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SLIDING MODE CONTROL VIA ALQR FOR UNCERTAIN SYSTEMS: AN LMI APPROACH

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Abstract. *This paper proposes a robust nonlinear control design strategy for a class of systems with parametric uncertainties subject to unknown exogenous inputs. Linear Matrix Inequality (LMI) conditions are derived in order to obtain a nonlinear controller that ensures stability and performance of the closed loop system. The design problem has been formulated using a consistent framework for robust Sliding Mode Control (SMC) via Amplified Linear Quadratic Regulator (ALQR). Typically, systems with standard ALQR controllers present enhanced stability margins. However, the standard ALQR does not ensure robust stability when model uncertainties are incorporated. In order to solve this problem, the ALQR is formulated via LMIs to cope with the parametric uncertainties. Exogenous inputs, on the other hand, are not dealt with by the ALQR. As a means to address this issue, a novel method using the SMC via ALQR is provided. The effectiveness of the design method was evaluated considering a mass-spring system with two carts connected to each other through a linear spring, with only one actuator. The control objective is to track a position reference and ensure performance and stability in spite of disturbances, noise and unmodeled dynamics (dry friction). The experiments showed that nonlinear controllers using SMC via ALQR based on LMIs may lead to advantageous results over classical design methods.*

Keywords: *Nonlinear Control, SMC, ALQR, LMI, Parametric Uncertainties*

1. Introduction

In recent years, nonlinear control systems have received considerable attention in the literature, as can be seen by a significant number of papers (Mohanty, 2008), (Joshi and Thakar, 2012), (Barzamini and Shafiee, 2012). This interest in the analysis and design of nonlinear control systems is due to unsatisfactory performance of linear controllers when applied to plants with accentuated nonlinearity or nonlinear plants operating over a wide operating range (Khalil, 2002). Therefore, nonlinear techniques that overcome this complexity have been developed. One potential approach, termed Sliding Mode Control (SMC) has been applied to a wide class of situations, for example, motor control, robotic control and flight control (Shyu *et al.*, 2000), (Qian *et al.*, 2007).

The sliding mode control technique is a variable structure control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to "slide" along a cross-section of its normal behavior (Utkin, 1978). However, this methodology proposed by Utkin (1978) showed some problems related to high gains and especially the existence of oscillations in high frequencies making its application infeasible in practice. To avoid such oscillations Slotine and Sastry (1983) proposed the smoothing of the switched term of control. With this modification, the sliding mode controller demonstrated to be robust and eliminated the problems related to some model uncertainty.

Several structure variants of sliding mode controllers have been investigated in order to increase the stability and consequently the robustness of the nonlinear controller (Slotine and Li, 1991), (Shyu and Shieh, 1996), (Utkin, 1977), (Hai Yu and Ozguner, 2006). In this context, the merit of this paper consists in extending the procedure of sliding mode controller design proposed by (Yu and Liu, 2005) using Amplified Linear Quadratic Regulator (ALQR) for uncertain systems. It is known that the standard ALQR using Riccati does not ensure robust stability when model uncertainties are incorporated (Prakash, 1990). Thus, sufficient conditions based on Linear Matrix Inequalities (LMIs) for the existence of a sliding mode control via ALQR controller to which parametric uncertainties are incorporated are provided. This numerical technique has several advantages. While the algebraic solution can only be applied to one system, the numerical procedure can take into account multiple systems, i.e., uncertain systems at different operation points (Feron *et al.*, 1992), (Boyd *et al.*, 1994), (Olalla *et al.*, 2009). This feature allows the practicing engineer to combine the properties of ALQR control with uncertainties. As a result, the controller is not optimal as in the nominal case, but provides a guaranteed cost. The envisioned application scenario presents a system with two carts connected to each other through a linear spring, with only one actuator. The control objective is to track a position reference and ensure performance and stability in spite of disturbances, noise and unmodeled dynamics (dry friction).

This paper is organized as follows. In Section 2 a solvability condition is established for existence of solution of the sliding mode control via ALQR controller to which parametric uncertainties are incorporated. In Section 3, the case study used to evaluate the effectiveness of the proposed method is presented. Section 4 contains the experimental results and discussions. Finally, Section 5 presents the conclusion.

2. Sliding mode control via ALQR for uncertain systems

Herein the SMC via ALQR method for uncertain systems is considered. The design consists of two sequential steps, the first step is characterized by calculating the constant ALQR-gain matrix, K_r, that specifies stability and robustness performance. In the second step, the sliding mode control is added to cope with unmodeled effects.

2.1 Amplified Linear Quadratic Regulator for Uncertain Systems - LMI Approach

Consider an uncertain linear time invariant system, described by the state equation

$$\dot{x}(t) = A(\alpha)\, x(t) + B(\alpha)\, u(t), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector and $A(\alpha)$, $B(\alpha)$ are not precisely known matrices, which belong to a convex bounded (polytope type) uncertain domain Θ given by,

$$\Theta = \left\{ A(\alpha):\ A(\alpha) = \sum_{i=1}^{N} \alpha_i A_i;\quad B(\alpha):\ B(\alpha) = \sum_{i=1}^{N} \alpha_i B_i, \alpha \in \Phi \right\} \tag{2}$$

with

$$\Phi = \left\{ \alpha \in \mathbb{R}^N,\ \sum_{i=1}^{N} \alpha_i = 1;\ \alpha_i \geq 0 \right\}. \tag{3}$$

Any uncertain pair of matrices $(A(\alpha), B(\alpha))$ in Θ can be described as a convex combination of the vertices (A_i, B_i), $i = 1, ..., N$ of the polytope (Boyd *et al.*, 1994). The optimal ALQR is obtained using the state-feedback gain K_r ($u = -K_r x$) that minimizes the performance index

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt \tag{4}$$

where $Q \in \mathbb{R}^{n\times n}$ is a symmetric and positive semidefinite ($Q \geq 0$) matrix and $R \in \mathbb{R}^{m\times m}$ is a symmetric and positive definite ($R > 0$) matrix (Prakash, 1990). Considering the state feedback with gain K_r, the performance index can be rewritten as

$$J = \int_0^\infty x^T \left(Q + K_r^T R K_r\right) x dt. \tag{5}$$

Since the $Tr(\cdot)$ possesses the property $a^T M b = Tr\left(M b a^T\right)$ (Olalla *et al.*, 2009), (5) can be recast as

$$J = \int_0^\infty Tr\left(\left(Q + K_r^T R K_r\right) x x^T\right) dt = Tr\left(\left(Q + K_r^T R K_r\right) P\right), \tag{6}$$

where $P = \int_0^\infty x x^T$ is a positive definite symmetric ($P > 0$) matrix that satisfies

$$(A_i + B_i K_r) P + P (A_i + B_i K_r)^T + x_0 x_0^T < 0, \tag{7}$$

in which x_0 is the initial state condition. Moreover, using the cyclic property of the trace operator:

$$Tr\left(K_r^T R K_r P\right) = Tr\left(P K_r^T R K_r\right) = Tr\left(K_r P K_r^T R\right). \tag{8}$$

Now, since the trace is invariant to a similarity transform, one may multiply the argument in (8) to the left by $R^{1/2}$ as and to the right by $R^{-1/2}$, and the following equality holds

$$Tr\left(K_r P K_r^T R\right) = Tr\left(R^{1/2} K_r P K_r^T R R^{-1/2}\right). \tag{9}$$

Therefore, in view of (8) and (9), the minimization of J given as in (6) can be restated as

$$\begin{array}{l} \min\limits_{P,K_r} Tr\left(QP\right) + Tr\left(R^{1/2} K_r P K_r^T R^{1/2}\right) \\ s.t \\ \left(A_i + B_i K_r\right) P + P\left(A_i + B_i K_r\right)^T + x_0 x_0^T < 0. \end{array} \tag{10}$$

The objective function in (10) involves the product of the variables P and K_r, thus complicating the minimization. In order to circumvent this hardship and obtain the amplification of the nominal LQR, a new variable Y is introduced, which satisfies $K_r = \frac{1}{\beta} Y P^{-1}$ for a fixed $\beta \in]0, 1[$. From this consideration the ALQR problem can be written as,

$$\begin{array}{l} \min\limits_{P,Y} Tr\left(QP\right) + Tr\left(R^{1/2} \frac{Y}{\beta} P^{-1} \frac{Y^T}{\beta} R^{1/2}\right) \\ s.t \\ A_i P + P A_i^T + \frac{B_i}{\beta} Y + Y^T \frac{B_i^T}{\beta} + x_0 x_0^T < 0. \end{array} \tag{11}$$

In addition, the nonlinear term $Tr\left(R^{1/2} \frac{Y}{\beta} P^{-1} \frac{Y^T}{\beta} R^{1/2}\right)$ can be replaced by a second auxiliary variable X, such that $X > R^{1/2} \frac{Y}{\beta} P^{-1} \frac{Y^T}{\beta} R^{1/2}$. This inequality can successively be decomposed by Schur's complement, resulting in

$$X > R^{1/2} \frac{Y}{\beta} P^{-1} \frac{Y^T}{\beta} R^{1/2} \Leftrightarrow \left[\begin{array}{cc} X & R^{1/2} \frac{Y}{\beta} \\ \frac{Y^T}{\beta} R^{1/2} & P \end{array}\right] > 0. \tag{12}$$

Therefore, the complete LMI formulation for ALQR problem is given by:

$$\begin{array}{l} \min\limits_{P,Y,X} Tr\left(QP\right) + Tr\left(X\right) \\ s.t \\ A_i P + P A_i^T + \frac{B_i}{\beta} Y + Y^T \frac{B_i^T}{\beta} + x_0 x_0^T < 0 \\ \left[\begin{array}{cc} X & R^{1/2} \frac{Y}{\beta} \\ \frac{Y^T}{\beta} R^{1/2} & P \end{array}\right] > 0, \quad P > 0. \end{array} \tag{13}$$

The ALQR controller can then be recovered making $K_r = \frac{1}{\beta} Y P^{-1}$. The main advantage of this strategy consists in the inclusion of uncertainties while the classical ALQR controller is only valid for systems without uncertainties. Moreover, the guaranteed multivariable margins of ALQR can be established, which correspond to the gain/phase perturbations that can be considered in all the outputs simultaneously or in an independent manner (Prakash, 1990) and (Cavalca and Kienitz, 2009). For the particular case with $Q > 0$, Prakash (1990) demonstrates that the ALQR loop will be stable within certain margins, even if the system has poles at the imaginary axis. For this case, the guaranteed multivariable margins are given by:

$$GM = \beta/2 \rightarrow \infty \tag{14}$$

$$PM = \pm \cos^{-1}(\beta/2) \tag{15}$$

in which, GM is the guaranteed gain margin and PM is the guaranteed phase margin.

2.2 Sliding mode control via ALQR

The exposition in this section closely follows (Yu and Liu, 2005). Suppose that the model of (1) in the nominal condition ($A(\alpha) = A$, $B(\alpha) = B$) is subject to additive disturbances $w(x, t)$:

$$\dot{x}(t) = Ax(t) + Bu(t) + w(x, t), \tag{16}$$

with B being a full column rank matrix. Thus, m sliding functions are defined as follows:

$$\Omega_i(x) = \xi_i x = \xi_{i1} x_1 + \xi_{i2} x_2 + \ldots + \xi_{in} x_n, i = 1, 2, \ldots, m, \tag{17}$$

where $\xi_i = \left[\begin{array}{cccc} \xi_{i1} & \xi_{i2} & \ldots & \xi_{in} \end{array}\right]$ is a row vector. The sliding vector is defined by:

$$\Omega\left(x\right) = \Xi x, \tag{18}$$

with $\Omega = \left[\begin{array}{ccc} \Omega_1 & \ldots & \Omega_m \end{array}\right]^T$, $\Xi \in \mathbb{R}^{m \times n}$, and satisfying the requirement $\det(\Xi B) \neq 0$.

From equations (16) and (18), the sliding vector dynamics is obtained as

$$\dot{\Omega} = \Xi \dot{x} = \Xi A x + \Xi B u + \Xi w(x, t). \tag{19}$$

Ensuring $\det(\Xi B) \neq 0$, the following control law is chosen

$$u = -(\Xi B)^{-1}\Xi A x - (\Xi B)^{-1}(\gamma + \sigma)\frac{\Omega}{\|\Omega\|}, \tag{20}$$

where $\gamma = \|\Xi\|\,\delta(x,t)$. Replacing u into (19) one obtains the following expression

$$\dot{\Omega} = -(\gamma + \sigma)\frac{\Omega}{\|\Omega\|} + \Xi w(x,t). \tag{21}$$

Multiplying Ω^T on both sides,

$$\begin{aligned} &\Omega^T\dot{\Omega} = -\sigma\|\Omega\| - \gamma\|\Omega\| + \Omega^T\Xi w(x,t),\\ &\Omega^T\dot{\Omega} = -\sigma\|\Omega\| - \gamma\|\Omega\|\left(1 - \tfrac{\Omega^T\Xi w(x,t)}{\gamma\|\Omega\|}\right),\\ &\Omega^T\dot{\Omega} < -\sigma\|\Omega\|. \end{aligned} \tag{22}$$

which will hold if $\gamma = \|\Xi\|\,\delta(x,t) > \|\Xi w(x,t)\|$.

The sliding condition

$$\Omega^T\dot{\Omega} < 0 \tag{23}$$

is satisfied for $\sigma > 0$, in which case the system enters into the sliding mode $\Omega = 0$ in limited time.

To design the sliding mode controller via ALQR, consider the sliding function

$$\Omega_T(x) = \Xi(x - x(0)) - \Xi(A - BK_r)\int_0^t x\,d\tau \tag{24}$$

$\Omega_T(x)$ is set to zero, to have a sliding surface in the feedback system. It is not necessary to consider whether the system can satisfy the approaching condition (22) or not (Yu and Liu, 2005). The Ξ matrix is chosen to satisfy $\Xi B \neq 0$. For the states to remain on the sliding surface, an extra control signal $-q\,sign(\Omega_T)$ should be added. This is necessary because it will lead to the sliding condition (23) (Yu and Liu, 2005). Thus, the total control value u_T of the SMC via ALQR is designed as

$$u_T = -K_r x - q\,sign(\Omega_T). \tag{25}$$

Taking the time derivative of (24), replacing $\dot{x}$ from (16) with $w = 0$, and making $u = u_T$ from (25) it follows that

$$\dot{\Omega}_T(x) = -q\,sign(\Omega_T). \tag{26}$$

Multiplying Ω_T, on both sides, results in

$$\Omega_T\dot{\Omega}_T(x) = -q\,|\Omega_T|. \tag{27}$$

The constant q must be positive to satisfy the sliding condition (23).

In the presence of an external disturbance w, (27) becomes

$$\Omega_T\dot{\Omega}_T(x) = -q\,|\Omega_T| + w\Omega_T. \tag{28}$$

Therefore, the choice of q must satisfy $q > |w|$ to ensure the sliding condition.

In order to avoid high gain and the chattering phenomenon (Slotine, 1984), the concept of the sliding layer is introduced in the control law (Yu and Liu, 2005), (Shyu and Shieh, 1996), (Utkin, 1977). Thus the saturation function $sat(\Omega_T, \upsilon)$ as defined below will be used instead of $sign(\Omega_T)$

$$sat(\Omega_T, \upsilon) = \begin{cases} 1, & \Omega_T > \upsilon \\ \Omega_T/\upsilon, & |\Omega_T| \leq \upsilon \\ -1, & \Omega_T < -\upsilon \end{cases}. \tag{29}$$

3. Case study and design

To illustrate the application of the control method, a case study was performed using a laboratory plant (Figure 1). The case study included the design steps as well as verification experiments with the actual plant. The system consists of two carts connected to each other through a linear spring, with only one actuator: the electrical motor. The cart with the electrical motor is deemed “active” cart and the other one, “passive” cart. The external force f driving the carts is composed by the motor force and the sum of the friction acting on both carts.

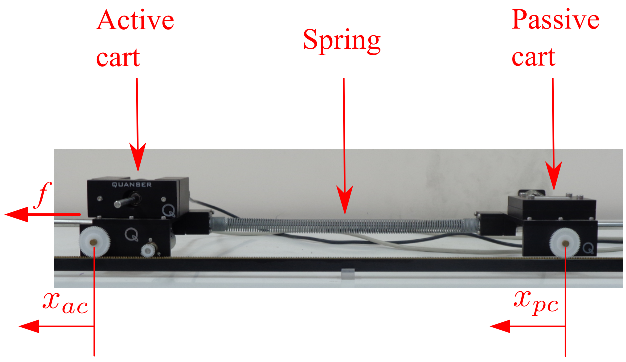

Figure 1. Mass-spring system used for the experiment (adapted from Colombo Jr. *et al.* (2014)).

The nonlinear control formulation adopted herein requires a linear model representing the system dynamics of the form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu_T(t) \\ y(t) &= Cx(t) + Du_T(t)\,. \end{aligned} \tag{30}$$

where $x = [x_{ac}\; x_{pc}\; v_{ac}\; v_{pc}]^T$ is the state vector composed of x_{ac}: active cart position, x_{pc}: passive cart position, v_{ac}: active cart velocity, v_{pc}: passive cart velocity. The control signal u_T is the voltage applied to the terminals of the motor.

In particular, for the mass-spring system in Fig. 1:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_s}{m_{ac}} & \frac{K_s}{m_{ac}} & -\frac{b_c + \frac{K_g^2 K_m K_t}{R_m r_{mp}^2}}{m_{ac}} & 0 \\ \frac{K_s}{m_{pc}} & -\frac{K_s}{m_{pc}} & 0 & -\frac{b_r}{m_{pc}} \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ \frac{K_g K_t}{R_m r_{mp}} \\ 0 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; D = 0_{2\times 1} \tag{31}$$

with the parameter values expressed in Table 1 obtained from the manual of the manufacturer of the equipment, as in Colombo Jr. *et al.* (2014). It is important to remark that the value shown for the spring stiffness constant K_s is the nominal one.

Table 1. Constants used to describe the laboratory plant (adapted from Colombo Jr. *et al.* (2014)).

Symbol	Physical meaning	Value
K_s	Spring stiffness coefficient	$142\,N/m$
m_{ac}	Active cart mass	$1.15\,kg$
m_{pc}	Passive cart mass	$0.54\,kg$
b_c	Viscous friction coefficient (active cart)	$5.4\,Ns/m$
b_r	Viscous friction coefficient (passive cart)	$2.2\,Ns/m$
K_g	Gearbox ratio	3.71
K_m	Counter-electromotive force constant	$7.67 \times 10^{-3}\,Vs/rad$
K_t	Motor torque constant	$7.67 \times 10^{-3}\,Nm/A$
R_m	Motor armature resistance	$2.6\,\Omega$
r_{mp}	Motor pinion radius	$6.35\,mm$

3.1 Control task

The control task is to move the carts from their starting position (which is considered to be zero) to $10\,cm$. It is desirable that the control signal stays within the interval $[-6, 6]\,V$, so that the motors are not damaged. This is ensured by a saturation of the control input. However, it is expected that the controller is tuned so that signal remains within the specified bounds.

From Colombo Jr. *et al.* (2014), it can be deducted that reasonable performance indices would be a rise time of less than $1.5\,s$ and a band of $\pm 5\%$ of error after settling around the reference, given the control constraints.

3.2 Controller design parameters

The design parameters adopted in the controller used in the experimental results are summarized in Table 2. It is important to remark that the assumed interval for variation of the spring stiffness constant K_s contains the nominal value shown in Table 1.

Table 2. Parameters in the controller design.

Parameter	Value
K_s	$[135, 450]\ N/m$
β	0.2
Q	$diag(1000, 1000, 0.01, 0.01)$ [1]
R	5
Ξ	$[20\ \ 20\ \ 100\ \ -0.1]$
υ	1×10^{-6}
q	3

The gain and phase margins associated with the choice of β for the ALQR are

$$GM = 0.1 \to \infty, \tag{32}$$

$$PM = \pm 84.3°. \tag{33}$$

The SeDuMi package (Sturm, 1999) was used to solve the semidefinite programming problem under Matlab® environment and the ensuing gain matrix of the ALQR was

$$K_r = [-78.1 \quad 91.3 \quad -1.09 \quad -0.353]. \tag{34}$$

4. Experimental results and discussion

Experimental results of the positions of the carts are depicted in Fig. 2. It can be noted that the carts approach the reference and after approximately $4\,s$ enter a band of $\pm 3\,mm$ around the reference, which is inside the 5% requirement. This small error was achieved without the use of integrators. The overshoot was of about 30% for the active cart and 26% for the passive one. As for the rise times from 0% to 100%, they were apporximately $1\,s$ for both carts. The imprecision on the spring constant value was dealt with by the ALQR via LMI with the assumption of possible variations on the value of the spring constant.

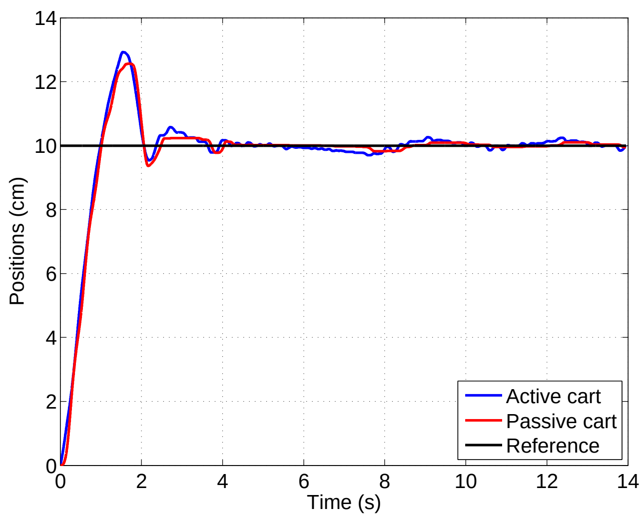

Figure 2. Position of the carts.

As for the control signal shown in Fig. 3, it can be seen that the $\pm 6\,V$ limits were respected throughout the maneuver. The instants when the SMC commutes its output can be clearly identified and are related to the oscillations of the positions

[1] $diag : \mathbb{R}^z \mapsto \mathbb{R}^{z \times z}$, such that $M = diag(a)$ means that $m_{ii} = a_i$, $1 \leq i \leq z$ and $m_{ij} = 0$, $i \neq j$.

around the reference. In particular, it can be noticed that during the initial phase of the maneuver, when both carts present positive velocity, the control signal from the SMC remains positive, and therefore compensates for the negative dry friction force. After that, the carts remain for some time above the reference, and the control signal from the linear state feedback alone would not be enough to steer them closer to the reference because of the dry friction. However, the SMC changes the sign of its output and conduces both carts closer to the reference despite the dry friction, demonstrating its robustness characteristic.

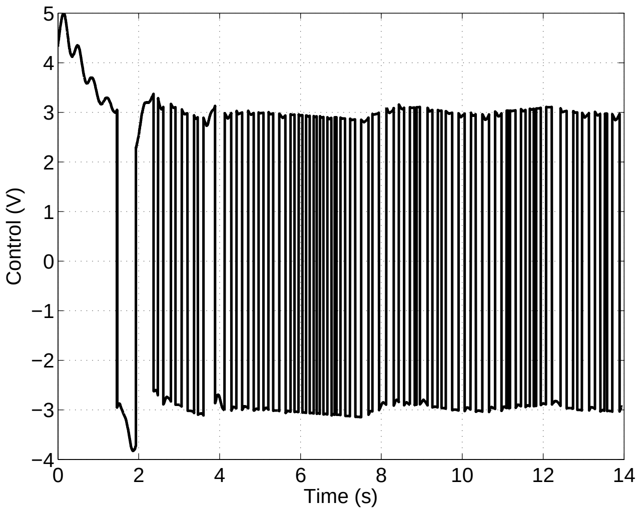

Figure 3. Control signal.

5. Conclusions

A methodology to design robust controllers combining the ALQR technique via LMI and SMC was proposed in this paper. Allying these techniques seems to be profitable as parameter uncertainty and unmodelled effects can be dealt by each of them accordingly.

As means to illustrate the application of the proposed technique, it was used to control the position of a mass-spring system with one active and one passive cart. The spring constant is assumed to be within certain known bounds, which should be considered for robustness of the controller. This is dealt with by the ALQR via LMIs. However, dry friction between the carts and the track over which they run cannot be directly accounted for in the design of the ALQR, therefore the need of a SMC controller. The experimental results showed that the proposed technique resulted in good performance regarding both the control objective, i. e., reaching a reference position, as well as the limits on the voltage applied to the motor, which must remain within certain bounds to avoid damaging the equipment.

It could be noticed that the voltage applied to the motor kept switching after the positions were in the $\pm 5\%$ band around the reference. Thus, a topic of interest for future evaluation would be a policy to switch off the SMC after the outputs reach an admissible band around the reference, to avoid persisting changes in the control signal.

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8. Responsibility notice

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