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ANALYTICAL SOLUTION MULTILAYER HEAT CONDUCTION: APPLICATION IN COATED TOOLS

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Abstract. *This work is dedicated to obtaining analytic solution in heat conduction through Green functions (FG), arising from a one-dimensional transient heat problem with multilayer medium thermal analysis of a machining tool with coating. It is concluded that the made the presence of the thermal properties of the coating material of cobalt carries in a temperature increase, while aluminum oxide (Al_20_3) coating and Titanium Nitride (TiN) induces a decrease of temperature in the tool-coating interface.*

Keywords: *Green functions, Multilayer, Conduction heat, Coated Tools*

1. INTRODUCTION

The thermal and mechanical behavior are extremely important in manufacturing processes. The tools are key to the success of any manufacturing process from the point of view of final quality of the finished material and the economic point of view which occupies in the production chain.

The technological evolution of the production of tools led to the development and application of coatings on tools facilitating the cutting by rubbing through performance tribological mechanisms. However, the application of an extra layer of a different material (coating) tool material (base or substrate) should also consider the thermal behavior, since it is the coating that will be in contact with the workpiece. Typically, these coatings have thicknesses of millimeters second (Machado *et al.*, 2011) and order your thermal analysis becomes delicate.

In this sense, the objective of the present work is the research and development of analytical solution for heat conduction in multi-layer media, or also called compounds, using the technique of FG and analyze the temperature at the interface in the interface (tool-coating).

It is proposed here to obtain the analytical solution for a two-layer thermal conduction problem using the Green's Functions (GF) method. An advantage in the use of integral solutions by GF is the possibility to build multidimensional solutions from 1D cases without additional difficulties. It can be observed, in this sense, that 2D and 3D solutions are completely equivalent to 1D equation, and therefore they can be obtained from product solutions in different directions 1D (Fernandes, 2009).

The use of analytic solutions is important in various aspects of modeling, since it offers robustness, accuracy, reduced computational expense and higher must really trust in its estimates of thermal parameters (Oliveira, 2015).

2. MATHEMATICAL MODEL

2.1 One-dimensional transient thermal problem $X2C12$

The problem of heat conduction $1D$ $X2C12$ shown in Fig. 1 describes a two layer plate accounting for the heat flux by using $g(x,t) = q(t)\delta(x-0)$ with isolate condition at the ends and can be written as

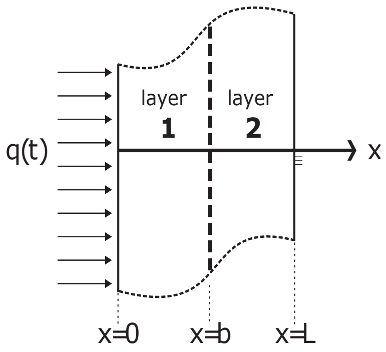

Figure 1. Two-layer plate subjected to the heat flux, $q(t)$ at $x = 0$, while the opposite surface, $x = L$, is isolated

The problem represented by Fig. 1 is given by:

$$\frac{\partial^2 T_1}{\partial x^2} + g(x,t) = \frac{1}{\alpha_1}\frac{\partial T_1}{\partial t} \tag{1a}$$

$$\frac{\partial^2 T_2}{\partial x^2} = \frac{1}{\alpha_2}\frac{\partial T_2}{\partial t} \tag{1b}$$

Subject to the boundary conditions

$$-k_1\frac{\partial T_1}{\partial x}\bigg|_{x=0} = 0; \qquad -k_2\frac{\partial T_2}{\partial x}\bigg|_{x=L} = 0 \tag{1c}$$

and to the continuity conditions

$$T_1|_{x=b} = T_2|_{x=b}\,; \qquad -k_1\frac{\partial T_1}{\partial x}\bigg|_{x=b} = -k_2\frac{\partial T_2}{\partial x}\bigg|_{x=b} \tag{1d}$$

And to the initial condition

$$T_1(x,0) = T_2(x,0) = F(x) = T_0 \tag{1e}$$

The solution in each region i is then written as

$$\begin{aligned} T_i(x,t) = \sum_{j=1}^{M} \Bigg\{ & \int_{x_j}^{x_{j+1}} G_{ij}(x,t|x',0)F_j(x')dx' \\ & + \alpha_j \int_0^t \int_{x_j}^{x_{j+1}} G_{ij}(x,t|x',\tau)\frac{g_j(x',\tau)}{k_j}dx'd\tau \Bigg\} \end{aligned} \tag{2}$$

where $x_j \leq x \leq x_{j+1}$, to $j = 1, 2, ..., M$, are the limits of each layer, and $G_{ij}(x,t|x',\tau)$ is the Green's function for the two-layer body.

If $M = 1$, the single layer solution can be given by Eq. (3). It means

$$T_{(1)}(x,t) = \alpha_{(1)} \int_0^t \int_{x_1}^{x_2} G_{(11)}(x,t|x',\tau)\frac{g_{(1)}(x',\tau)}{k_{(1)}}dx'd\tau \tag{3}$$

If $M = 2$, $0 \leq x \leq b$ and $b \leq x \leq L$, the solutions for T_1 e T_2 given by Eqs. (4) and (5) respectively are given by

$$\begin{aligned} T_1(x,t) = {} & \alpha_1 \int_0^t \int_{x_1}^{x_2} G_{11}(x,t|x',\tau)\frac{g_1(x',\tau)}{k_1}dx'd\tau \\ & + \alpha_2 \int_0^t \int_{x_2}^{x_3} G_{12}(x,t|x',\tau)\frac{g_2(x',\tau)}{k_2}dx'd\tau \end{aligned} \tag{4}$$

and

$$T_2(x,t) = \alpha_1 \int_0^t \int_{x_1}^{x_2} G_{21}(x,t|x',\tau)\frac{g_1(x',\tau)}{k_1}dx'd\tau + \alpha_2 \int_0^t \int_{x_2}^{x_3} G_{22}(x,t|x',\tau)\frac{g_2(x',\tau)}{k_2}dx'd\tau \tag{5}$$

It can be observed that $g(x,t) = q(t)\delta(x-0)$, therefore the second term of the Eq.(4) and Eq.(5) are null.
The Green function G_{ij} is given by Haji-Sheikh and Beck (2002)

$$G_{ij}(x,t|x',\tau) = \sum_{n=1}^{\infty} e^{-\lambda_n^2(t-\tau)}\frac{1}{N_x}X_{in}(x)X_{jn}(x'), \tag{6}$$

where X_{in} and X_{jn} are the eigenfunctions and λ_n eigenvalues.
The N_n norm is defined by

$$N_x = \sum_{j=1}^{M}\int_{x_j}^{x_{j+1}} \left[X_{jn(x')}\right]^2 dx' \tag{7}$$

Therefore, the temperature solution in the range $[x_1, x_2]$ has the following form:

$$\begin{aligned} T_1(x,t) &= \frac{\alpha_1}{k_1}\sum_{n=1}^{\infty}\frac{X_{1n}}{N_x}\int_0^t e^{-\lambda_n^2(t-\tau)}\int_{x_1}^{x_2} X_{1n}(x')q(t)\delta(x'-0)dx'd\tau \\ &= \frac{\alpha_1}{k_1}\sum_{n=1}^{\infty}\frac{X_{1n}(x)X_{1n}(0)}{N_x}\int_0^t q(t)e^{-\lambda_n^2(t-\tau)}d\tau \end{aligned} \tag{8}$$

and in the range $[x_2, x_3]$:

$$\begin{aligned} T_2(x,t) &= \frac{\alpha_1}{k_1}\sum_{n=1}^{\infty}\frac{X_{2n}}{N_x}\int_0^t e^{-\lambda_n^2(t-\tau)}\int_{x_1}^{x_2} X_{1n}(x')q(t)\delta(x'-0)dx'd\tau \\ &= \frac{\alpha_1}{k_1}\sum_{n=1}^{\infty}\frac{X_{2n}(x)X_{1n}(0)}{N_x}\int_0^t q(t)e^{-\lambda_n^2(t-\tau)}d\tau \end{aligned} \tag{9}$$

The eigenfunctions $X_1 = X_{1n}(x)$ and $X_2 = X_{2n}(x)$ and respective eigenvalues are obtained using separation of variables method. In this sense, assuming a separation of $T(x,t)$ in two spatially and time dependent functions of a single variable each in the form

$$T_1(x,t) = X_1(x)\Gamma_1(t) \;\; and \;\; T_2(x,t) = X_2(x)\Gamma_2(t) \tag{10a}$$

Substituing Eq. (10a) in Eqs. (1a) and Eq. (10a) in Eqs. (1b) yelds

$$\frac{\partial^2 X_1}{\partial x^2} + \gamma^2 X_1 = 0 \;\; and \;\; \frac{\partial^2 X_2}{\partial x^2} + \eta^2 X_2 = 0 \;\; where \;\; \gamma^2 = \frac{\lambda^2}{\alpha_1} \;\; and \;\; \eta^2 = \frac{\lambda^2}{\alpha_2} \tag{11}$$

And the solutions for X_1 and X_2 are

$$X_1 = Acos(\gamma x) + Bsen(\gamma x) \;\; e \;\; X_2 = Ccos(\eta x) + Dsen(\eta x) \tag{12a}$$

It can be observed that Eq. (12a) must satisfy the boundary condition at $x = 0$

$$-k_1 \frac{\partial X_1}{\partial x}\bigg|_{x=0} = 0 \tag{13}$$

After substituting the eigenfunction X_1 Eq (12a) in Eq (13) and solving the expression, we obtain $B = 0$ and without loss of generality it is concluded that $A = 1$ (Özişik, 1993). Soon,

$$X_1 = cos(\gamma x) \tag{14}$$

Then the boundary conditions at $x = b$ must be satisfied , it means

$$X_1|_{x=b} = X_2|_{x=b}\,; \qquad -k_1 \left.\frac{\partial X_1}{\partial x}\right|_{x=b} = -k_2 \left.\frac{\partial X_2}{\partial x}\right|_{x=b} \tag{15}$$

After substituting X_1 and X_2 in equation (15) yelds

$$cos(\gamma b) - C cos(\eta b) - D sen(\eta b) = 0 \tag{16}$$

and

$$-\left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) + C sen(\eta b) - D cos(\eta b) = 0 \tag{17}$$

As the boundary condition at $x = L$ is

$$-k_2 \left.\frac{\partial X_2}{\partial x}\right|_{x=L} = 0 \tag{18}$$

after substituing X_2 in Eq. (18) , Eqs., (16), (17) and (18) , can be written in matrix form as

$$\begin{bmatrix} cos(\gamma b) & cos(\eta b) & sen(\eta b) \\ -K sen(\gamma b) & sen(\eta b) & -cos(\eta b) \\ 0 & -\eta sen(\eta L) & \eta cos(\eta L) \end{bmatrix} \begin{bmatrix} 1 \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \;; where K = \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) \tag{19}$$

Coefficients C and D are then determined by solving the linear system given by equation (19). It means

$$C = cos(\eta b) cos(\gamma b) + \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) sen(\eta b) \tag{20a}$$

$$D = cos(\gamma b) sen(\eta b) - \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) cos(\eta b) \tag{20b}$$

Since all coefficient have been calculated the eigenfunctions coefficients X_1 and X_2 are given by

$$\begin{aligned} X_1 &= cos(\gamma x) \\ X_2 &= \left[cos(\eta b) cos(\gamma b) + \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) sen(\eta b)\right] cos(\eta x) \\ &\quad + \left[cos(\gamma b) sen(\eta b) - \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) cos(\eta b)\right] sen(\eta x) \end{aligned} \tag{21}$$

And temperature solution can be given by

$$\begin{aligned} T_1(x,t) &= \frac{\alpha_1}{k_1} \sum_{n=1}^{\infty} \frac{X_{1n}}{N_x} \int_0^t e^{-\lambda_n (t-\tau)} \int_{x_1}^{x_2} X_{1n}(x') q(t) \delta(x'-0) dx' d\tau \\ &= \frac{\alpha_1}{k_1} \sum_{n=1}^{\infty} \frac{X_{1n}(x) X_{1n}(0)}{N_x} \int_0^t q(t) e^{-\lambda_n (t-\tau)} d\tau \\ &= \frac{\alpha_1}{k_1} \sum_{n=1}^{\infty} \frac{cos(\gamma x) cos(0)}{N_x} \int_0^t q(t) e^{-\lambda_n (t-\tau)} d\tau \end{aligned} \tag{22a}$$

$$\begin{aligned} T_2(x,t) &= \frac{\alpha_1}{k_1} \sum_{n=1}^{\infty} \frac{X_{2n}}{N_x} \int_0^t e^{-\lambda_n (t-\tau)} \int_{x_1}^{x_2} X_{1n}(x') q(t) \delta(x'-0) dx' d\tau \\ &= \frac{\alpha_1}{k_1} \sum_{n=1}^{\infty} \frac{X_{2n}(x) X_{1n}(0)}{N_x} \int_0^t q(t) e^{-\lambda_n (t-\tau)} d\tau \\ &= \frac{\alpha_1}{k_1} \sum_{n=1}^{\infty} \frac{1}{N_x} \left\{ \left[cos(\eta b) cos(\gamma b) + \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) sen(\eta b)\right] cos(\eta x) \right. \\ &\quad \left. + \left[cos(\gamma b) sen(\eta b) - \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) cos(\eta b)\right] sen(\eta x) \right\} cos(0) \\ &\quad \times \int_0^t q(t) e^{-\lambda_n (t-\tau)} d\tau \end{aligned} \tag{22b}$$

where the norm, N_x is given by:

$$\begin{aligned} N_x &= \int_{x1}^{x2} [X_{1n}(x')]^2 dx' + \int_{x2}^{x3} [X_{2n}(x')]^2 dx' \\ &= \int_0^b [cos(\gamma x')]^2 dx' + \int_b^L \left\{ \left[cos(\eta b) cos(\gamma b) + \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) sen(\eta b) \right] cos(\eta x') \right. \\ &+ \left. \left[cos(\gamma b) sen(\eta b) + \left(\frac{k_1}{k_2}\right)\left(\frac{\gamma}{\eta}\right) sen(\gamma b) cos(\eta b) \right] sen(\eta x') \right\}^2 dx' \end{aligned} \tag{23}$$

The equation for determination of the eigenvalues is obtained from the requirement that in Eq. (19) the determinant of the coefficients should vanish. Then, the eigenvalues are roots of the following transcendental equation:

$$\begin{vmatrix} cos(\gamma b) & cos(\eta b) & sen(\eta b) \\ -K sen(\gamma b) & sen(\eta b) & -cos(\eta b) \\ 0 & -\eta sen(\eta L) & \eta cos(\eta L) \end{vmatrix} = 0 \;\; or \;\; tan(\gamma b) = -K tan[\eta(b-L)] \tag{24}$$

Equation (24) can be solved by using various mathematical methods. (Beck, 1992) and (Haji-Sheik and Beck, 2000) present solutions to the transcendental equation based on asymptotic approximations. For each eigenvalue, there are well-defined upper and lower values for η within which only one eigenvalue is located. These limits are the ordered locations of the asymptotes for the right side and for the left side of each equation. The asymptotes will be located where the denominators of the right side and of the left side become zero. Figure (2), shows the behavior of these asymptotes.

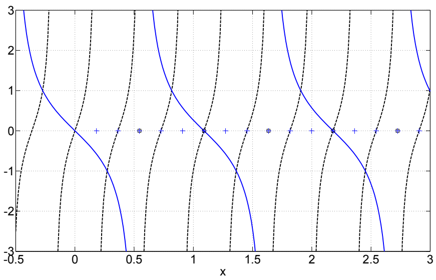

Figure 2. Representation of the asymptotes curves for η

3. VERIFICATION OF ANALYTICAL SOLUTION

The solution of Eq. (22) is shown in Fig. 3 considering the heat flux prescribed equal 4×10^5 $[W/m^2]$, $T_0 = 0$ $[^oC]$, $L = 5 \times 10^{-2}$ $[m]$, $b = L/2$ $[m]$, $\alpha_1 = 18,8 \times 10^{-6}$, $\alpha_2 = 117 \times 10^{-6}$, $k_1 = 64$ $[W/mk]$ and $k_2 = 401$ $[W/mk]$.

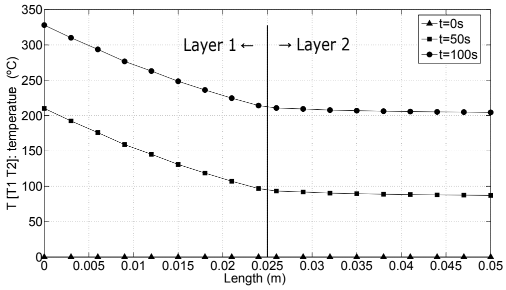

Figure 3. Spatial temperature for three different times. Two-layer body

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The two-layer solution can be verified by considering the one layer solution (Fig. 4). It means if both layers have the same thermal properties the thermal problem can be described as a one single layer problem. For comparison, a single layer solution is presented (Fig. 4) . These results are then compared with the solution of two-layer case, but considering the same thermal properties (Fig. 4). In both cases the same parameter are used. It means, $q(t) = 4 \times 10^5$ $[W/m^2]$, $T_0 = 0$ $[^oC]$, $L = 5 \times 10^{-2}$ $[m]$, $b = L/2$ $[m]$, $\alpha_1 = 18,8 \times 10^{-6}$, $\alpha_2 = 18,8 \times 10^{-6}$, $k_1 = 64$ $[W/mk]$ and $k_2 = 64$ $[W/mk]$.

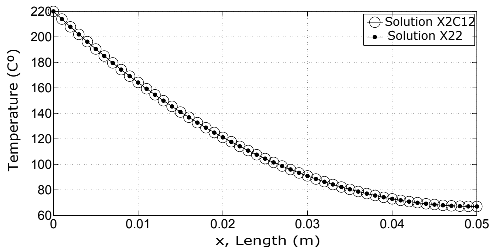

Figure 4. Spatial temperature for three different times. Single layer body.

The absolute deviate between the solutions of the problem $X22$ and the problem $X2C12$ (Fig. 4) is shown in (Fig. 5). It can be observed a maximum difference of $0,14(C)$ with a percentage error of $0,04\%$.

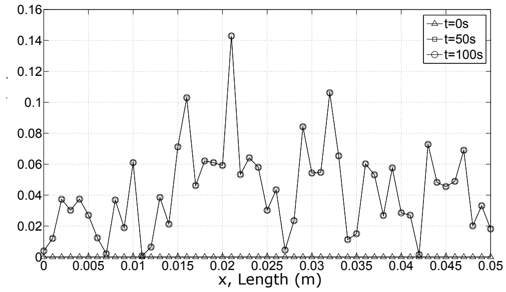

Figure 5. Absolute error between the problems $X22$ and $X2C12$.

4. THERMAL ANALYSIS TOOL-COATING INTERFACE

To perform this analysis is considered a hard metal tool ISO K 10 whose thermal properties are in (1). It is also considered three types of coatings. The goal of this thermal analysis is to verify the physical term parameters involved in the analysis, enabling a better temperature distribution on the interface tool/flooring, decreasing the stresses present in the process and in order to increase tool life.

It presents, in table (1), the thermal properties of the tools used in these analyses and the three types of coatings, TiN, cobalt and Al_20_3 . These properties are, respectively, in (Brito *et al.*, 2009), (Rech *et al.*, 2004) and (Du *et al.*, 2000).

Table 1. Term physical properties of the tool and of TiN, cobalt and Al_20_3 .

Properties	Tool	TiN	Cobalt	Al_20_3
$\alpha \times 10^{-5}$ $[m^2/s]$	$4,36$	$0,7$	$2,66$	$0,76$
k $[W/mK]$	130	21	$99,2$	36

As the usual thickness of a coating varies between $4\mu m$ and $12\mu m$ according to (Machado *et al.*, 2011), This analysis will consider the coating thickness equal to $10\mu m$ and the conditions of: constant heat flux $q = 25 \times 10^5$ $[W/m^2]$, board length $L = 0.030$ $[mm]$ and a time 10 to seconds.

Fig. (6) shows the temperature profile of the uncoated tool and the three types of coating tools throughout the domain.

It is observed that the thermal behavior differences between uncoated tool and coated tools are more pronounced in the covering. This difference, however, decreases enough to approach the material base of the tool.

The Tabs. (2)-(3)-(4) show these differences in a few distinct points of the domain, including for $x = 0.010$ $[mm]$ which is the tool/coating interface.

It is observed that the TiN coating has the highest thermal insulation effect, leading to a temperature difference in

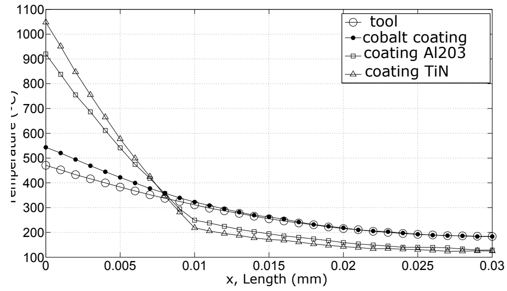

Figure 6. Thermal analysis between tool and coated tool coated with three different coat types.

Table 2. Temperature difference between tool and coated with $Al_2 0_3$.

Cases	Positions Approximate [mm]	Temperature of the Tool (C)	Temperature of the Coated tool (C)	Difference of Temperature (C)
1	0	$470,6$	$919,9$	$449,3$
2	$0,010$	$311,5$	$249,3$	$62,2$
3	$0,015$	$265,4$	$202,8$	$4,1$
4	$0,030$	$183,7$	$128,8$	$54,9$

Table 3. Temperature difference between tool and coated cobalt coated.

Cases	Positions Approximate [mm]	Temperature of the Tool (C)	Temperature of the Coated tool (C)	Difference of Temperature (C)
1	0	$470,6$	$543,4$	$72,8$
2	$0,010$	$311,5$	$322,6$	$11,1$
3	$0,015$	$265,4$	$269,5$	$3,6$
4	$0,030$	$183,7$	$183,6$	$0,1$

Table 4. Temperature difference between tool and coated with TiN coating.

Cases	Positions Approximate [mm]	Temperature of the Tool (C)	Temperature of the Coated tool (C)	Difference of Temperature (C)
1	0	$470,6$	$1048,0$	$577,4$
2	$0,010$	$311,5$	$218,4$	$93,1$
3	$0,015$	$265,4$	$177,8$	$87,6$
4	$0,030$	$183,7$	$124,9$	$58,8$

relation to the base metal $577,4[C]$ Tab. (4)). Although this effect decreases enough to approach the interface, it is still present causing a difference of temperature $93,1[C]$ Tab. (4)).

This behavior, however is not observed by the Cobalt coating. Thermal insulation effect is not present on the interface. Table (3) show that coating interface/tool temperature, for the case of this coating, it's bigger than that for uncoated tool.

5. CONCLUSIONS

A temperature solution for a two-layer body has been presented. The analytical solution has been verified by using one layer case. Calculating the spatial eigenvalues is the main difficulty that one must consider when dealing with problems of this type. The procedure presented here can be extended for multilayer bodies.

It has been shown that the cobalt coatings, presentead increase in temperature, while the aluminum oxido ($Al_2 0_3$) and titanium (TiN) showed a decrease os temperature in the tool-coating interface. One of the causes that led to this result is due to thermal conductivity of materials.

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