A DISCUSSION ON MECHANICAL TRANSMISSION AND THE STUDY OF A MATHEMATICAL METHOD FOR PROPER DEVICE SELECTION

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Abstract. *This article explores mechanical transmission of torque and speed and tries to do a brief summary of its features and applications, and then group them. This work aims at a comprehensive and enlightening look on mechanical transmissions and torque reduction, making clear the different applications and methods of choice for the design engineer. In order to make a more didactic discussion, the term transmission is used to refer to the mechanical transmission of torque and speed. Although mentioned in the article, clutches, electrical and hydraulic transmission of torque and speed and differentials will not be discussed here. Rather than just making a reliquary of the formulas used in the different types of transmissions, the author's effort is directed at the discussion of the development of a mathematical model for the proper selection of mechanical transmission devices. Thus, deviating from the reliance of the design engineer's experience, the method should focus on the more efficient device available for the proposed application. The conclusion will be the baseline for the further study on the mathematical formulation of the method.*

Keywords: *mechanical transmission, selection method, axiomatic design*

1. INTRODUCTION

Mankind is discovering new ways of maximizing the use of force since the wheel discovering in the 4th millennium B.C. Building new machines such as the ones that helped civilizations build the Lighthouse of Alexandria, the pyramids and the roman aqueducts were only possible by the use of force maximization and force transmission (Kirby, 1990). Although these machines have changed a lot with the use of new materials and the advent of science studies, it is of an extraordinary significance the success of those millenary systems many still in use today, e.g. gears, pulleys and levers.

The main function of mechanical transmission of torque and power is to suit the work scheme of the drive component to the necessities given by the performance study of the driven component. In vehicles driven by internal combustion engines, a transmission system is designed so that the relation between drive and driven systems can be changed during its operation (Heldt, 1955). This thought can be applied to all kind of motorization types or source of torque and power. Although the terminology transmission is used in such systems, transmitting is not its primary function. Converting torque and power to adequate the momentary needs for the operation is its main role.

A transmission system is a vital part on the development of driven mechanical systems. Applied to guarantee the continuous optimum torque required to a determined function, transmission systems have evolved in the last decades to many different construction types and mechanical principles. For each application there are different system options and the selection of a determinate type can be an engineer's choice or may be the consequence of references inhered to the project. This restricts the options to select an ideal transmission system, which can lead to undesired results and to the use of old solutions, thus propagating old technologies. A good example is the manual transmission system used in the automotive industry and the bicycle gear set. Both systems are more than five decades old, used in large scale and have received very few improvements over the years.

An effective selection criterion can provide many benefits to the planning phase of a machine design. The method to be proposed has to provide a fast way for selecting the appropriate transmission system in order to help the design engineers create smart responses to the customer necessities. As product design time is a common problem for any business, the study and product development phase has to be able to quickly define specific analysis and parameters that will guide the product (Madureira, 2010). Therefore, a mathematical model that helps the project engineer in selecting a proper transmission system during the development phase is a welcoming idea.

One way to approach the creation of a selection model is through the axiomatic design, which quantifies complex systems. It can be defined as the creation of synthesized solutions to form products, processes or systems that satisfy the perceived needs through the mapping of the customer desires in functional requirements, whereby project parameters are obtained (Suh, 2005). Functional requirements represent the goals of the project; hence the project parameters express how the functional requirements will be satisfied.

Independently of the chosen system, mechanical transmissions will always consider the torque as its main variable. Thus, torque on the dimensional system is;

$$\frac{Mass \times Length^2}{Time^2} = ML^2T^{-2} \tag{1}$$

According to the Fundamental Law of Gears, the transmission ratio will be;

$$i = \frac{T'}{T} \tag{2}$$

or

$$i = \frac{n}{n'} \tag{3}$$

Where, i – transmission ratio; T - input torque; T' - output torque; n - input rotation; n' - output rotation

The equations that govern each type of these transmissions are extensive and richly documented in the literature. Therefore, this paper will not focus on them; it will rather emphasize the discussion on the study of a device selection method.

Being one of the most complex industrial mechanical systems, from transportation to consumer goods, the mechanical transmission subject has become vast and very rich in information. Given the complexity of the subject, the following sections of this study will detail some of the main systems and comment their characteristics and attributes, to later correlate it with the complexity theory and how could axiomatic design help to propose a transmission selection method.

2. TRANSMISSION SYSTEMS

To start the study on a selection method, the authors proposes to organize transmission systems in groups. This approach will be responsible for the criteria used to determine which transmission type is the best suitable for the project, a discussion that will be further developed in the subsequently sections of this work. There are many ways of classifying and arrange transmission systems, e.g. by their construction or by their outputs. Some authors have given their understanding to this matter as seen the arrangement proposed by Bartz (1993) in figure 1. He discretizes transmission systems in two groups, which embrace many different types in its definition. Although the example of figure 1 is a good way to classify transmission systems, it goes against the creation of a system selection method.

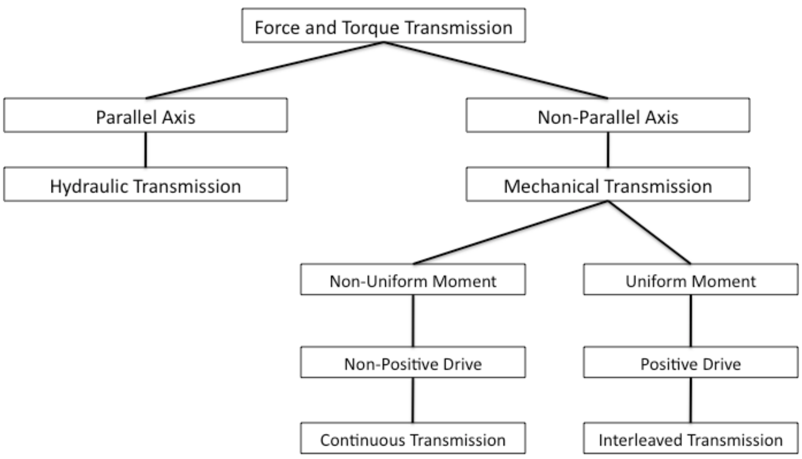


Figure 1: Transmission system classification; adapted from Bartz (1993).

In this work the author proposes a different approach for the classification and arrangement of the mechanical transmission systems in order to facilitate the study of a methodology capable of selecting a proper system to a specific need. First the main characteristics of each system are separated and arranged in small groups, as seen in figure 2.

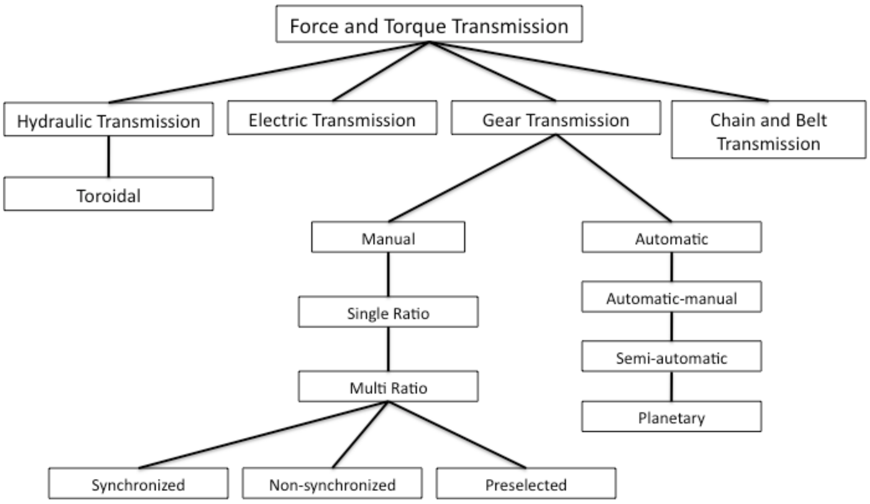


Figure 2: Proposed transmission system classification

Electrical transmission will be mentioned as learning proposes only, for this paper will not develop further discussions on electrical transmission of torque. Also, it is the authors understanding that there are more hydraulic transmission types, not just the one discussed here. It is chosen to discuss this specific systems in order to give a first glimpse of the method that can be then further extended to other transmission systems.

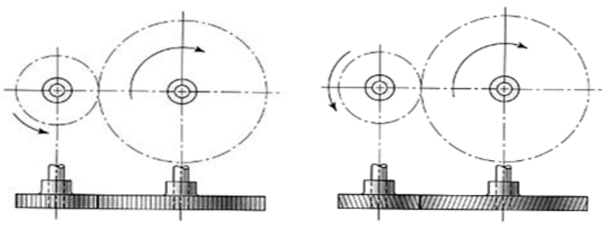
2.1 Gears

Torque transmission using gears is given by the coupling of two or more gears connected directly by their teeth. The oldest reference of this type of transmission is dated from 100 B.C. from an essay of Hero from Alexandria (Norton, 1999). According to the Fundamental Law of Coupling, the angular velocity of the gears has to be constant during the coupling. These sets are governed by the rate of torque in respect to the input and output velocity or by the number of teeth from the drive and driven gears (Merritt, 1947).

2.1.1 Manual Transmission

In this work, manual transmission will be referred to gear transmissions that need the user's intervention to operate. Although it is the author's knowledge that belt and chain transmissions can be called manual, for this research they will not be dealt as such.

Single rate transmission boxes (fig. 3a and 3b) are frequently used in operation machines in industry as on drive motors, mechanical reduction in press and in naval operations. These reductions can be fixed or with the ability to decouple during operation.



(a) (b)

Figure 3: Single rate transmission with spur gears (a) and helical gears (b) (Budagas and Nisbert, 2009)

A multi ratio gearbox can be differentiated in three types; synchronized, non-synchronized and preselected.

- Synchronized: the name of these transmissions is due to the synchronizer that has the function of adding a determined gear pair to transmit torque. The most common gearbox has three shafts, being the entrance, the secondary and the exit shaft. The gears of the secondary shaft are fixed, while the gears of the entrance and exit shaft are subjected to the action of the synchronizer (Stone and Ball, 2004).

- Non-synchronized: in those gearboxes the pairs are not previously coupled. The coupling is given with the movement of the secondary shaft.
- Preselected: here the shift lever is used to pre-select the gear to be used, a separate control is then used to engage the gear. This system avoids the need for the user to control a manual clutch, resulting a faster shifting and being able to operate high input power with a lighter mechanism.

2.1.2 Automatic

This transmission shifts gear via a pre-established control that doesn't need a direct action of the user. For this work, automatic transmission will be referred to gear transmissions. A special note on CVT will be addressed later on.

In the automatic-manual transmission, the selection is given by a pre-defined control or by the direct action of the user. In this case, the user can choose if he wants to control the shifting or not. Generally both actions operate in parallel, although a direct action of the control unit is needed to guarantee the integrity of the operation.

In the planetary transmission (fig. 4), the torque is transmitted by two loading paths. Due to its compact size and reliability, this system is largely used in automatic transmissions for cars, planes and helicopters.

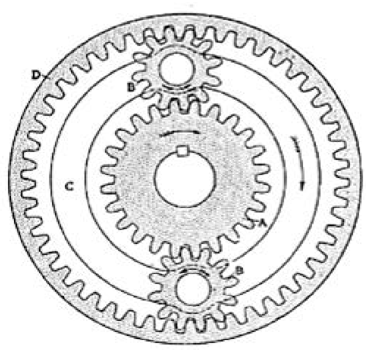


Figure 4: Planetary transmission (Heldt, 1955).

Planetary gear trains have great advantages since they divide the load between the gears due to their concentric entrance and exit shafts. This results in the radial resultant of the forces being zero at the entrance and exit shaft, which guarantees a reduced package and weight when compared to gear boxes with parallel shafts. This coaxial aspect is important since it allows high speed shifting without decoupling the gears. The smaller package results in smaller and more rigid components, lesser noise and vibration and a higher efficiency. Since the planetary transmission transmits only torque, the bearings' project is simplified, allowing, in some cases, a nearby coupling to the drive and driven components.

2.2 Chain and Belt

Reduction boxes that use chain or belt as an element of transmitting torque from the drive to the driven components will be classified under the same criteria due to their similar characteristics like the ability of driving more than one element at time, operating in multiple power ranges and the ability to operate systems that are far away from the power source.

In a belt-drive transmission, torque is transmitted thought the friction between the belt and the pulley; with the exception of the synchronized belt, in which its teeth create a positive coupling with the pulley (Erickson, 1987). Used in a variety of machines, from small mechanisms with less than 20cm in belt length up to large belt with over 3m long, these transmissions are capable of delivering an operating power from 0,1kW up to many thousands of kW. Its main characteristics are predictable performance, visual indication of failure, versatile velocity ratio, low maintenance, low noise levels and efficient operation.

Chain-drive transmission provides a constant ratio without slip and the torque is transmitted by the chain through the sprocket with the least articulation angle as possible. The lubrication is essential to obtain a long life; therefore a superficial oil bath is needed (Budynas and Nisbett, 1999). It can drive more than one shaft using a single source of power and it shares many advantages of a timing-belt. Never the less, positive drives are capable of transmitting higher torques than belt-drives and they rarely fail because they lack tensile strength – failures normally occur due to great service hours or due to wear of the rollers or fatigue of the roller's surface.

2.3 Electrical transmission

Electrical transmission consists in a generator moved by a driving source with both equipment directly connected to a driven element. This type of converter produces a continuous acceleration for all velocity spectrums, which eliminates stride and losses of torque common in mechanical transmissions (Heldt, 1955). This characteristic facilitates its use in conditions that demand a high velocity rate and allows a higher equipment and operation reliability.

The main disadvantages, when compared to other types of transmissions, are weight and the high production cost.

2.4 Hydraulic transmission

In a hydrostatic transmission the mechanical energy of the drive shaft is transformed in pressure energy in an almost incompressible fluid and is then converted in to mechanical energy on the driven shaft. Essentially, the hydrostatic transmission consists in a bomb on the drive shaft that delivers fluid to the driven shaft (Thoma, 1964). The flow, produced with a determined entrance velocity, is variable and depends on the selected displacement. Afterwards, a constant displacement that provides the output shaft its corresponding velocity according to the flow is fad.

The characteristic of this transmission is that the energy is transferred though the static energy of the fluid and not by the cinematic energy associated with the fluid's velocity (Thoma, 1964). It means that the static pressure is way higher than the dynamic pressure corresponding to the fluid's high velocities.

An advantage is that with a simple adjustment on the input flow, the mean velocity can be continuously selected. This means that the output velocity can be maintained independent of the output torque.

The limitations of these transmissions are vast because of its operational method regarding the conversions from rotational energy in to pressure energy and vice-versa.

Special Note: Continuously Variable Transmission or CVT

CVT’s are all transmissions that provide an infinite number of ratios in a space between two finite ratio limits (Carbone, et. al., 2006). Even though a continuously variable transmission can also be hydraulic and electric, not just belt and chain drive, the authors chooses to study only belt, V-belt and chain belt CVT. A further look on hydraulic and electric CVT’s will be discussed on another paper. Belt and chain CVT’s employ a system that use a centrifugal sensor in one or in both pulleys, torque sensors can also be used in the driven pulley, according to the need of the application. In toroidal CVTs, hydraulic pistons are used to actuate the rolls, changing the transmission ratio.

The necessity of reducing the carbon foot print imposed by legislations all over the world, and by users, has boosted the use of CVT in motor vehicles for they provide reductions in fossil fuels consumption to up to 10%. Especially in hybrid vehicles, the use of a CVT can be an excellent choice to deal with the torque variations imposed by the power sources. The use of V-belt CVT is very common in motorcycles, especially in low power motors and in scooters.

3. SELECTION CRITERIA METHOD

The selection of a transmission system is not at all obvious. A proper transmission has to be designed considering input power and torque, the need for changing the velocity ratio and the desirable output properties. As seen in the previous sections, there are many applications for these systems and there are many types that can be used in each of the applications discussed. So, transmission systems require a great knowledge of the design engineer. This translates to engineers that are specialists in one type or one application, since it is very difficult for an engineer to understand all systems to the button.

Therefore, studies in selection methods or models are held to improve this subject. One study in particular discusses the selection of laws that govern the motion on electric drive systems and a method to select the proper drive train for it. In these studies, Cusimano proposes that the input and output moment of inertia be considered as the main criteria to select a transmission system for a robotic arm (Cusimano, 2001, 2011, 2013 and 2015). He establishes that for each motor and transmission, in each moment, there is a rotational balance at the joint of the arm as in equation 4. He then suggests resolving the selection problem with the creation of a diagram that graphically shows which motor can overpower the required load to move the mechanical arm as seen in figure 5. Cusimano's work is precise, but it is only concerned with a specific application.

$$\frac{M_m(t)}{\tau} = \frac{J_M}{\tau^2}\alpha_L(t) + M_L(t) \tag{4}$$

Where M_m - motor’s torque; M_L - load’s torque; τ - transmission ratio; α_L - load’s acceleration; J_M - motor’s moment of inertia

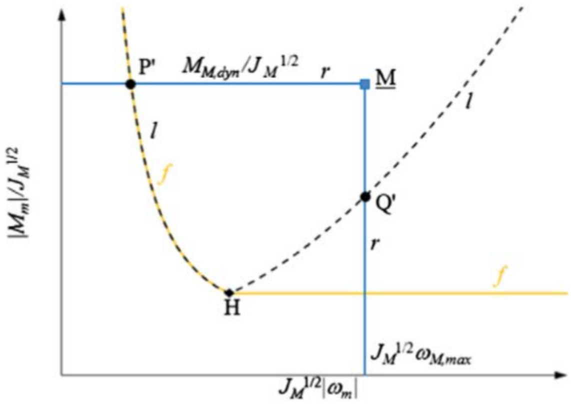


Figure 5: Torque peak curve l and rectangular dynamic operating range of the drive system.
L – torque peak curve; PQ – feasibility arc; M – specific motor (Cusimano, 2015).

So by considering a constant acceleration law, the appropriate choice of a load´s motion law is written as.

$$M_m = \left(\frac{J_M}{\tau} + J_L \tau \right) \alpha_L \tag{5}$$

Where J_L - load's moment of inertia

To consider different applications, a selection method has to be able to answer the output needs and the power train restrictions. Therefore, considering axiomatic design, this work proposes a further study on how such method should work. The procedure is to be made from the input data and has to consider coupling, load requirements, transmission efficiency and available space issues (fig. 6). So that the output data be the transmission type with its formulations and, if possible, indicate other possible transmissions that could deliver similar outputs.

In the axiomatic design, complexity is the function of the relationship between project and system variance, which is affected by the relationship between functional requirements (Suh, 2005). That can be seen in figure 7 and, as in Cusimano's work, can help in the decision making of the proper transmission. But, axiomatic design can also help understanding the needs for further investigation on the design project. Since it differentiates complexity in Real Complexity, Imaginary Complexity, Combinatory Complexity and Periodic Complexity, it is possible to have a more embracing look on the problem.

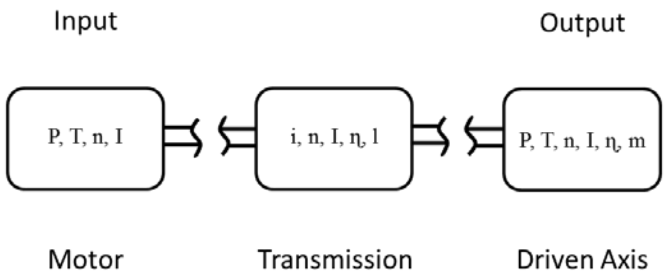


Figure 6: Proposed system architecture

- Real Complexity: it is when the probable design function of the system variance is not completely inside the project variance; there is a finite complexity called real complexity. Real complexity is equal to the information content (Suh, 2005).

$$C_R = I_{sys} \tag{6}$$

Where, C_R - real complexity of the system and I_{sys} - information content

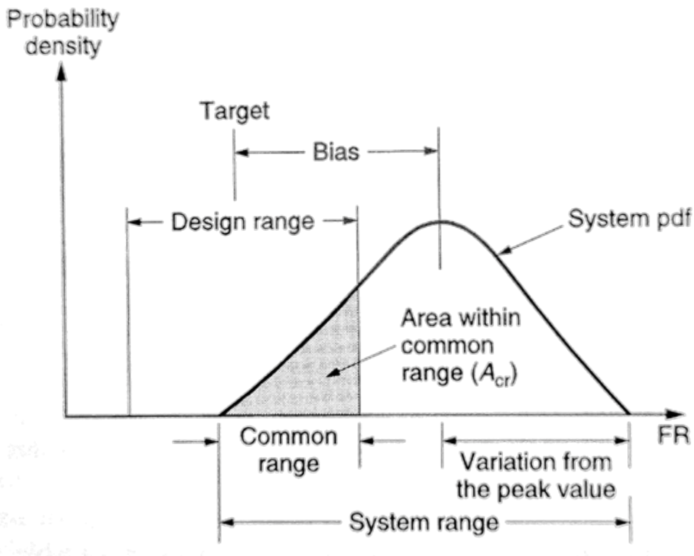


Figure 7: Probability density of a functional requirement (Suh, 2005).

- Imaginary Complexity: is not real, but it exists due to the lack of understanding on the system, its architecture and/or its behavior. Imaginary complexity can exist even when real complexity is equal zero.
- Combinatory Complexity: exists when the system variance changes with time. It can enter a chaotic state.
- Periodic Complexity: given a function (geometric, chemic, etc) can create itself periodically and re-initiate the system.

Suh (2005) states that, "design involves interplay between 'what we want to achieve' and 'how we choose to satisfy the need.'" So, the design consists of four domains, customer, functional, physical and process domain. These are set in an order so that the engineer is able to map from the left domain to a domain on its right (fig. 8).

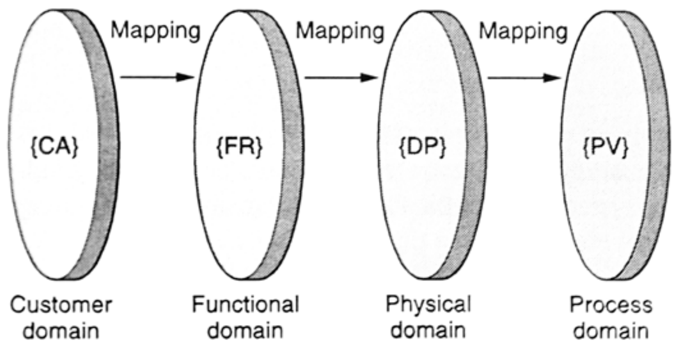


Figure 8: Four domains of the design (Suh, 2005).

In the study here proposed, the attributes of the transmission will be characterized by the customer domain, i. e. the answers the method will deliver. In the functional domain, the system input and output will be considered to satisfy the system requirements and constrains. Design parameters will be addressed in the physical domain following to the process domain, where system variables will be characterized.

Functional requirements are defined as the minimum set of independent requirements that completely characterize the functional needs of the product in the functional domain, whereas complexity is the function of the relationship between design and system variance – which is affected by the relationship between functional requirements (Suh, 2005). The first axiom, or Independent Axiom, formulates that the functional requirements independence has to be always maintained (Kulag, et. al., 2010). It is important to emphasize the need to perform a careful investigation of the independent requirements in order to minimize future errors related to functional requirements (Thompson, 2013).

Expressing the process mathematically, it is possible to define the system vectors. Hence a set of functional requirements for a specific system constitutes the $\{FR\}$ vector in the functional domain. The same can be applied to the design parameters (DP) and to the design matrix.

$$\{FR\} = [A]\{DP\} \qquad (7)$$

Where $[A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$ (8)

Where, $\{DP\}$ - design parameter vector; $[A]$ - design matrix

4. CONCLUDING REMARKS

As being one of the classical studies in mechanical engineering, mechanical transmission of torque and power has been discussed and developed for centuries. And, as the authors opinion, it will still be discussed for a long time. As a mechanism that has to be adapted for many applications and usages, the mechanical transmission will always face new designs and redesigns to properly adjust to new machine requirements.

In the face of a new era, where time is not on the engineer's side, a methodology that helps on the decision making for the best transmission to be used is more than welcome. Although many approaches to this issue can be addressed, the complexity theory is one that can give a mathematic answer to the selection method. Thus, empowering the decision making with a universal answer that can be used anywhere.

A deeper study on the mathematical procedures that will govern the system vectors and the elements of the design matrix has to be further developed. Also, the hierarchy that will preside the criteria for the proper transmission system selection is still the author's ongoing work, but the knowledge and tools exist as shown in this work.

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