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CHAOS ANALYSIS OF THE PIV MEASUREMENT QUALITY ON THE LIQUID VELOCITY FLUCTUATION IN A BUBBLE COLUMN SECTION

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Abstract. *Bubble columns are widely used in chemical processes for providing various advantages during the process of mass and heat transfer. They are used in petroleum, chemical, biochemical and metallurgical industry. Knowledge of the fluid dynamics behavior of these reactors is important in order to control heat transfer, mass transfer and chemical reaction rate. Particle Image Velocimetry (PIV) has become a very attractive technique in experimental analysis of multiphase flows. The performance of PIV technique is compromised due to the nonlinear hydrodynamic and the high concentrations of gas bubbles in large flow rates. This work aims to evaluate the PIV measurement quality on the liquid velocity fluctuation in a bubble column section for different gas flow rates (1.5, 5.0 e 40.0 L/min) using chaos analysis. The PIV uncertainty and the correlation dimension were used as a quality indicator. The PIV uncertainty was estimated by correlation statistic and the correlation dimension was determined in the chaos analysis.*

Keywords: *Bubble column, PIV; Nonlinear dynamic; Chaotic Invariant.*

1. INTRODUCTION

Bubble columns are used in several processes in the oil, chemistry and biochemistry industry. The study of fluid dynamics behavior in bubbles column reactors is important for controlling of parameters related to the rate of chemical reaction, heat and mass transfer (Lain *et al*., 1999). The Particle Image Velocimetry (PIV) is a non-intrusive measurement technique used to determine velocity fields in two or three-dimensional domain in single or multiphase flow. The PIV performance depends on the quality of the recorded images and processing of these images. Currently, there is still a gap concerning the best quality indicator and uncertainty quantification for PIV measurement. The majority of studies rely on simulation of synthetic data to identify different sources of error, but it usually underestimates the errors in real experiments (Wieneke, 2014).

An alternative to evaluate the quality of PIV measurements can be found in nonlinear analysis of the velocity fluctuations. The chaos analysis can be used to qualitatively and quantitatively characterize the hydrodynamic behavior and regime transitions in a bubble column (Letzel, *et al*., 1997; Vial *et a*l., 2000; Lin *et al*., 2001; Gourich *et al*., 2006; Nedeltchev *et al*., 2007; Ajbar *et al*., 2009; Nedeltchev *et al*., 2011). This approach has shown great utility when extracting time series information arising from systems whose behavior has nonlinear characteristics. According to Abarbanel (1996), the behavior is described as deterministic chaotic when it is aperiodic and sensitive to small changes in the initial conditions. Calculating of the correlation dimension is one of the most widely used methods to analyze the nature of time series data. The correlation dimension allows estimating the number of independent variables or degrees of freedom that describe the system. The main problem in estimating the size in time series is related to the fact that these are contaminated by noise. However, Schouten, *et al.*, 1994 proposed a method of calculation that is derived from the assumption that the data are obtained from a system constrained by amplitude noise.

This work aims to evaluate the PIV measurement quality on the liquid velocity fluctuation in a bubble column section for different gas flow rates (1.5, 5.0 e 40.0 L/min) using chaos analysis. The PIV uncertainty and the correlation dimension were used as a quality indicator. The PIV uncertainty was estimated by Wieneke (2014) method and the correlation dimension was determined using the Schouten, *et al.*, 1994 method.

2. BACKGROUND

2.1 Fluorescent Particle Image Velocimetry – Fluorescet PIV

PIV is a non-intrusive technique that uses a light source to illuminate tracer particles in the flow in order to make the flow pattern visible through a CCD or CMOS sensor. This technique calculates the instantaneous velocity field of the flow by displacing small groups of particles in small areas of investigation (interrogation windows) using cross-correlation (Fig. 1a) in recording mode double-frame / single-exposure (Prasad, 2000). The PIV cross-correlation plane is the probability distribution of the shift pattern of the particle images between consecutive frames (Raffel *et al*., 2007). The Signal-to-Noise Ratio (SNR) or peak ratio is defined as the ratio between the highest correlation peak to the second tallest peak and it is a practical measure of the quality of a cross-correlation. Numerical investigations indicate that a correlation peak height of about 0.5 and a SNR of about 2.0 reliably avoid spurious vectors (Hain and Kahler, 2007). The evaluation of PIV measurement errors has been investigated thoroughly in the last two decades either via theoretical modelling of the processing algorithm (Westerweel, 1997) or more frequently by Monte Carlo simulations (Scarano and Riethmuller, 2000; Lecordier *et al*., 2001). A method presented by Wieneke (2014) is found in the software Davis 8.2 (LaVision) and provides the uncertainty of the PIV displacement field using a generic post-processing method based on statistical analysis of the correlation process using differences in the intensity pattern in the two frames (Wieneke, 2014).

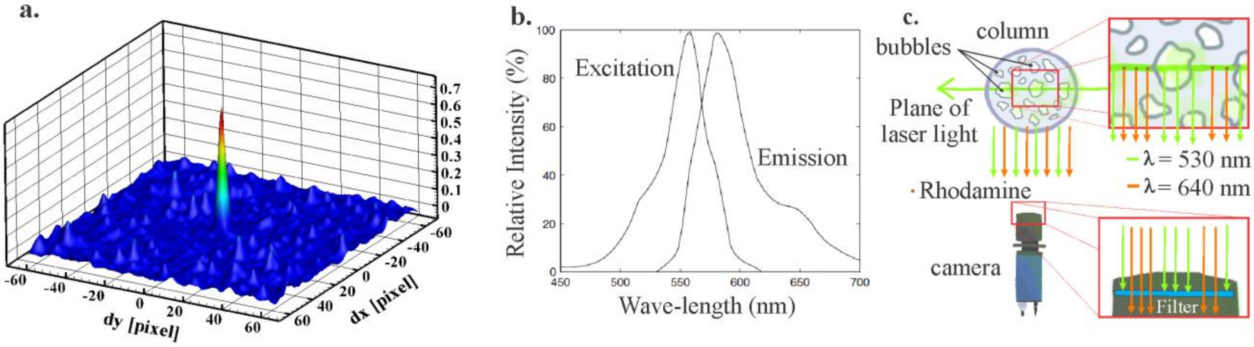

Figure 1 - (a) PIV cross-correlation plain for an interrogation window, (b) excitation and emission spectrum of the Rhodamine B (Adapted from Wilson *et al.*, 1986) and (c) representation of a Fluorescent PIV system.

The experimental error is crucial particularly when PIV measurements are used to validate computational fluid dynamics (CFD) results (Shitano *et al*., 2014). A major problem in PIV applications for bubble column is the large scattering of laser light caused by the bubbles that may damage the camera sensor. In order to avoid the problem, fluorescent tracer particles that absorbs the green light of laser (530 nm) and emits oranges (640 nm) (Fig. 1b) are combined with high-pass filter on camera that allowing for the pass of the light only scattered by tracer particle (Fig. 1c) (Lindken and Merzkirch, 2002).

2.2 Chaos analysis: Correlation dimension

The sensitivity of the chaos invariants, such as correlation dimension, is affected in the presence of noise, resulting in the main disadvantage of using them to identify the chaos particularly when using the algorithm proposed by Grassbeger e Procaccia (1983). The definitions of these algorithms involve the threshold of small length scales implying on limitations in measuring technique. Thus, it is justified to apply the maximum likelihood method developed by Schouten et al. (1994) for obtaining chaotic invariant. This method is applied in experimental signals in order to reduce noise and solve the problems associated with the theoretical prediction capacity of the system behavior.

The correlation integral is defined as

$$C(l)=\frac{1}{N^2}\lim_{N\to\infty}\sum_{\substack{i,j=1\\ i\neq j}}^{N}\theta[l-|\mathbf{x}_i-\mathbf{x}_j|] \tag{1}$$

where θ(x) is the Heaviside function. In this case $x = (l - |\mathbf{x}_i - \mathbf{x}_j|)$, that is x ≥ 0 implies that θ(x) = 1 and if x ≤ 0 implies that θ(x) = 0. The correlation integral counts the number of pairs of points ($\vec{x}_i$, $\vec{x}_j$) separated in the attractor whose distance is less than l . If the time series is characterized by an attractor, the correlation integral follows a power law with

$$C(l) \approx l^D \text{ with } l \to 0 \text{ and } N \to \infty \tag{2}$$

where the exponent D is defined as the correlation dimension of the attractor and can be obtained from the graph slope of $\ln\left[C(l)\right]$ versus $\ln(l)$.

The correlation integral provides reliable values when the data is noise free. However, this method presents failures for noisy data, such as experimental ones. Schouten, *et al.*, 1994 proposed a method of determination of the correlation integral based on the maximum norm considering that the noise induces a deviation in the trajectories distances represented in Eq. (3), this method proposes a redefinition of the distances of the attractor points with the possibility of limiting the range of noisy data.

$$C(r) = \left[\frac{r - r_n}{1 - r_n}\right]^D, r_n \le r \le 1 \tag{3}$$

where $r = l_z / l_0$ e $r_n = l_n / l_0$ are normalized distances relative to the cutting length l_0. In this case, l_z is the maximum distance from the vectors in the presence of noise, l_n is the maximum distance or amplitude noise and l_0 can be obtained from the mean absolute deviation of the time series. According to Schouten, *et al.*, 1994, when l_n is known *a priori*, all distances are resized and the correlation dimension may be estimated from the slope of $\ln[C(l)]$ vs $\ln[l - l_n]$ or $\ln[C(l)]$ vs $\ln[r - r_n]$. Alternatively, by applying the maximum likelihood method on the Eq. (3) an estimate of the correlation dimension is obtained by Eq. (4).

$$D_{ML,n} = \left[-\frac{1}{M}\sum_{i=1}^{M}\ln\left(\frac{r_i - r_n}{1 - r_n}\right)\right]^{-1}, \ r_n \le r_i \le 1 \tag{4}$$

However, in most practical situations, l_n is not known or not known with sufficient accuracy. In these cases, assuming that the function $C(r)$ is known, D and r_n can be determined using the method of least squares by Levenberg-Marquardt (Schouten *et al*., 1994). It was found that the solution of Eq. (4) is not trivial, besides it requires a high computational time, this method requires the solution of two non-linear algebraic equations as well as the storage of all distances of r_n (Schouten *et al*. 1994a). Alternatively, the maximum likelihood method can be applied in correlation integral (Eq. 1) without resizing distances of vectors corrupted by noise.

The correlation dimension D_{ML} according to Takens (1984) is given by

$$D_{ML} = \left[-\frac{1}{M}\sum_{i=1}^{M}\ln(r_i)\right]^{-1} \tag{5}$$

The vector $\{r_1, r_2, ..., r_M\}$ is obtained from a random sample of M observations of a cumulative distribution function whose probability (Gaussian, for example) is given by Eq. (5). The standard error is given by Eq. (6).

$$s\left(\frac{1}{D_{ML}}\right) = \frac{\sigma(1/D_{ML})}{1/D_{ML}} = \frac{1}{\sqrt{M}} \tag{6}$$

The relative error in D_{ML} for large samples (M) is approximated by $1 / (M)^{1/2}$ and for smaller samples the estimation has a tendency to higher values.

3. EXPERIMENTAL

Figure 2a shows the layout of the acrylic bubbles column of 14.5 cm diameter and 1 m height. The column was filled with water at 25 ° C and the gas phase used was air with a flow rate of 1.5, 2.5, 5, 7.5, 10, 20 and 40 L/min. The investigation area was located 55.2 cm from the distributor and had a size of 14.5 x 10 cm (Fig. 2a and 2b). A plane plate with 21 holes (1 mm in diameter) distributed equally in a square array (Fig. 2c) was used as a distributor. The region immediately below the plate was filled with glass beads with an average diameter of 2 mm in order to better distribute the air entrance.

The PIV system used was developed by LaVision and has a CCD sensor (1376 x 1040 pixels and pixel size of 6.45 μm) with a maximum frequency of 30 Hz and a Nd: YAG (200 mJ / pulse and λ = 532 nm) laser system with a maximum frequency of 5 Hz. The cameras and the laser were controlled with a programmable time unit (PTU) and Davis 8.2 software. The recording frequency used was 4.2 Hz. A Nikon objective lens with a focal length of 60mm (f / 2.8D) was used in camera. A lens with a focal length of -20 was used in the laser source to expand the light beam in order to get the laser sheet with defined thickness (1 mm). A box with a slit of 1 mm thickness located 1.20 m from the laser (Fig. 2d and 2e) was used in order to provide a homogeneous intensity profile (top-hat).

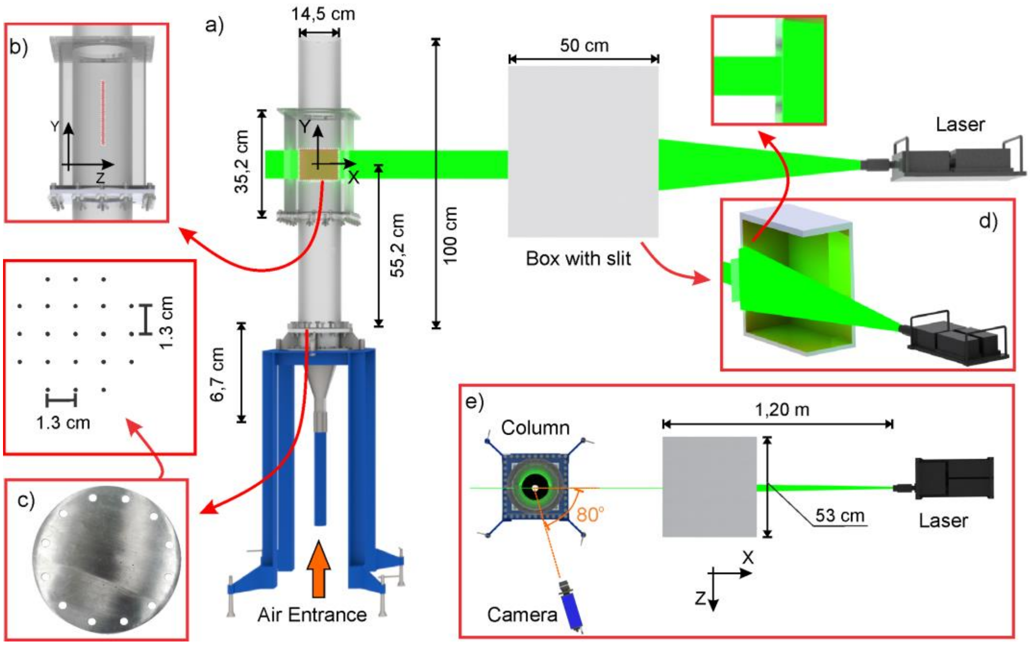

Figure 2. (a) Column, laser system and box with slit. (b) Investigation area. (c) Air distributor with 21 holes. (d) Arrangement of the PIV system in the column.

Rhodamine B (diameter of 20 - 50 μm) was used as tracer particle. A high-pass filter on the camera was employed to allow only the passage of light emitted by the tracer (λ = 620 nm). A calibration procedure was performed in order to correct the image distortions caused by the column curvature. The calibration error was 0.981 pixel. Furthermore, the use of the box with the slit facilitates the alignment of the laser light sheet with the calibrated plan. The CCD camera was positioned at 80° from the laser light plane. The resolution of the imaging system was 12.5 pixels / mm and the particle image diameter was 2-3 pixels.

The inter frame time was 500 μs for flow rates of 1.5 and 2.5 L/min, and 1000 μs for those larger than 2.5 L/min producing, a maximum particle image displacement of 15 - 20 pixels. After the images recording, a preprocessing was conducted with the sequential application of the RMS and SSM filters. A pixel intensity lower than 10 counts was defined as 0. An upper limit of pixel intensity 50 was used. For the PIV processing, the standard cross-correlation was made using an interrogation strategy with two steps: the first using an interrogation window size of 256 pixels (50% overlap of adjacent windows) and the second of 128 pixels (75% overlap of adjacent windows). The median test (Westerweel and Scarano, 2005) was used in the PIV post processing to remove false vectors of the vector field. According to the authors, a removal bound of 2 is suitable for detecting false vectors in a neighborhood of 3 x 3 pixels. The quality of PIV measurements were analyzed using the correlation coefficient, signal-to-noise ratio (SNR) and uncertainty distribution estimated by Wieneke (2014) method.

The chaos analysis used as an alternative method for determining the PIV measurement quality was applied in time series from the velocity module for flow rates of 1.5, 5.0 and 40.0 L/min, in radial positions r/R = -1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75 and 1. The correlation coefficient, SNR and uncertainty estimated by Wieneke (2014) method were compared with the results of the chaotic analysis. The RRChaos (Shouten *et al*., 1993) software was used to calculate the correlation dimension. The uncertainty for the time series was defined as the difference in the convergence of correlation dimension obtained by Eq. (3) with Eq. (5). This comparison aims to identify regions of the velocity field in which it has a higher concentration of noisy data.

4. RESULTS AND DISCUSSION

Figure 3 shows the distributions of time-averaged velocity V (a), its uncertainty estimated by Wieneke (2014) method (b), correlation coefficient (c) and signal-to-noise ratio (SNR) (d) for flow rates of 1.5, 2.5, 5.0, 7.5, 10, 20 and 40 L/min. The position of measurement was across the radius and 55.2 cm from the gas distributor.

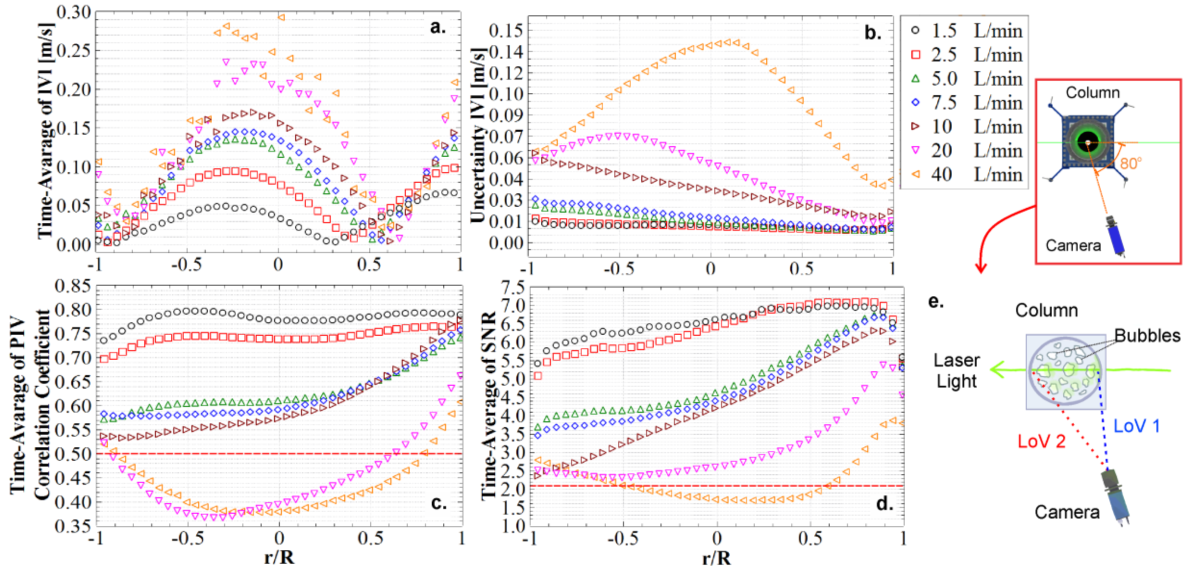

Figure 3. Distribution of time-averaged of velocity V (a), its uncertainty estimated by Wieneke (2014) method (b), correlation coefficient (c) and signal-to-noise ratio (SNR) (d) Lines of vision (LoV) 1 and 2 traveled by light of the tracer particles (e).

Figure 3a shows that the profile of time-averaged velocity is not symmetrical and, as the gas flow rate increases, the velocity at each interrogation window (each point on the graph) shows no consistency with its neighborhood along the radius. This is clearly observed for flow rates of 20 and 40 L / min and it can be analyzed by high uncertainty of the velocity in Fig.3b. The Wieneke (2014) method considers the shape of the PIV correlation peak, disregarding the PIV correlation coefficient and SNR. The values of the correlation coefficient and SNR found by Hain and Kahler (2007) do not ensure the quality of the measurement (line on the graph in Fig 3c and 3d). The uncertainty of time-averaged velocity V increases from r / R = 1 to -1 and with the increase of gas flow, except for flow rates of 20 and 40 L / min. The concentration of bubbles in the column increases with the increase of gas flow causing the non-uniform illumination of the laser light in the region of interest (in the radius direction r / R = -1 to 1) due to the scattering of light.

The light emitted by the Rhodamine is scattered by the bubbles localized in front of the investigation, which compromises the recording of the particle images on the CCD sensor. The trajectory taken by the light emitted by the tracer also becomes important in the analysis of the PIV measurement quality. The light emitted by the tracer particles traveling in LoV1 (Line of Vision 1) trajectory travels a shorter path than the LoV2 trajectory, interacting with less bubbles that are located in front of the light sheet. This effect can be seen in the vector field of time-averaged velocity for flow rates of 1.5, 5 and 40 L / min in Fig. 4. One way to correct the false vectors is by making the comparison of an interrogation window with its neighborhood in a PIV post processing. The PIV Sum of correlation or ensemble correlation method is used to estimate the time-averaged velocity field and can provide reliable measurements at low SNR conditions, where the standard cross correlation techniques fails (Meinhart *et al.*; 2000). In Fig. 4 it is observed that the average field without and with PIV post processing calculated with the standard cross-correlation are similar for the cases of 1.5 and 5 L/min flows. However, the post processing interferes strongly for the 40 L/min flow. In other words, increasing the gas flow it is observed that the number of false vectors increases and consequently decreasing the performance of the PIV post processing. Simulations made by Keane and Adrian (1990;1992) show that the amount of tracer illuminated in the flow affects the quality of the vector field. Additionally, the authors show that to obtain a PIV measurement with good quality it is necessary to have at least five particle image pairs correlated in the interrogation window.

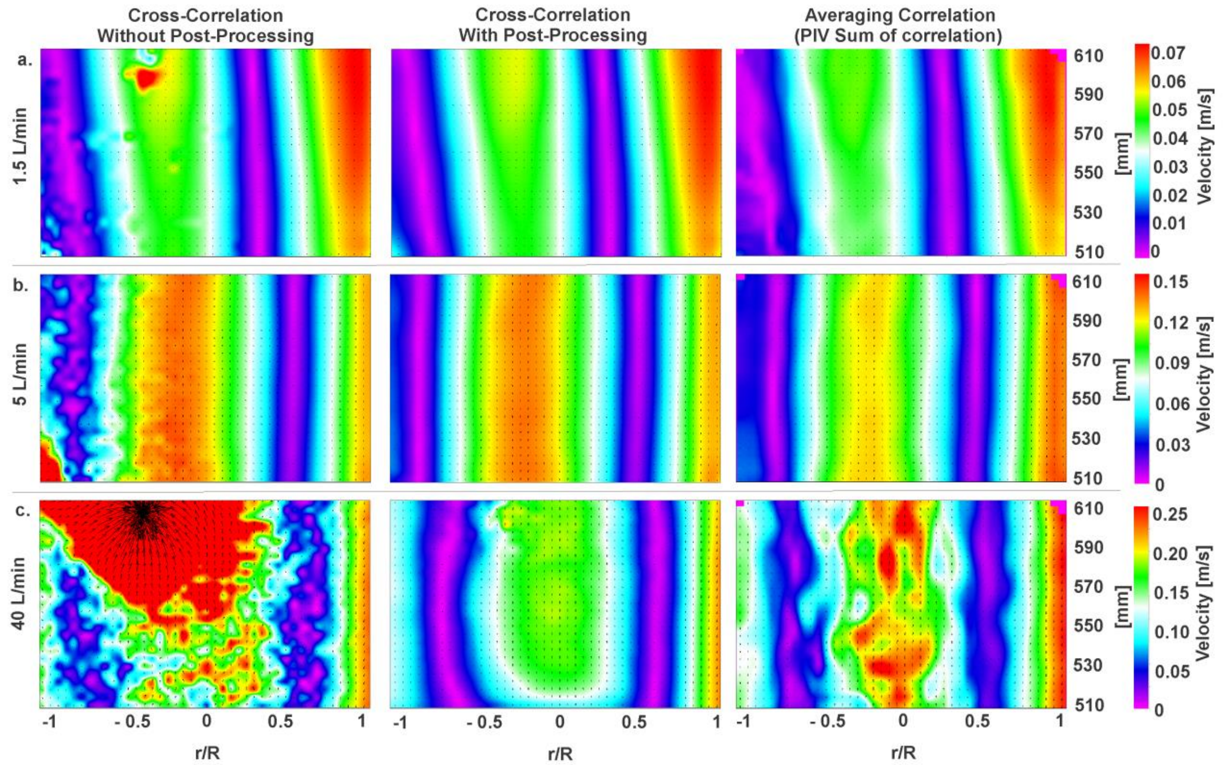

Figure 4. Comparison between the field average velocity calculated using the standard PIV cross correlation (with and without PIV post processing) and the ensemble correlation (PIV Sum of correlation) for (a) 1.5 (b) and 5 (c) 40 L/min using 4000 images.

Figure 5 shows the distribution of the correlation dimension in the radial positions. The correlation dimension behaviors for flow rate of 1.5 L/min (Fig. 5a) ,without and with PIV post processing, follows the same trend, with only a small difference at the radial position r/R = -1. This implies that the degree of freedom for the time series of the velocity module without and with the PIV post processing for 1.5 L/min did not suffer significant variations. This analysis support that the complexity of this system for 1.5 L/min flow is consistent with the Fig. 3, in which the neighborhood coherence of velocity profile is maintained (Fig. 3a) with high value of the correlation coefficient across the radius (Fig. 3c). However, for the flow of 5 L/min (Fig. 5b) it is observed that there is a difference in system complexity level in the radial direction r/R = 0 to -1, and this fact is aggravated when the flow rate is increased to 40 L/min (Fig. 5c). In Fig. 5c for a flow rate of 40 L/min, it is observed that there is a difference in the degree of freedom for the velocity fields without and with PIV post processing through the cross section, except for r/R = 1. The correlation dimensions calculated for the fields with PIV post processing varied little along the radius in respect to the fields without PIV post processing.. A possible explanation for this arises from the spatial interpolation of the interrogation windows that does not present vectors.

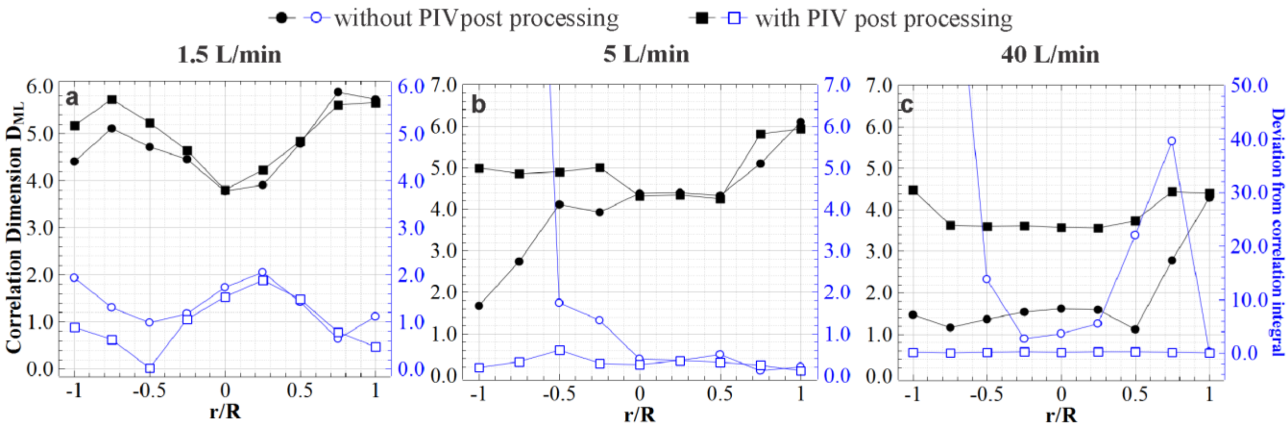

Figure 5. Correlation Dimension and desviation from correlation integral for the flow of (d) 1.5, (e) 5 and (f) 40 L/min without and with PIV post processing.

In addition, the PIV post processing filter developed by Westerweel and Scarano (2005), which uses the principle of neighborhood connectivity to correct the areas presents false vectors resulting in possible erroneous interpretations of the flow. This behavior was observed for 40 L/min (Figure 4c). The chaos analysis shows a greater sensitivity in PIV post processing effect on the velocity field for the low flow rates.

The difference between the correlation dimension values without and with PIV post processing is directly related to the noise level in the time series. Analysing the Figure 5, it is noted that there is indeed noisy components, where the uncertainty for points without PIV post processing accompanies the correlation dimension behavior. In this case, the uncertainty was defined as the difference between the two ways of calculating the correlation dimension, that is, the difference between the solution of Eq. (3) and Eq. (5). Otherwise, according to Eq. (6), the uncertainty value would be nearly constant, varying only with the number of pairs of data points in the time series. This definition affects the resizing of time series without and with PIV post processing, resulting in a sensitive parameter the presence of noise, thus ensuring the reliability and equivalence of velocity fields. When the curves for the velocity field without and with PIV post processing are close, it is indicated that there is a convergence in both dimensions (D e D_{ML}), that is, the resizing of the series considering the noise amplitude when calculating D is equivalent to the solution with the application of the maximum likelihood method (D_{ML}) starting from Eq. (5).This implies that the noise level has a low value. For example, the convergence of the dimensions for the time series in the points r/R = 0.75 and 1 for the flow rate of 40 L/min without PIV post processing is shown in Fig. 6. In this case, there is a distinct state of non-convergence and convergence, where the region r/R = 0.75 presents greater uncertainty than the region r/R = 1.

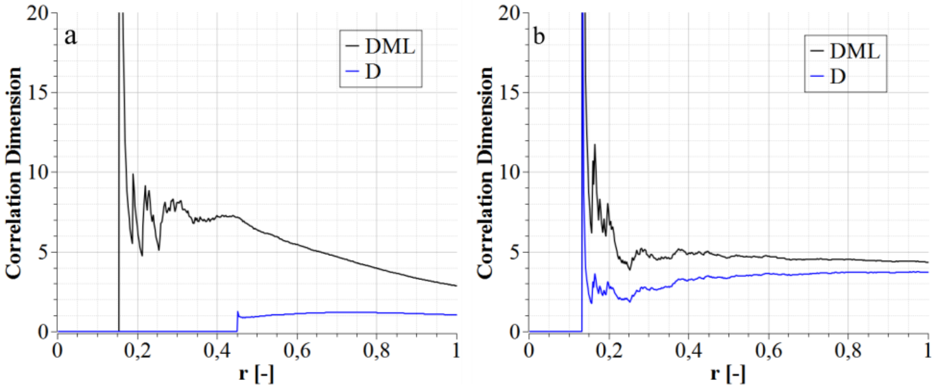

Figure 6 – Comparison of the points r/R = 0.75 and 1 for the convergence of the correlation integral at 40 L/min without PIV post processing.

5. CONCLUSION

This work focused on evaluation of PIV measurement quality of the liquid velocity fluctuation in a bubble column section using PIV correlation statistic and chaos analysis. The PIV measurement quality was analyzed for three different flow rates (1.5, 5.0 and 40.0 L/min) and nine radial positions using PIV correlation statistic (PIV cross-correlation coefficient, Signal-to-Noise Ratio and the Wieneke (2014) method) and chaos analysis (correlation dimension). The results of the correlation dimension (chaos analysis) have shown consistency with the PIV correlation coefficient. At low flow rates considered, the velocity fields without and with PIV post processing have presented high discrepancy in the regions between r/R = -1 and 0. This fact is directly related to the trajectory of the light emitted by the tracer particles and the presence of bubbles in the investigation area inside the column. For higher flow rate (40 L/min) the degree of freedom (correlation dimension) differs considerably in all radial regions of the column. For the correlation dimension, the uncertainty in the calculation has been considered as the difference between resizing time series and its estimative by maximum likelihood method. The previous discussion shows that chaos analysis presents greater sensitivity than PIV approach to the presence of noise. On top of that, the current investigation has shown significant differences between velocity fields in which the PIV post processing had been considered and those where the PIV post processing was not considered. In fact for both analysis discussed in this paper the vector field obtained for low flow rates (1.5 and 5.0 L/min) are acceptable. On the hand the results obtained for high flow rates (40L/min) require further investigation in order to improve the quality of the experimental results.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE

The authors Helder Lima de Moura, Rodrigo de Lima Amaral, Diana Isabel Sánchez Forero, Guilherme José de Castilho, Sávio Souza Venâncio Vianna and Milton Mori are the only responsible for the printed material included in this paper.