical results using commercial software COMSOL for a time/duration of 1200s.
3. Obtain the temperature at sensor Y shown in Fig. 3.
4. The "experimental" Y temperatures were used in the correlation method to estimate the location and magnitude of the generation term.
5. The magnitude of tumor metabolic heat generation was estimated assuming that the tumor is at each preset position/interval (0 mm to 1 mm, 0,1 mm to 1,1 mm, 0,2 mm to 1,2 mm, ... (L-1) mm, (L) mm).
6. Once the generation terms were obtained for each assumed preset position, new temperature profiles for the outer surface were calculated (using the direct problem).

7. The temperature profile corresponding to the real location of the tumor will give the best agreement with the "experimental" temperature.

8. This agreement is verified by calculating the Pearson correlation coefficient of the estimated and "experimental" temperature profiles [10].

9. Thus the location and magnitude of the tumor (were/can be) obtained.

5.1 Numerical Results

In following the problem where the tumor is located in the interval (25:26) mm in the body was analysed. Using the procedure as in the previous case, the temperartures Y were obtained from the direct problem using the *COMSOL*, and then the location and metabolic heat generation was estimated using the hypothetical intervals of location of the tumor. The core temperature, initial temperature, ambient temperature is and convective heat transfer are equal to $27C$, and convective heat transfer is 9,8 W/m^2K.

Table 2 shows the results of the estimated location and intensity for the tumor using different heat generation values imposed on *COMSOL* at position [25,26] mm.

Table 2. Estimated location and intensity of the tumor to different heat generation values located at a = 25 mm b = 26 mm (1200s)

Heat generation imposed (W/m^3)	Estimated location [a:b] mm	Estimated metabolic heat (W/m^3)
10000	[25:26]	10000
25000	[25:26]	25000
50000	[25:26]	50000
100000	[25:26]	100000
500000	[25:26]	500000

It is noted that for the numerical case, the estimation of parameters is obtained with efficiency, regardless of the intensity of the tumor located in the region of this problem.

5.2 Experimental Results

In following the problem has the same conditions proposed before, however now holds up a 1D experiment. Where *PVC - Polyvinyl chloride* (k is 0,15 W/mK, ρ is 1488 kg/m^3 and c = 861 J/kgK) is breast tissue, the electrical resistance is responsible for internal heat generation, to ensure the core temperature of the body was used a thermal bath. In this experiment, the blood perfusion term is not included, as well as heat sources for generation of healthy tissue. Figure 4 shows the configuration of this experiment.

Figure 4. 1D experimental model using PVC

Table 3 shows the results obtained for the estimate of 1 mm tumor localization in seven different experiments. Where the value of the heat generation imposed by eletric resistance was 255555 W/m^3 and the localization in the experiment is a = 25 mm and b = 26 mm.

It is noticed that the results obtained to estimated the tumor localization using experimental data showed some errors (a maximum of 7,8%). However, it can be affirmed that the correlation method used in this problem has great potential.

Table 3. Estimated tumor location for different experiments where heat generation is located at a = 25 mm and b = 26 mm

Experimental	Estimated location [a:b] mm	Error (%)
1	[21:22]	7,8
2	[23:24]	3,9
3	[23:24]	3,9
4	[24:25]	1,9
5	[26:27]	1,9
6	[24:25]	1,9
7	[21:22]	7,8

To enhance the experimental results still is necessary to better control the prescribed temperature, as the thermal bath was unable to keep it exactly constant.

6. CONCLUSION

The estimation of the location and magnitude of metabolic heat generation of the tumor using the correlation method was satisfactory. Thus this technique has potential for application in vitro experimental cases. With the possibility also of application in vivo, once the analysis considered the low sensitivity regions, as in real cases.

7. ACKNOWLEDGEMENTS

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