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MATHEMATICAL MODELING AND ANALYSIS OF THERMODYNAMIC PROCESSES OF AN IRREVERSIBLE MILLER CYCLE WORKING ON A PISTON ROTARY ENGINE

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Abstract. *To develop high energy-density power generation engine with variable geometrical compression ratio, the performance of an irreversible air standard Miller cycle in a piston rotary engine denominated Kopelrot Rotary Engine (KRE), was analyzed using thermodynamic numerical simulations. With consideration of the characteristics of the working processes in the KRE, corresponding differential equations were established and then simplified by period features. Application of Miller cycle in conventional engines can be realized by either early or late intake valve closing (EIVC or LIVC); in KRE engine this can be achieved varying the position of inlet port. Thermodynamic modeling shows that the Miller cycle has the potential to increase fuel efficiency compared to a conventional Otto-cycle engine, however with the cost of a reduced engine power density.*

Keywords: *thermodynamic model, swing piston engine, Atkinson and Miller cycle, Late Intake Valve Closure (LIVC)*

1. INTRODUCTION

From last two centuries to present, conventional reciprocating piston internal combustion engine are still successful used widely as the principal source of mechanical power used in automobiles. According to Heywood (1988) engine developments, perhaps less fundamental but nonetheless important have continued ever since in many research facilities (Agnew, 1978). One more relatively recent major development has been the rotary internal combustion engine. Although a wide variety of experimental rotary engines have been proposed over the years (Wankel, 1965). The first practical rotary internal combustion engine, the Wankel, was not successfully tested until 1957. That engine, which evolved through many years of research and development, was based on the designs of the German inventor Felix Wankel (Ansdale and Lockley, 1969; Yamamoto, 1981). Despite of big efforts and resources to solve many problems, the Wankel rotary engine is the only one which achieved limited commercial success.

For decades, researchers and individual inventors all over the world proposed other types of rotary engines towards achieve a more direct energy conversion without the use of a reciprocating piston. Oledzki (2010), proposed a rotary positive displacement internal combustion engine, according to author, the invention is based on a specific form of four bar mechanism and features unique simplicity, compactness and strength that make the engine similar in these and some other aspects to gas turbines. Kauertz (1964), proposed a radial-piston or radial-vane internal combustion engine. Bakhtine (2001), develop a oscillatory scissor type rotary engine. Omori (2010), claim a cat-and-mouse type internal combustion engine of concentric two-rotor/six-piston type.

Kopelowicz (2003), proposed a spark-ignition internal combustion rotary engine with variable speed displacers. In this engine, four displacers rotate concentrically, in an annular space, at variable and phased angular velocities, thus creating distinct variable-volume working spaces between them. A notable feature of this proposal is a mechanism that offers a convenient way to change the compression ratio. In this work, the characteristics of the working processes of this engine (KRE) are analyzed, and then mathematical models of the thermodynamic process are also established which provide effective analysis methods for the structural design.

2. STRUCTURE AND PRINCIPLE

Figure 1 shows the two pairs of pistons and respective rotors of Kopelrot engine, toroidal cylinder and intake-exhaust ports. The steady rotation of engine power shaft and a novel mechanisms produces pistons of different rotors moving towards or away from each other. This relative motions between rotors form chambers of variable volume among the adjacent pistons. The Kopelrot rotary engine avoids piston reversal movement observed in conventional reciprocating combustion engine.

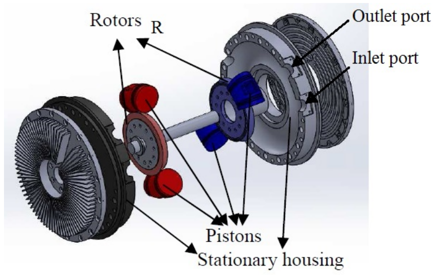

Figure 1. Pair of pistons and rotors of engine

The rotors attached to pistons mentioned above are linked to a differential velocity drive mechanism and output power shaft. The mathematical model of the KRE for the position, velocity, and acceleration was established (Deng *et al.*, 2013). Figure 2 shows the variations of toroidal section volumes among adjacent pistons.

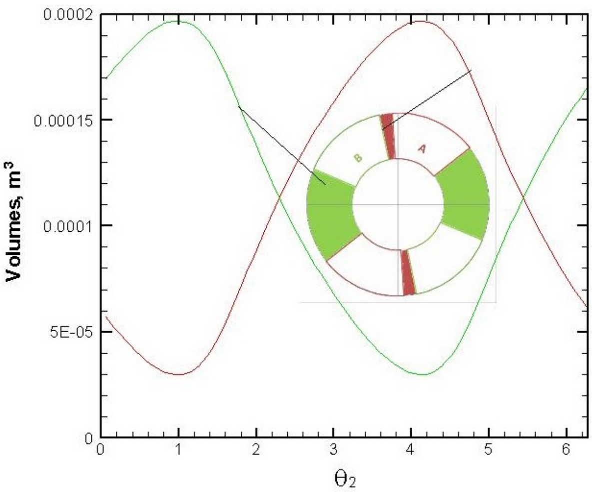

Figure 2. Variation of volumes in the toroidal cylinder chamber at maximum compression ratio, r_c

Figure 3 shows the variations of pistons rotational speeds relative to output power shaft ($\dot{\theta}_2$).

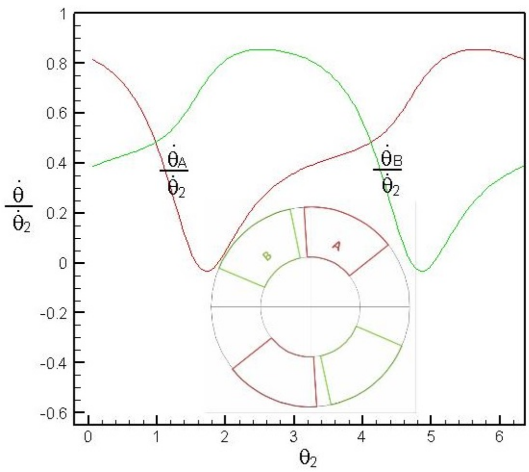

Figure 3. Pistons rotational speeds relative to shaft engine rotational speed, $\dot{\theta}_2$

3. THERMODYNAMIC MODEL

To permit analysis of critical features and our basic understanding of the thermodynamic process in KRE, in this work was used an appropriate combinations of assumptions and mathematical equations.

3.1 Basic assumptions

The first approach of this work was consider a simplified thermodynamic model called zero-dimension model (Lavoie *et al.*, 1970), this unveils the essential features of internal combustion engine. The model describes the thermodynamic states of the burned and unburned gases as a function of time. This model has the following assumptions:

1. The working substance is an ideal gas;
2. The mixture air-fuel in combustion chamber is uniformly mixed;
3. The system state parameter is function of time regardless of the position and space;
4. The loss of kinetic energy at inlet ports can be ignored.

3.2 Basic equations

We consider a control volume enclosing all the gases mixture in the toroidal cylinder chamber volume V. This working volume change according like to harmonic motion. Mass of gas-fuel mixture enter the control volume through the inlet port at flow rate, $\dot{m}_e$. Similarly, mass leave through the outlet port, leaks included, at flow rate, $\dot{m}_s$. The first law of thermodynamics and mass conservation equation for this control volume may be written in compact form as

$$\begin{bmatrix} \dfrac{dp}{dt} \\ \dfrac{dh}{dt} \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \begin{bmatrix} \dot{m}_e - \dot{m}_s - \rho\dfrac{dV}{dt} \\ \dot{m}_e h_e - \dot{m}_s h_s + \dot{Q}_c - \dot{Q}_w + \rho h\dfrac{dV}{dt} \end{bmatrix} \tag{1}$$

Where, matrix $\mathbf{E}$ of ordinary differential equations system ODEs, is compute from thermodynamic derivative relations $\psi = (\frac{\partial\rho}{\partial p})_h$ and $\xi = (\frac{\partial\rho}{\partial h})_p$, thermophysical properties enthalpy h and density ρ, and working volume V.

3.3 Heat release model

In order to approximating the burn rate in internal combustion engines, analytical functions are useful an inexpensive tools for engine cycle simulations. The best known of these is the Wiebe function, which is used to predict the burn fraction and burn rate in internal combustion engines operating with different combustion systems and fuels (Ghojel, 2010). The Wiebe functions for non-dimensional burn fraction x and its derivative w (burn rate) as function of time can be written as

$$x = 1 - e^{6.908\left(\frac{t}{t_d}\right)^{m+1}} \tag{2}$$

$$w = \frac{dx}{dt} = \frac{6.908(m+1)}{t_d}\left(\frac{t}{t_d}\right)^m e^{6.908\left(\frac{t}{t_d}\right)^{m+1}} \tag{3}$$

Where t are measured from start of combustion (t=0), the above equations (2) and (3) can be written as function of degrees crank angle as

$$x = 1 - e^{6.908\left(\frac{\varphi-\varphi_g}{\varphi_d}\right)^{m+1}} \tag{4}$$

$$w_\varphi = \frac{dx}{d\varphi} = \frac{6.908(m+1)}{\varphi_d}\left(\frac{\varphi-\varphi_g}{\varphi_d}\right)^m e^{6.908\left(\frac{\varphi-\varphi_g}{\varphi_d}\right)^{m+1}} \tag{5}$$

Where φ is the crank shaft angle; φ_g denotes the angle at which burning start; φ_d is the duration of combustion; m is designate the shape parameter.

The fuel mass m_f, fuel calorific value LHV and mass fraction burned x, rules the change of heat release.

$$\frac{dQ_c}{d\varphi} = m_f LHV \frac{dx}{d\varphi} \tag{6}$$

3.4 Heat transfer model

In internal combustion engines, heat loss from combustion gases through the cylinder wall to the coolant strongly influences the thermodynamics of the engine cycle, this heat loss is an important part of the energy balance. The instantaneous heat transfer rate, $\dot{Q}_w$, shown in Eq.1 is calculated applying usually Nusselt, Reynold and Prandtl numbers correlations as follows.

$$\dot{Q}_w = \frac{h_t A_w (T - T_w)}{2\pi N_e} \tag{7}$$

Where $T - T_w$ is the mean gas temperature difference in the cylinder between gas and wall cylinder temperatures. A_w is the instantaneous surface area available for heat transfer, and h_t is the heat transfer coefficient which can be determined using Woschni's correlation (Woschni, 1967). N_e is the engine speed.

4. Numerical simulations

Ordinary differential equations ODE was solved by a Fortran VODE solver (Brown *et al.*, 1989) that uses a multistep methods with fully variable-step sizes and coefficients. Thermophysical and transport properties was computed using GasMix sub-library from FLUIDPROP thermodynamic property library (Colonna and Van der Stelt, 2009). GasMix implements the ideal gas model for fluids and mixtures.

4.1 Preliminary results

To start the simulations, for convenience, the compression starting point was selected as the initial value of ordinary differential equations. Then for a working cycle, numerical calculation is made step-by-step. Unlike conventional engines, the intake and exhaust of valveless KRE engine ports are all open. In this engine, Miller or Otto cycle can be achieved varying the position of intake port. Figure 4 shows the variation of mean effective pressure and enthalpy of a Miller cycle.

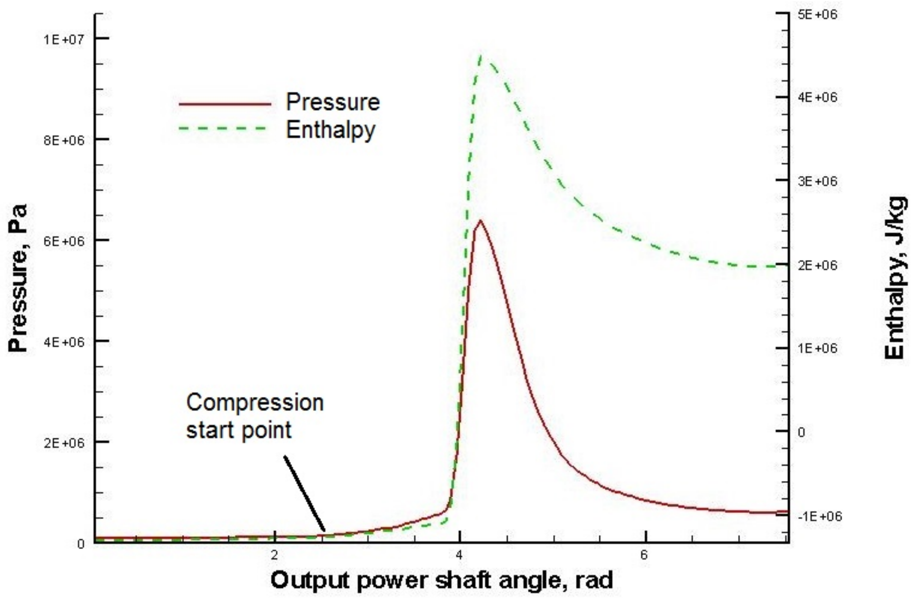

Figure 4. Variation of mean effective pressure and enthalpy of a Miller cycle

Comparison of Otto and Miller cycles are shown in Fig. 5. This figure also illustrates the work loss that can be avoided from the Miller cycle (shaded). Heywood (1988) showed that significant increases in engine efficiency can be achieved in over-expanded cycles, especially at low compression ratios.

According to Mikalsen *et al.* (2009), the compression ratio $r_c = V_b/V_a$ will be equal for the Otto and Miller engines. For the Otto cycle engine, the expansion ratio is equal to the compression ratio, whereas for the Miller cycle engine, one can define a 'Miller cycle ratio', ε, expressing the relative difference between expansion ratio and compression ratio:

$$\varepsilon = \frac{r_e}{r_c} = \frac{V_c/V_a}{V_b/V_a} = \frac{V_c}{V_b} \tag{8}$$

where r_e is the Miller cycle expansion ratio.

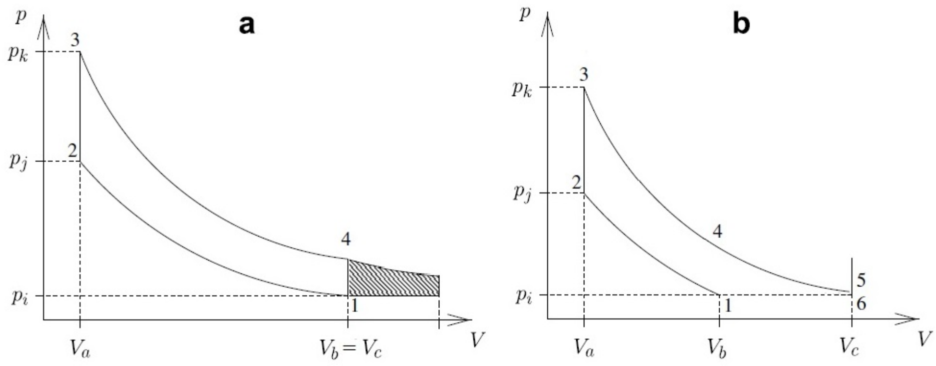

Figure 5. Comparison of Otto (a) and Miller cycles (b)

Figure 6 shows the efficiency advantage that can be achieved in a Miller cycle engine compared to the equivalent Otto cycle, which is consistent with reports from other authors (Heywood, 1988). In efficiency ratio, η_M and η_O are the efficiency of Miller and Otto cycle.

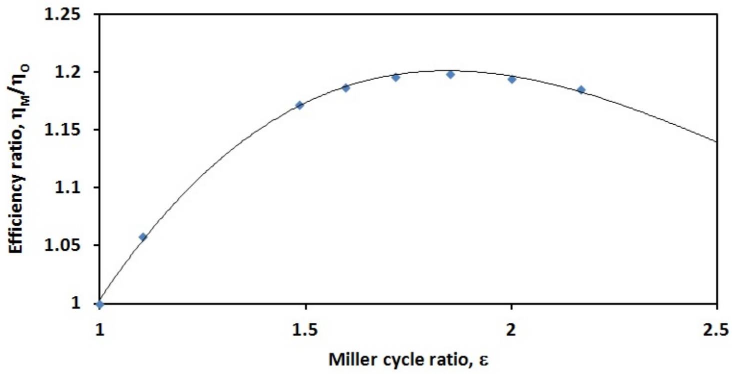

Figure 6. Potential efficiency advantage of the Miller cycle engine

Figure 7 shows the power reduction in the Miller cycle with constant mean piston speed. A significant reduction of power can be seen, as expected.

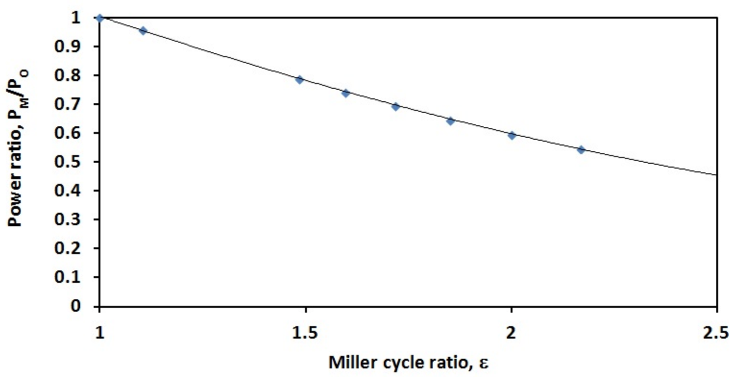

Figure 7. Power reduction in the Miller cycle engine with constant mean piston speed

5. Conclusions

Concerning the characteristics of the KRE working process, ordinary differential equations of its thermodynamic process was established considering a zero-dimension model. Mathematical models of the thermodynamic process provide effective analysis methods for the structural design. As expected from other authors, Miller cycle as a potential efficiency advantage compared to Otto cycle engine, however, with cost of a reduced engine power density. It was found that efficiency ratio (η_M/η_O) has a maximum at Miller ratio value $\varepsilon = 1.85$. This work is an initial effort and further work is required to validate the findings presented in this paper.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Agnew, W.G., 1978. “Fifty years of combustion research at general motors”. *Progress in Energy and Combustion Science*, Vol. 4, No. 2, pp. 115 – 155. ISSN 0360-1285. doi:http://dx.doi.org/10.1016/0360-1285(78)90007-2. URL `http://www.sciencedirect.com/science/article/pii/0360128578900072`.

Annand, W., 1963. “Heat transfer in the cylinders of reciprocating internal combustion engines”. In *Proc. Instn Mech. Engrs*. Vol. 177 of *36*, pp. 973–980.

Ansdale, R. and Lockley, D., 1969. *The Wankel RC engine: design and performance*. A. S. Barnes.

Bakhtine, V., 2001. “High-output robust rotary engine with a symmetrical drive and improved combustion efficiency having a low manufacturing cost”. US Patent 6,305,345.

Brown, P.N., Byrne, G.D. and Hindmarsh, A.C., 1989. “Vode - a variable-coefficient ode solver”. *Siam Journal on Scientific and Statistical Computing*, Vol. 10, No. 5, pp. 1038–1051. ISSN 0196-5204. doi:10.1137/0910062. URL `<Go to ISI>://WOS:A1989AM02600014`. Brown, pn byrne, gd hindmarsh, ac.

Colonna, P. and Van der Stelt, T., 2009. “Fluidprop: A program for the estimation of thermo physical properties of fluids”. URL `http://www.fluidprop.com`.

Deng, H., Pan, C.y., Wang, X.c., Zhang, L. and Deng, L., 2013. “Comparison of two types of twin-rotor piston engine mechanisms”. *Journal of Central South University*, Vol. 20, No. 2, pp. 363–371. ISSN 2095-2899. doi: 10.1007/s11771-013-1496-9. URL `<Go to ISI>://WOS:000314700900011`. Times Cited: 35.

Ghojel, J.I., 2010. “Review of the development and applications of the wiebe function: a tribute to the contribution of ivan wiebe to engine research”. *International Journal of Engine Research*, Vol. 11, No. 4, pp. 297–312. ISSN 1468-0874. doi:10.1243/14680874jer06510. URL `<Go to ISI>://WOS:000281794500004`. Times Cited: 2021.

Heywood, J., 1988. *Internal Combustion Engine Fundamentals*. Automotive technology series. McGraw-Hill. ISBN 9780071004992.

Hohenberg, G., 1979. “Advanced approaches for heat transfer calculations”. In *SAE Technical Paper 790825*. Detroit, USA. doi:10.4271/790825. URL `http://papers.sae.org/790825/`.

Kauertz, E., 1964. “Rotary radial-piston machine”. US Patent 3,144,007.

Kopelowicz, J., 2003. “A system for the construction of pumps, compressors, and motor engines, formed by a rotary chamber and pistons which are driven in the same direction at varying velocities alternatively opposite to each other, inside a fixed open or closed structure”. WO Patent App. PCT/BR2002/000,111.

Lavoie, G.A., Heywood, J.B. and Keck, J.C., 1970. “Experimental and theoretical study of nitric oxide formation in internal combustion engines”. *Combustion Science and Technology*, Vol. 1, No. 4, pp. 313–326. ISSN 0010-2202. doi:10.1080/00102206908952211. URL `http://dx.doi.org/10.1080/00102206908952211`.

Mikalsen, R., Wang, Y. and Roskilly, A., 2009. “A comparison of miller and otto cycle natural gas engines for small scale {CHP} applications”. *Applied Energy*, Vol. 86, No. 6, pp. 922 – 927. ISSN 0306-2619. doi:http://dx.doi.org/10.1016/j.apenergy.2008.09.021. URL `http://www.sciencedirect.com/science/article/pii/S0306261908002456`.

Oledzki, W., 2010. “Internal combustion two stroke rotary engine system”. US Patent App. 12/218,959.

Omori, T., 2010. “Cat-and-mouse type internal combustion engine, and its correlation type crank”. US Patent App. 12/602,901.

Wankel, F., 1965. *Rotary piston machines, by Felix Wankel*. Iliffe books.

Woschni, G., 1967. “A universally applicable equation for the instantaneous heat transfer coefficient in the internal combustion engine”. In *SAE Technical Paper 670931*. Detroit, USA. doi:10.4271/670931. URL `http://papers.sae.org/670931/`.

Yamamoto, K., 1981. *Rotary engine*. Sankaido.

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