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SLIDING MODE CONTROL OF A PSEUDOELASTIC TWO-BAR TRUSS

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Abstract. *The two-bar truss is largely employed to investigate the stability of framed structures as well as of flat arches, since its nonlinear dynamics may present some interesting and complex behavior like snap-through behavior and chaotic dynamics. In this paper, a sliding mode controller is applied to a shape memory two-bar truss for vibration suppression. A constitutive model is employed in order to describe the shape memory alloy behavior, presenting close agreement with experimental data. This system exhibits both constitutive and geometrical nonlinearities. The control system performance is evaluated by numerical simulations, reducing vibration and avoiding undesired behaviors.*

Keywords: *Smart structures, two-bar truss, sliding mode control (SMC), shape memory alloy.*

1. INTRODUCTION

Smart structures are being increasingly applied in many areas of technology to geometry control, vibration and stiffness control, dissipation, among others. For these applications, a greater emphasis is given to shape memory alloys (SMAs), which are used in situations where high loads, high strain and low frequency load are necessary. In order to achieve these results, smart structures are most often equipped with sensors to evaluate the deformation of its elements and actuators to perform control actions.

SMAs are well known due to the shape memory effect (SME) and the pseudoelasticity presented by them. The SME is the phenomenon which the material can recover its shape after a permanently strain through a specific heating. The pseudoelasticity consists of recover all the material strain imposed on it through a hysteresis loop in a loading-unloading cycle. These phenomena are associated with martensitic transformations which occur in SMAs thermomechanical cycles. These characteristics make them very applicable in different areas. Specifically, SMAs have been used in the manufacture of actuators (Mohd Jani *et al.*, 2014), in the biomedical field (Machado *et al.*, 2003), aerospace engineering (Lagoudas, 2008) and robotics (Paiva and Savi, 2006).

Many authors have proposed different models to simulate the thermomechanical behavior of SMAs. Falk (1980) and Müller and Xu (1991) proposed a model based on a polynomial to describe the thermomechanical behavior of SMA bars. Despite its simplicity, the model provides a good description of system dynamic response, but can not represent the hysteresis of the material, which is a major feature of SMAs. Fremond and Miyazaki (1996) considered a three-dimensional model for the thermomechanical response of them, whereas the martensitic transformations are described with the aid of two internal variables that represent the volume fractions of the two variants of detwinned martensite ($M+$ and $M-$), and must satisfy restrictions regarding the coexistence of three distinct phases, with austenite (A) being the third one. Based on Fremond and Miyazaki (1996), Savi *et al.* (2002b) developed a one-dimensional model with internal constraints, but including twinned martensite (M) in their formulation. Furthermore, different parameters were adopted for each phase and thermal expansion and plastic strain were also included. Others works (Baêta-Neves *et al.*, 2004; Paiva *et al.*, 2005; Savi and Paiva, 2005; Monteiro *et al.*, 2009; Oliveira *et al.*, 2010) have used the model proposed by Savi *et al.* (2002b) to develope other ones. On the other hand, some authors have been using the finite element method (FEM) to describe the thermomechanical behavior of SMAs (Masud *et al.*, 1997; Auricchio and Sacco, 1999; Auricchio and Petrini, 2002; Cava *et al.*, 2004).

Stability analysis of structures is an aim of much research, mainly due to complex behaviors they present. Because of the complexity associated with more elaborate structures, archetypes models are generally used to study general aspects related to the dynamic behavior of structures, among that models a greater emphasis is given to the two-bar truss or von Mises truss, as is well known (Bazant and Cedolin, 2010). An important property of this structure is that it can present two displacement configurations when an increasing external mechanical load is applied on it, the displacement of its center point passes from one configuration to another and this is known as snap-through behavior. The two-bar truss behavior is even richer when its material presents a nonlinear behavior, as SMAs, which will be considered in this work. Savi *et al.* (2002a) and Savi and Nogueira (2010) studied the behavior of SMA two-bar truss apllying different models to simulate the thermomechanical response of SMAs, one simplified, based on a polynomial, and another sophisticated, which consider the internal constraints of phases transformations, respectively. Bessa *et al.* (2013) developed a fuzzy sliding mode controller to reduce vibration in a SMA two-bar truss using the simplified model proposed by Savi *et al.* (2002a) to simulate the thermomechanical behavior of the structure. The sliding mode control technique has been used successfully in the control of SMA actuators (Song *et al.*, 2003; Romano and Tannuri, 2009; Lee *et al.*, 2013), due mainly to its ability to deal with model inaccuracies and nonlinearities present in the SMA models.

In this work, a smooth sliding mode controller is proposed for vibration suppression in a SMA two-bar truss. A sophisticated constitutive model (Savi and Nogueira, 2010) is assumed to simulate the thermomechanical behavior of its elements, which presents close agreement with experimental data, and a simplified constitutive model (Savi *et al.*, 2002a) is used to develope the control law. The control law is based on the methodology sliding mode, using a smooth function to perform switching of the compensation term for model inaccuracies when the system reaches the sliding surface. Numerical simulations are carried to show the robustness and effectiveness of the control developed even in the presence of inaccuracies in the system. In this regart, we are investigating an SMA structure which needs an apropriate control, using external actuators. It is important to highlight which SMA properties are being used to achieve others goals than control. This situation is common in distinct applications which include aerospace systems as self-erectable structures.

2. DYNAMIC MODEL

The two-bar truss is formed by two identical bars, free to rotate around their supports and at the main join, as shown in Fig. 1. The two identical bars have length L and cross-sectional area A and they form an angle φ with a horizontal line and are free to rotate only on the plane formed by the two bars.The critical Euler load of both bars is assumed to be sufficiently large so that buckling will not occur in the simulations reported here.

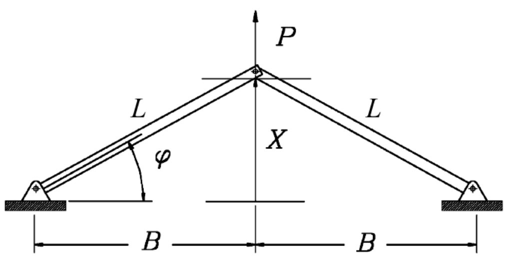


Figure 1: Schematic representation of a two-bar truss.

In this work, we consider a SMA two-bar truss where each bar present the shape memory and pseudoelasticity effects. The mass m of the structure will be considered as concentrated at the central join of the two bars and only vertical and symmetrical movements is allowed, transforming an infinite points system in a one-dimensional discrete dynamic system. Despite the simplicity, the response is able to display important complex behaviors for structure analysis. Also the model presents geometric and constitutive non-linearities. The symmetric, vertical displacement is denoted by X. Therefore, the balance of momentum in the central join is expressed through the following equation

$$-2F\sin\varphi - c\dot{X} + P = m\ddot{X} \tag{1}$$

where F is the force on each bar, which is related to the thermomechanical behavior of the material bars, P is the external disturbance and $c\dot{X}$ is a linear viscous damping term used to represent all dissipation mechanisms. In this work, we used a sophisticated model proposed by Savi and Nogueira (2010) to simulate the thermomechanical behavior of the SMA bars in a two-bar truss, which considers phase transformations on the material. These transformations is assumed to be homogeneous through the truss.

In order to present this model, let us consider strain (ε), temperature (T), and three state variables associated with the volume fractions of each phase in the SMA: β_1 is associated with tensile detwinned martensite, β_1 is related to compressive detwinned martensite and β_3 represents austenite. Actually, a fourth phase β_4 related to twinned martensite is considered, which can be obtained from the phase coexistence condition

$$\beta_4 = 1 - (\beta_1 + \beta_2 + \beta_3) \tag{2}$$

With this assumption, the complete set of constitutive equations which describes the thermomechanical behavior of SMAs is given by:

$$\sigma = E\varepsilon + [\alpha + E\alpha_h]\,(\beta_2 - \beta_1) - \kappa\,(T - T_0) \tag{3}$$

$$\dot{\beta}_1 = \frac{1}{\bar{\eta}_1}\big\{\alpha\varepsilon + \Lambda + [2\alpha_h\alpha + E\alpha_h^2](\beta_2 - \beta_1) + \alpha_h[E\varepsilon - \kappa(T - T_0)] - \partial_1 J_\pi\big\} + \partial_1 J_\chi \tag{4}$$

$$\dot{\beta}_2 = \frac{1}{\bar{\eta}_2}\big\{ - \alpha\varepsilon + \Lambda - [2\alpha_h\alpha + E\alpha_h^2](\beta_2 - \beta_1) - \alpha_h[E\varepsilon - \kappa(T - T_0)] - \partial_2 J_\pi\big\} + \partial_2 J_\chi \tag{5}$$

$$\begin{aligned}\dot{\beta}_3 =& \frac{1}{\bar{\eta}_3}\left\{ - \frac{1}{2}(E_A - E_M)[\varepsilon + \alpha_h(\beta_2 - \beta_1)]^2 + \Lambda_3 + (\kappa_A - \kappa_M)(T - T_0)[\varepsilon + \alpha_h(\beta_2 - \beta_1)] - \partial_3 J_\pi\right\} + \\ &+ \partial_3 J_\chi\end{aligned} \tag{6}$$

where $E = E_M + \beta_3(E_A - E_M)$ is the elastic modulus and $\kappa = \kappa_M + \beta_3(\kappa_A - \kappa_M)$ is related to the thermal expansion coefficient. Note that subscript 'A' refers to the austenitic phase, while 'M' refers to martensite. Parameter α_h is used in order to define the horizontal width of the stress−strain hysteresis loop, while α helps vertical hysteresis loop control on the stress−strain diagrams. The parameters $\Lambda = \Lambda(T)$ and $\Lambda_3 = \Lambda_3(T)$ are associated with phase transformation stress levels as follows:

$$\Lambda = \begin{cases} -\bar{L}_0 + \dfrac{\bar{L}}{T_M}(T - T_M) & \text{if} \quad T > T_M \\ -\bar{L}_0 & \text{if} \quad T \leq T_M \end{cases} \tag{7}$$

$$\Lambda_3 = \begin{cases} -\bar{L}_0^A + \dfrac{\bar{L}^A}{T_M}(T - T_M) & \text{if} \quad T > T_M \\ -\bar{L}_0^A & \text{if} \quad T \leq T_M \end{cases} \tag{8}$$

where T_M is the temperature below which the martensitic phase becomes stable in a stress-free state. Besides, $\bar{L}_0$, $\bar{L}$, $\bar{L}_0^A$ and $\bar{L}^A$ are parameters related to the critical stress for phase transformation.

The terms $\partial_i J_\pi$ $(i = 1, 2, 3)$ are sub-differentials of the indicator function J_π with respect to β_i (Rockafellar, 1970). The indicator function $J_\pi = J_\pi(\beta_1, \beta_2, \beta_3)$ is related to a convex set π, which provides the internal constraints related to the phases' coexistence. With respect to evolution equations of volume fractions, $\bar{\eta}$ represents the internal dissipation related to phase transformations, it is possible to consider different values for the internal dissipation parameter $\bar{\eta}_i$ $(i = 1,\ 2,\ 3)$: $\bar{\eta}_i^L$ and $\bar{\eta}_i^U$ during the loading and unloading processes, respectively (Paiva *et al.*, 2005; Savi and Paiva, 2005). Moreover $\partial_i J_\chi$ $(i = 1, 2, 3)$ are sub-differentials of the indicator function J_χ with respect to $\dot{\beta}_i$ (Rockafellar, 1970). This indicator function is associated with the convex set χ, which establishes conditions for the correct description of internal subloops due to incomplete phase transformations and also avoids phase transformations $M+ \to M$ or $M- \to M$.

Now, the following strain definition is considered,

$$\varepsilon = \frac{L}{L_0} - 1 = \frac{\cos\varphi_0}{\cos\varphi} - 1 \tag{9}$$

with L_0 and φ_0 representing the nominal values of L and φ, respectively. Defined the strain, the equation of motion may be rewritten as following:

$$m\ddot{X} + c\dot{X} + 2A\frac{X}{(X^2 + B^2)^{1/2}}\left\{E\left[\frac{(X^2 + B^2)^{1/2}}{L_0} - 1\right] + [\alpha + E\alpha_h](\beta_2 - \beta_1) - \kappa(T - T_0)\right\} = P(t) \tag{10}$$

where B is the horizontal projection of each truss bar (Fig. 1).

Considering a periodic excitation $P(t) = P_0\sin(\omega t)$, Eq. (10) may be written in non-dimensional form as

$$\begin{aligned} x' &= y \\ y' &= \gamma\sin(\Omega\tau) - \xi y - \mu_E\left[1 - \frac{1}{(x^2 + b^2)^{1/2}}\right]x - [(\bar{\alpha} + \mu_E\alpha_h)(\beta_2 - \beta_1) - \bar{\kappa}\mu_\kappa(\theta - \theta_0)]\,\frac{x}{(x^2 + b^2)^{1/2}} \end{aligned} \tag{11}$$

Moreover, the following non-dimensional parameters are considered:

$$\begin{aligned} &x = \frac{X}{L}\,, \quad \gamma = \frac{P_0}{mL_0\omega_0^2}\,, \quad \omega_0^2 = \frac{2E_M A}{mL_0}\,, \quad \Omega = \frac{\omega}{\omega_0}\,, \quad \tau = \omega_0 t\,, \quad \bar{\kappa} = \frac{\kappa_M T_M}{E_M}\,, \quad \xi = \frac{c}{m\omega_0} \\ &\mu_E = \frac{E}{E_M}\,, \quad \mu_\kappa = \frac{\kappa}{\kappa_M}\,, \quad \bar{\alpha} = \frac{\alpha}{E_M}\,, \quad \theta = \frac{T}{T_M}\,, \quad b = \frac{B}{L_0}\,, \quad \text{and} \quad (\cdot)' = \frac{\mathrm{d}(\cdot)}{\mathrm{d}\tau} \end{aligned} \tag{12}$$

3. CONTROLLER DESIGN

According to the control theory, parametric uncertainties and unmodeled dynamics can have strong adverse effects on nonlinear control systems. The designed control system must deal with them, therefore, the sliding mode control technique is recommended for these cases, since it is successfully employed in problems where parametric uncertainties and unmodeled dynamics are present in the constitutive model (Ling *et al.*, 2012; Slotine and Li, 1991).

In order to ensure that the state vector $\mathbf{x} = [\ x \quad y\ ]$ reaches the desired states $\mathbf{x}_d = [\ x_d \quad y_d\ ]$ and the tracking error $\tilde{\mathbf{x}} = [\ \tilde{x} \quad \dot{\tilde{x}}\ ] = [\ x - x_d \quad y - y_d\ ] \to 0$ as $\tau \to \infty$, a smooth sliding mode controller is applied assuming a simplified model to simulate the response of the von Mises truss based on Savi *et al.* (2002a). Despite the simplicity of the model which will be considered to control the system and the external disturbance, the control proposed can be successfully employed. The control action is effected by a linear actuator which is supposed to be installed vertically at the junction between the two bars, as illustrated in Fig. 2.

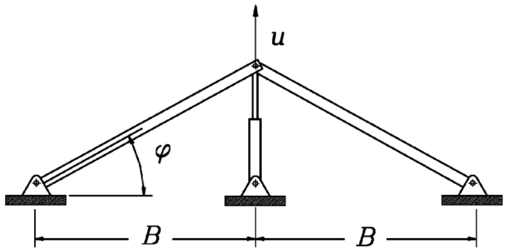


Figure 2: Schematic representation of an actuated two-bar truss.

The combination of linear actuators with shape memory elements enables the development of variable geometry trusses that also have the ability of self-attenuate their vibration levels. This kind of adaptive structure could be very useful, for example, in aerospace applications. In this work is not specified any particular actuator, since this selection process would only be possible with a broad knowledge of the system behavior for a given operating condition, i.e., the selected actuator would depend on the rise time, stiffness and size of the structure, range of the associated loads, among others. Even without selecting a specific actuator, as options to control the two-bar truss we could apply an electro-hydraulic or a pneumatic actuator or even SMA bars.

On this basis, the related control variable u must be added to the non-dimensional equation of motion Eq. (11), which for control purposes could be simply rewritten as

$$\begin{aligned} x' &= y \\ y' &= f + d + u \end{aligned} \tag{13}$$

where u is the control action, $d = \gamma \sin(\Omega\tau)$ is an external disturbance assumed to be unknown, and f is the dynamics system. The block diagram of the control scheme is presented in Fig. 3. For the proposed control f is given by a simplified model to simulate the dynamics of the structure (Savi *et al.*, 2002a), i.e. by an estimate of f given by $\hat{f}$ as follows:

$$\begin{aligned} \hat{f} = & -\hat{\xi} y + x\big\{ -[(\theta-1) - 3\alpha_2 + 5\alpha_3] + [(\theta-1) - \alpha_2 + \alpha_3](x^2+b^2)^{-1/2} + \\ & -[3\alpha_2 - 10\alpha_3](x^2+b^2)^{1/2} + [\alpha_2 - 10\alpha_3](x^2+b^2) + 5\alpha_3(x^2+b^2)^{3/2} - \alpha_3(x^2+b^2)^2 \big\} \end{aligned} \tag{14}$$

where $\hat{\xi}$ is a non-dimensional viscous damping coefficient. The dissipation due to hysteretic effect on the sophisticated model may be considered by assuming an equivalent viscous damping related to this parameter. Moreover, α_2 and α_3 are non-dimensional material constants, which are given by:

$$\alpha_2 = \frac{a_2}{a_1 T_M} \quad \text{and} \quad \alpha_3 = \frac{a_3}{a_1 T_M} \tag{15}$$

with a_2 and a_3 representing material constants. Regarding the development of the control law, the following assumptions must be made:

Assumption 1. *The state vector* $\mathbf{x}$ *is available.*

Assumption 2. *The function* f *is unknown but bounded by a known function of* $\mathbf{x}$, *i.e.* $|\hat{f}(\mathbf{x}) - f(\mathbf{x})| \le \mathcal{F}(\mathbf{x})$ *where* $\hat{f}$ *is an estimate of* f.

Assumption 3. *The disturbance* d *is unknown but bounded, i.e.* $|d| \le \mathcal{D}$.

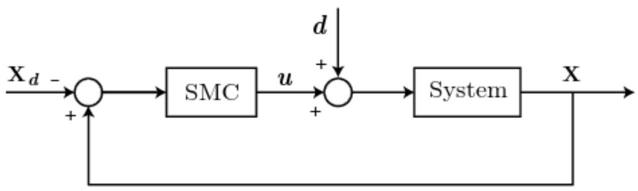


Figure 3: Block diagram of the control scheme.

Now, according to the control strategy described in Slotine and Li (1991) and considering the switching variable as $s = \tilde{y} + \lambda\tilde{x}$, the sliding mode controller for the SMA two-bar truss can be defined with the combination of an equivalent control $u = \hat{f} - \hat{d}(s) - \lambda\tilde{y}$ and another term, $K\text{sat}(s/\phi)$, to confer robustness to the system:

$$u = -\hat{f} - \hat{d}(s) - \lambda\tilde{y} - K\text{sat}(s/\phi) \tag{16}$$

where $\hat{d}$ is an estimate of d, K is a positive gain, ϕ is a strictly positive constant that represents the boundary layer thickness and sat(s/ϕ) is defined as

$$\text{sat}(s/\phi) = \begin{cases} \text{sgn}(s/\phi) & \text{se} \quad |s/\phi| \geq 1 \\ s/\phi & \text{se} \quad |s/\phi| < 1 \end{cases} \tag{17}$$

Considering Assumptions 2 and 3, the robustness of the adopted smooth sliding mode controller against parametric uncertainties, modeling inaccuracies and external disturbances is assured by defining the gain K according to (Slotine and Li, 1991):

$$K \geq \eta + \mathcal{F} + \mathcal{D} + |\hat{d}| \tag{18}$$

where η is a strictly positive constant related to the time required to reach the sliding surface.

4. NUMERICAL SIMULATIONS

Numerical simulations are now in focus exploring the controller capability to perform vibration reduction in a pseudoelastic two-bars truss. An iterative numerical procedure based on the operator split technique (Ortiz *et al.*, 1983), the orthogonal projection algorithm (Savi *et al.*, 2002b) and the classical fourth order Runge−Kutta method was developed to deal with nonlinearities in the formulation. In all simulations, we have used the material properties presented in Tab. 1. These values were chosen in order to represent a typical SMA behavior as shown in Fig. 4.

Table 1: SMA constitutives parameters.

E_A (GPa)	E_M (GPa)	α_h	α (MPa)
54	54	0.052	150
$\bar{L}$ (MPa)	$\bar{L}_0$ (MPa)	$\bar{L}_0^A$ (MPa)	$\bar{L}^A$ (MPa)
41.5	0.15	0,63	185
κ_A (MPaK^{-1})	κ_M (MPaK^{-1})	T_M (K)	T_0, T_A (K)
0.74	0.17	291.4	307.7
	$\bar{\eta}^C$ (MPa s)	$\bar{\eta}^D$ (MPa s)	
	10	27	

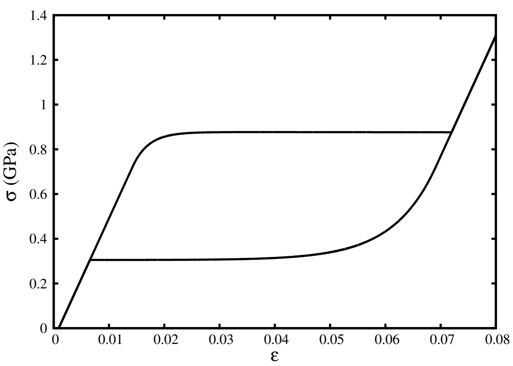
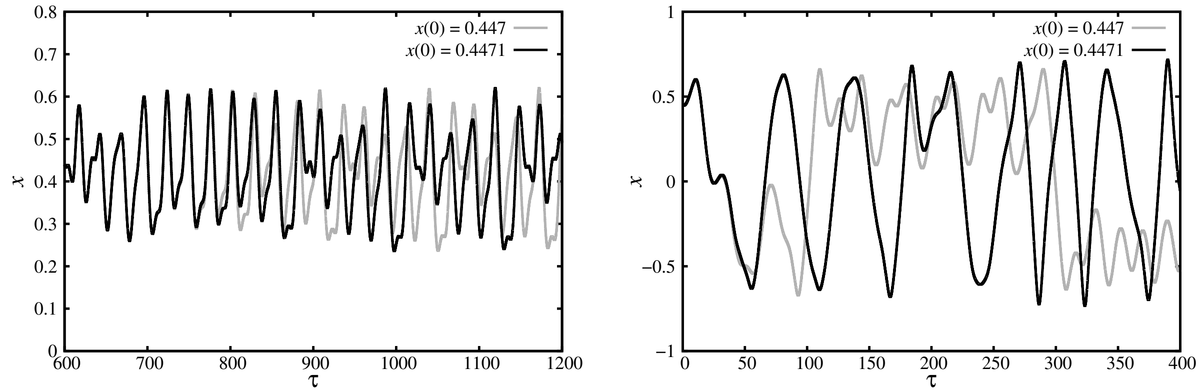


Figure 4: Stress-strain curve of a typical SMA ($T = 373$ K).

Savi and Nogueira (2010) considered the bifurcation diagram which represents stroboscopically sampled displacement values, x, under the slow quasi-static increase of a system parameter. First, the driving frequency, Ω, is considered, assuming a fixed forcing amplitude. After, they considered different frequency values in order to analyze the system response for an especific value of frequency. Phase space plots and Poincaré sections were presented for each set of parameters. Both bifurcation diagrams show regions related to a cloud of points related to chaotic motion and also regions represented by a discrete number of points associated with periodic motions. The differences are noticeable between different frequency and amplitude values.

In order to present the chaotic motion of the uncontrolled structure, we simulate its behavior using specific values for Ω and γ according to Savi and Nogueira (2010) that will present chaotic motion. For the data in Table 1, the parameters defined in Eq. (11) assume the values: $x(0) = 0.447$, $b = 0.894$ and $\xi = 0$. The chaotic response x related to τ is represented in Fig. 5 for different values of Ω and γ and submitted to a high temperature which the pseudoelasticity phenomenon is present. Time steps are chosen according to $\Delta\tau = \pi/1000\Omega$.



(a) Response for $\gamma = 0.008$ and $\Omega = 0.475$ (b) Response for $\gamma = 0.01$ and $\Omega = 0.3347$

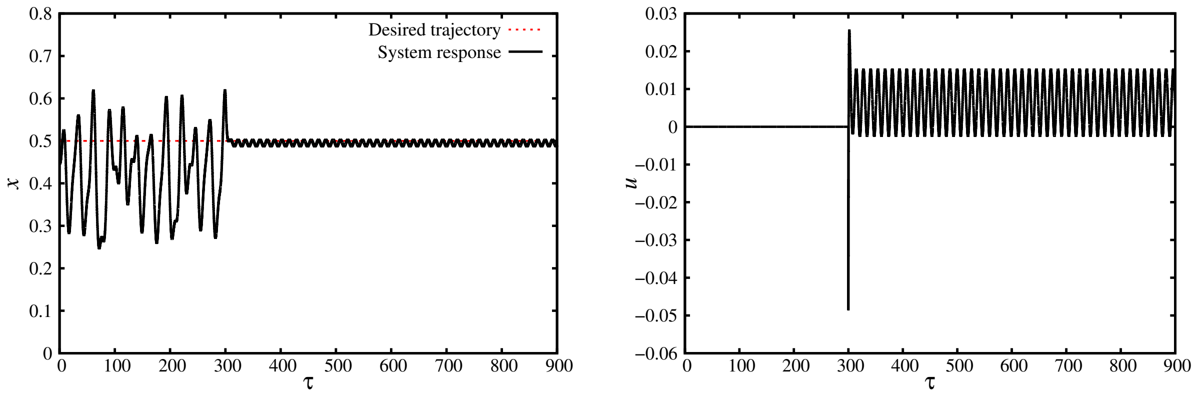
Figure 5: Uncontrolled response of the system (T = 373 K).

The chaos phenomenon is associated with the unpredictability of the system response with small variations of the initial conditions, however small are these variations (Khalil, 2002). From the Figure 5 we can see that the responses of the system are similar until a certain period of time and soon after begin to diverge from each other up to the point where the curves have completely different behaviors. The SMA two-bars truss is very sensitive to its parameters, in the Fig. 5a the response start to become unpredictable around $750\,\tau$, without presenting the snap-through behavior, while in the Fig. 5b the response starts to be unpredictable in $50\,\tau$, but presenting the snap-through behavior.

The controller capabilities are now investigated. Sampling rates of $200\Omega/\pi$ for control system and $1000\Omega/\pi$ for dynamical model are assumed. In order to demonstrate that the adopted control scheme can deal with both modeling inaccuracies and external disturbances, a simplified model to simulate the SMA behavior was considered (Savi *et al.*, 2002a). Moreover, the external disturbance $\gamma\sin(\Omega\tau)$ is not taken into account within the design of the control law. On this basis, according to (Savi *et al.*, 2002a) the values of the constants a_1, a_2 and a_3 are chosen in order to match experimental data obtained by Sittner *et al.* (1995) for a Cu-Zn-Al-Ni alloy at 373 K. For the proposed controller, the constants assumed the following values: $a_1 = 523.29$ (MPa/K), $a_2 = 1.868 \times 10^7$ (MPa) and $a_3 = 2.186 \times 10^9$ (MPa). With theses values, the non-dimensional parameters of of the Eq. (14) are $\alpha_2 = 1.225 \times 10^2$ and $\alpha_3 = 1.430 \times 10^4$. Besides, the following parameters are adopted: $\mathcal{F} = 0.05$; $\mathcal{D} = 0.05$; $\phi = 0.1$; $\lambda = 0.6$ and $\eta = 0.03$.

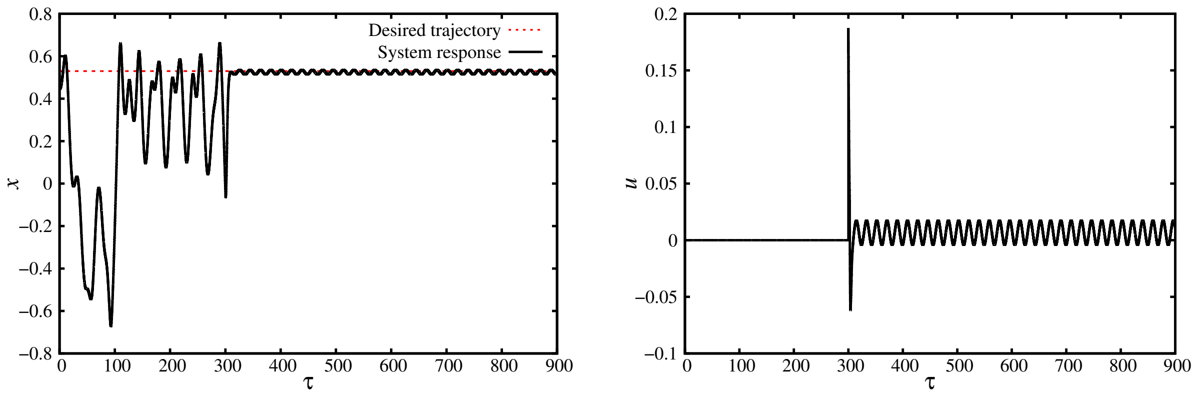
The stabilization of the state vector in the neighborhood of one of the equilibrium points of the shape memory two-bar truss is carried out. This approach shows that the adopted control scheme can significantly reduce the vibration level and also avoid the undesired snap-through behavior as shown in Fig. 6 and Fig. 7. The obtained results are considering $x(0) = 0.447$, $\mathbf{x}_d = [\ 0.50 \quad 0.0\ ]$ and $\mathbf{x}_d = [\ 0.53 \quad 0.0\ ]$, respectively, and the hysteretic effect of the system in Eq. (14) was considered by an equivalent viscous damping $\hat{\xi} = 0.01$. Note that the chaotic behavior with large amplitudes of the uncontrolled respose is replaced by a regular behavior with small amplitudes around the desired state when the controller is activated ($\tau = 300$).

The robustness of the designed controller can be seen in Fig. 6 and Fig. 7 that even in the presence of modeling inaccuracies, which were associated with the simplified model used in the proposed control, and external disturbance, it is able to provide the desired stabilization with a small error associated. The error is present because of the term ϕ which soften the control action and makes a tracking with null error and presenting chattering in a tracking with guaranteed accuracy without chattering (Bessa, 2009). This is an important point to treat complex systems as SMA two-bars truss since it allows the use of simple constitutive models for control purposes.



(a) System Response x (b) Control action u

Figure 6: Controller performance for $\gamma = 0.008$ and $\Omega = 0.475$.



(a) System Response x (b) Control action u

Figure 7: Controller performance for $\gamma = 0.01$ and $\Omega = 0.3347$.

5. CONCLUDING REMARKS

In this paper, a sliding mode controller is considered for vibration reduction in a pseudoelastic two-bar truss. A constitutive model with internal constraints is assumed to describe the thermomechanical behavior of the bars and a simplified one is assumed to the designed control. Despite the simplicity, this model can be used to provide a description of the system to the controller. Numerical simulations show the robustness of the sliding mode controller against modeling inaccuracies and external disturbances. Undesirable behaviors such as snap-through and chaotic responses are avoided by the controlled structure. It should be highlighted that the controller robustness to modeling inaccuracies is an important issue that allows the use of a simple constitutive model for control purposes.

6. ACKNOWLEDGEMENTS

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