23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

NUMERICAL MODELING OF THERMOACOUSTIC HEAT TRANSFER INCLUDING PRESSURE LOSSES

Heitor Herculano de Barros, heitorhb@yahoo.com.br
Leonardo S. de B. Alves, leonardo.alves@mec.uff.br
Laboratório de Mecânica Teórica e Aplicada, Programa de Pós-Graduação em Engenharia Mecânica, Universidade Federal Fluminense, Rua Passo da Pátria 156, bloco E, sala 216, Niterói, RJ, 24210-240, Brazil

Abstract. *Thermodynamic and hydrodynamic models have been applied to model the behavior of thermoacoustic waves inside a one-dimensional cavity containing compressible gas or supercritical fluid in zero gravity. These thermoacoustic waves entrapped within cavity walls propagate and reflect several times inducing a rapid heating of the entire fluid, resulting in a homogeneous increase of its bulk temperature. The fast temperature relaxation phenomenon is known today as piston effect and it has been extensively studied. The original thermodynamic model developed for supercritical heat transfer under microgravity conditions is essentially the heat conduction equation with a source term proportional to the bulk temperature time derivative. This source term models the adiabatic compression mechanism responsible for the piston effect. A fully analytical solution of this thermodynamic model is obtained through a combination of the Generalized Integral Transform Technique and the Matrix Exponential Method. The hydrodynamic model utilized consists of the compressible unsteady Navier-Stokes equations solved by utilizing a flux-splitting algorithm for convection and a second order Explicit Runge-Kutta time discretization. Time scales present in this model range from acoustic (micro to miliseconds) to thermal diffusion (minutes to days), but steps are restricted to the acoustic scales. As consequence, this model has a high computational cost for long times permitting only one-dimensional modeling. In our previous works, we have modeled the pressure losses that occur when the thermoacoustic waves hit the walls by means of wall impedance. These losses were modeled through specific pressure boundary conditions in both models, where it was possible to obtain the same qualitative behavior of the experimental results. The motivation of this work is to improve the actual models to better describe the real behavior of the thermoacoustic heat transfer. For this, this study intends to improve our thermodynamic model to take in account the actual wall heating period, previously assumed instantaneous. We present also the results obtained through the hydrodynamic model compared with the experimental data available in the literature. Finally, we perform a quantitative evaluation of the thermoacoustic heat transfer including acoustic impedance using both models. This analyses indicates that without the wall impedance, the models do not describe the real behavior of the pressure along the time inside the cavity.*

Keywords: *thermodynamic critical point, piston effect, integral transform, heat transfer, analytic solution*

1. INTRODUCTION

When a compressible fluid within a closed enclosure is subjected to rapid temperature increase at a solid surface, the fluid in the immediate vicinity of the boundary is heated by conduction and tends to expand (Trilling, 1954). However, the sudden expansion of the fluid due to the energy input is constrained by the inertia of the unperturbed media and creates a local pressure disturbance, which then leads to the production of a pressure wave. The pressure wave that is generated from the hot wall travels at approximately the speed of sound within the enclosure and impinges on the opposite wall and then is reflected back. The wave repeatedly traverses between the walls, and its amplitude eventually damps out due to the viscous and thermal losses within the fluid and wall boundary (Aktas, 2004). Although the heat-transfer effects of such waves are usually not appreciable under standard conditions, they may be significant when the fluid is near to the thermodynamic critical point or when other ways of convection are not strong (Lin e Farouk, 2008).

All fluids are subject to an universal divergence of their thermodynamic properties as their critical points are approached. Even though the thermal diffusion relaxation time diverges near the critical point, a very fast temperature equilibration is still observed in cavity samples on both ground-based (Dahl and Moldover, 1972) and microgravity (Nitsche and Straub, 1987) experiments. Such a critical speeding-up has been explained using simulations of a thermodynamic model (Boukari *et al.*, 1990) as well as a hydrodynamic model (Zappoli *et al.*, 1990). It is caused by the ability of small temperature perturbations to create severe compression in near-critical fluids, which generates thermoacoustic waves. As above cited, when entrapped within cavity walls, their propagation and reflection induces a rapid heating of the entire fluid, resulting in a homogeneous increase of its bulk temperature. This fast temperature relaxation phenomenon is known today as piston effect and it has been the subject of reviews in the literature (Carlès, 2010).

In the present work, the piston effect is modeled using a thermodynamic model developed for supercritical heat transfer under microgravity conditions and a hydrodynamic model that consists of the unsteady compressible Navier-Stokes equations.

The original thermodynamic model developed for supercritical heat transfer under microgravity conditions (Onuki and Ferrel, 1990) is essentially the heat conduction equation with a source term proportional to the bulk temperature time derivative. This source term models the adiabatic compression mechanism responsible for the piston effect. It becomes dominant whenever $\gamma \gg 1$, i.e. closer to the critical point. On the other hand, it vanishes in the incompressible limit, where $\gamma = 1$. Recently, the Generalized Integral Transform Technique was applied to a variation of this thermodynamic model where all fluid properties have a linear dependence on both temperature and pressure (Alves, 2013), confirming that variable properties have little impact on the bulk temperature evolution (Masuda *et al.*, 2002). This hybrid technique analytically transforms the thermodynamic model into an unsteady system of ordinary differential equations, which is then numerically marched forward in time.

The hydrodynamic model consists of the unsteady compressible Navier-Stokes equations solved to describe the velocity, pressure and temperature fields in the enclosure. These equations are solved numerically using a highly accurate, explicit time-integration technique that has minimal numerical diffusion. For the convective terms, it was utilized the Flux Splitting algorithm.

The main purpose of this study is to model the pressure losses that occurs when the wave hits the wall by means of a wall impedance. In several reviews (Huang e Bau, 1995; Farouk *et al.*, 2000; Aktas, 2004), the wall is considered rigid and the pressure wave reflection is perfect, even though the existence of a wall damping was quickly cited (Lin and Farouk, 2008). Finally, the wall impedance was simulated in the two models where it was possible to perform the pressure evolution along the time and a comparative study between the numerical results and the experimental measurements.

2. MATHEMATICAL MODEL

Appropriate pressure boundary conditions were implemented to simulate the thermoacoustic wave reflection at the walls. Since millions of reflections occurs during brief intervals of physical time, a minimal loss during the wave reflection phenomena can provide different numerical results for the pressure behavior. Typically, the specific acoustic impedance is used to describe the acoustic properties of materials and considering a wall reflection impedance Z and the three first reflections, a equation can be deduced in the form

$$\begin{aligned} P_1 &= Z\,P_0 \\ P_2 &= Z\,P_1 \\ P_2 &= Z^2\,P_0 \\ P_3 &= Z^3\,P_0 \\ &\dots \\ P_n &= Z^n\,P_0 \end{aligned}$$

where P_0 stands for first termoacoustic wave, P_1 for first reflected wave, P_2 for second one, and so on. The discrete number of reflections can be approximated by a continuous by $n = t/t_a$ and the acoustic time

$$t_a = L/c \tag{1}$$

where L stands for characteristic length and c for the speed of sound.

The pressure boundary condition assumes different forms for the hydrodynamic and thermodynamic models detailed in next subsections.

2.1 Hydrodynamic Model

The hydrodynamic model consists in unidimensional Navier-Stokes equations that are written in conservative form as

$$\frac{\partial \mathbf{Q}}{\partial t} = \frac{\partial \mathbf{E_v}}{\partial x} - \frac{\partial \mathbf{E_i}}{\partial x} \tag{2}$$

where independent variables t and x represent the physical time and spatial coordinate, respectively. The conservative dependent variable vector is defined as $\mathbf{Q} = \{\rho,\, \rho u,\, \rho E\}$ and the inviscid and viscous flux vectors are defined as

$$\mathbf{E}_i = \left\{\rho u,\, \rho u^2 + P, \left(\rho E + P\right) u\right\} \tag{3}$$

$$\mathbf{E}_v = \left\{0,\, \frac{4\mu}{3}\frac{\partial u}{\partial x}, \frac{4\mu}{3}\frac{\partial u}{\partial x} + k\frac{\partial T}{\partial x}\right\} \tag{4}$$

respectively . In above definitions, ρ stands for density, u for velocity, $E = e + u^2/2$ for total energy per unit mass, P for pressure, μ for dynamic viscosity, k for thermal conductivity ant T for temperature. In the present study, the equation of state utilized, the isothermal compressibility and isobaric thermal expansion coefficient are, respectively, defined as

$$\delta\rho = \rho\,\kappa_T\,\delta P - \rho\,\alpha_P\,\delta T \quad , \quad \kappa_T = \frac{1}{\rho}\left.\frac{\partial \rho}{\partial P}\right|_T \quad \text{and} \quad \alpha_P = -\frac{1}{\rho}\left.\frac{\partial \rho}{\partial T}\right|_P \quad . \tag{5}$$

The appropriate boundary condition on the complex amplitude $\hat{p}$ that satisfies the Helmholtz equation is given by Eq. (6):

$$i\omega\rho\hat{P} = -Z_S\Delta\hat{P}\cdot n \tag{6}$$

where i stands for the complex number $\sqrt{-1}$, ω represents the the angular velocity of thermoacoustic wave, $\Delta\hat{P}$ is the difference between the incident wave on the surface and the consequent reflected wave, and n is the normal vector pointing out of the material into the fluid. For a unidimensional cavity, we can write:

$$\Delta\hat{P}\cdot n = \pm\frac{\partial P}{\partial x} \tag{7}$$

where the signal $\pm$ is positive or negative according to the sense of the unitary normal vector n of the wall under analyses (+ to right wall and - to left wall). Using the pressure as a difference by unit of length, we have:

$$\hat{P} = \frac{P - P_0}{L} \tag{8}$$

From the Eq. (6) and replacing the above results yields:

$$i\omega\rho\frac{P - P_0}{L} = \pm Z_S\frac{\partial P}{\partial x} \tag{9}$$

Doing $Z^* = \frac{Z_S}{i\omega\rho}$, we obtain the wall pressure boundary condition:

$$Z^*\frac{\partial P}{\partial x} \pm \frac{P - P_0}{L} = 0 \tag{10}$$

A surface that is perfectly rigid has $|Z_S| = \infty$. The other extreme, where $|Z_S| = 0$, corresponds to the ideal case of a pressure release surface.

With the objective of normalize the acoustic impedance Z_S, the below transformation equation was used to allow work on a range $0 - 1$.

$$Z_S = \log\frac{1}{1 - Z} \Longrightarrow Z = 1 - e^{-Z_S} \tag{11}$$

The temperature at the walls are prescribed as the excitation temperature T_1 at the left and the initial temperature T_0 at the right wall. For the velocity obviously we have zero velocity $u = 0$ in both walls.

2.2 Thermodynamic Model

The thermodynamic model implemented neglects the fluid flow, assumes all the properties are constant and considers an energy balance in the form of

$$\rho C_P\frac{\partial T}{\partial t} - \rho\left(C_P - C_V\right)\frac{\kappa_T}{\alpha_P}\frac{\partial P_n}{\partial t} = k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \tag{12}$$

where the thermodynamic pressure, assumed spatially uniform, is obtained by deriving the equation of state with respect to time and simplifying to

$$\frac{dP_T}{dt} = \frac{\alpha_P}{\kappa_T}\frac{\int_0^L \frac{\partial T}{\partial t}dx}{\int_0^L dx} \quad \Rightarrow \quad P_T(t) = \frac{\alpha_P}{\kappa_T}T_b \tag{13}$$

which represents a mass balance inside the cavity and T_b stands for the bulk temperature. The real pressure over time P_n inside the cavity can be evaluated taking into account the wall impedance Z:

$$P_n - P_0 = Z^n(P_T - P_0) \tag{14}$$

where P_0 stands for the initial pressure inside the cavity. Deriving the above equation with respect to time, leading to:

$$\frac{dP_n}{dt} = \frac{1}{t_a}\log[Z]Z^{t/ta}(P_T - P_0) + Z^{t/ta}\frac{dP_T}{dt} \tag{15}$$

Then, we have a system formed by the Eqs. (12), (13) and (15). The three equations have to be solved as a system of Partial Differential Equation (PDE), because there is a interdependence between the terms that must be evaluated at the same time step. The Eq. (12) is solved using the Generalized Integral Transform Technique (GITT) and from this temporary result, the other equations are evaluated, solving the complete system.

In the thermodynamic model, the acoustic impedance Z assumes the value $Z = 1$ for a perfect wall where the pressure release surface and values $Z > 1$ for the real wall, where $Z = 0$ means that the pressure wave is totally absorbed.

All fluids properties are assumed constant and evaluated at the initial state, the thermal diffusivity is defined as $\alpha_0 = k_0/\left(\rho_0\, C_{P0}\right)$ and the bulk temperature of the fluid is calculated using definition

$$T_b(t) \;=\; \frac{1}{L}\int_0^L T(x,t)\,dx \tag{16}$$

and the initial boundary conditions for temperature are

$$T(x,t=0) \;=\; T_0 \quad , \quad T(x=0,t) \;=\; T_1 \quad \text{and} \quad T(x=L,t) \;=\; T_0 \quad , \tag{17}$$

respectively. Density can now be obtained from equation of state:

$$\frac{\rho}{\rho_0} = 1 + \kappa_{T_0}(P - P_0) - \alpha_{P_0}(T - T_0) \tag{18}$$

where P_0 and ρ_0 are the initial pressure and density.

3. GAS PROPERTIES

In all previous work reported in the literature, the gas properties were assumed constant. This hypothesis is valid only when the disturbed wall temperature is a little bit greater the gas initial temperature ($T_w - T_0 = 0.01$). All the experimental data about thermoacoustic wave available for compressible gases were obtained using a wall temperature between 100 and 500 degrees Kelvin. Then, in order to simulate some cases comparable with the experimental data available it is necessary to describe correctly the gas behavior, once the gas properties are strongly dependent on the temperature.

The gas properties assumed dependent on the temperature are: specific heat at constant volume (C_v), specific heat at constant pressure (C_p), viscosity and thermal conductivity. The figures below trace the behavior of the properties studied over temperature.

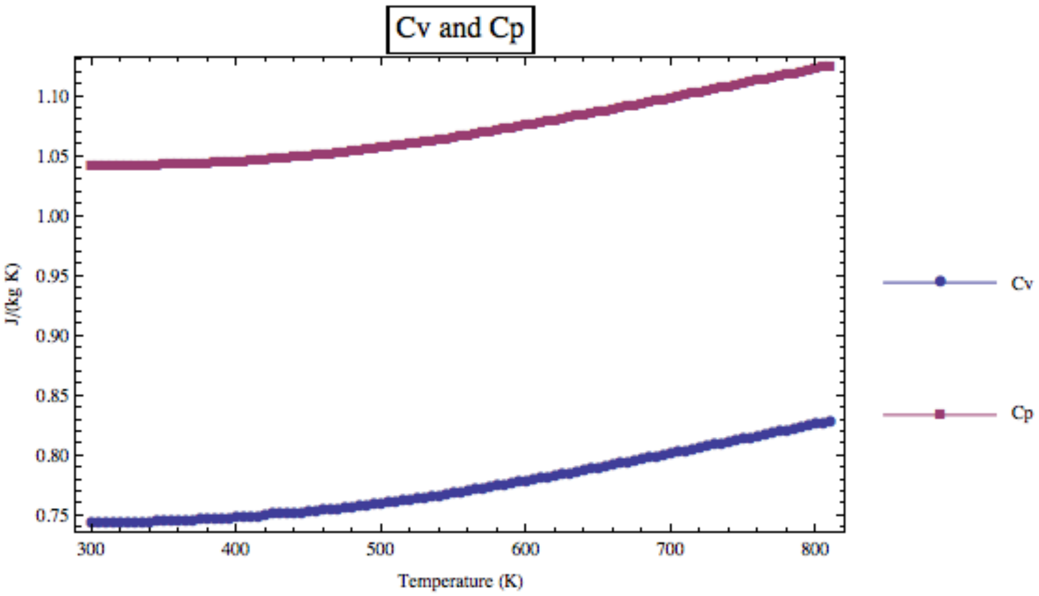

Figure 1. Evolution of the specific heats over temperature.

From the data showed on the pictures, it was possible to evaluate the properties dependence over temperature and fit equations for each property. The equations were used in the hydrodynamic code for correctly describe the fluid behavior as explained.

4. RESULTS AND DISCUSSIONS

A initial comparative analysis using wall impedance Z = 1 between the numerical results obtained from the hydrodynamic model and the thermodynamic model was done. This was performed in Fig. 4, which shows the dimensionless time for γ = 2, 5, 10 and 15. Lines represent the thermodynamic model analytic solution whereas all points represent the

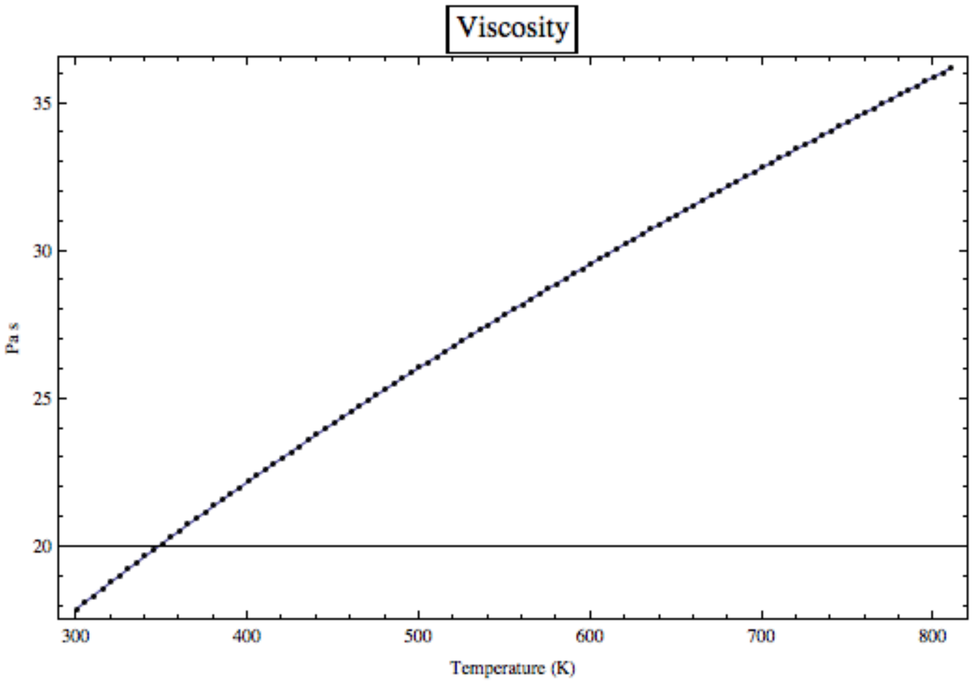

Figure 2. Evolution of the viscosity over temperature.

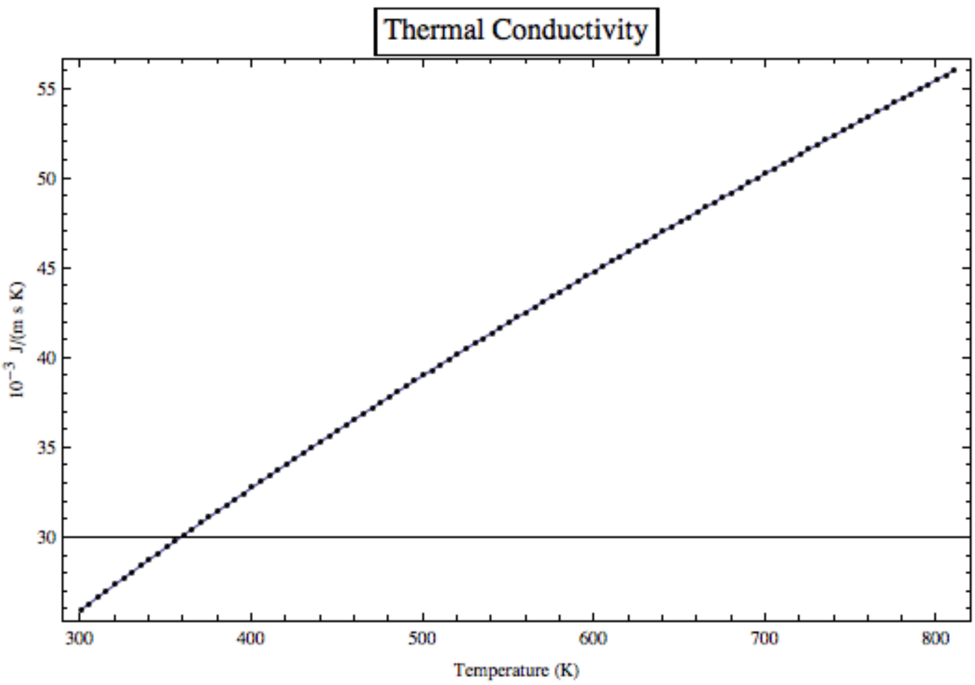

Figure 3. Evolution of the viscosity over temperature.

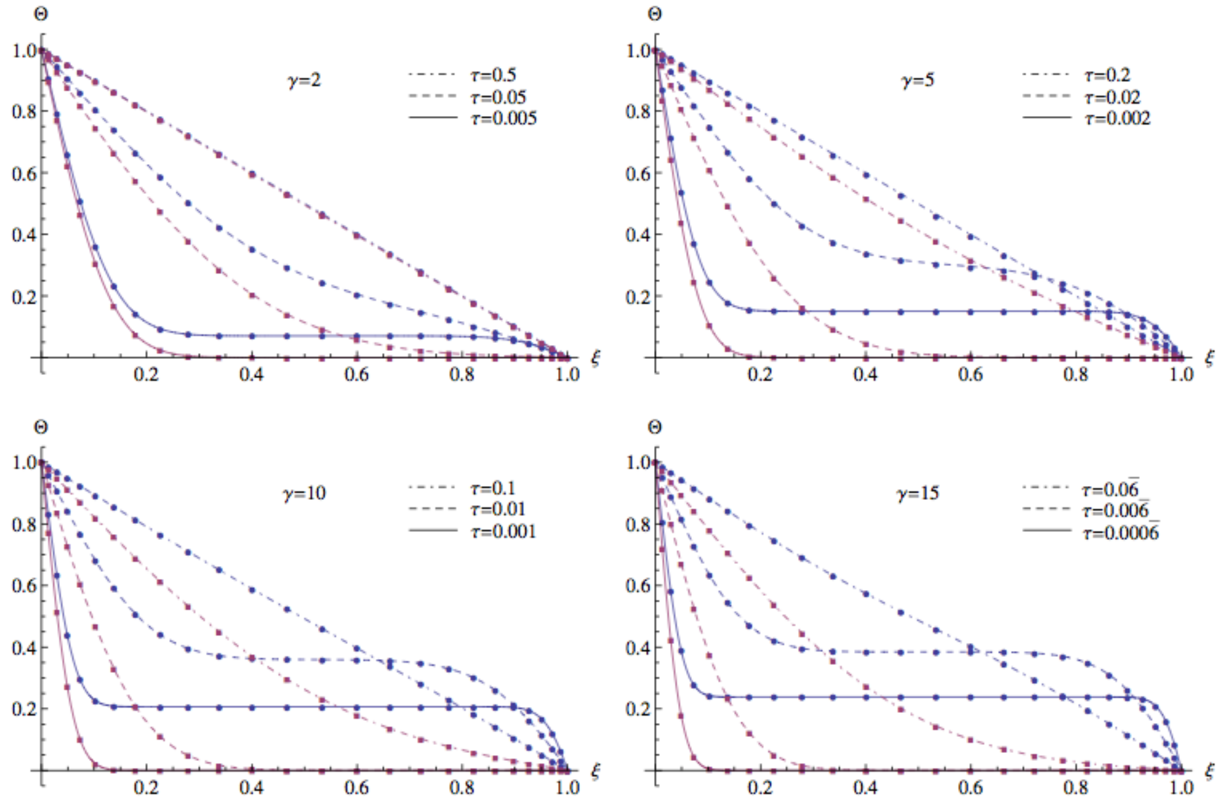

Figure 4. Spatial distribution of the dimensionless bulk temperature at three different dimensionless times for γ = 2, 5, 10 and 15 using wall impedance Z = 1.

hydrodynamic model numerical solution. Circular points, and their respective lines, are solutions that include the piston effect. On the other hand, square points, and their respective lines, are solutions that include thermal diffusion only.

There is an excellent agreement between both methods in the analyzed cases. However, a small deviation can be observed at the later time including the piston effect when γ = 5. The strong early temporal gradients lead to bigger errors, which cumulate over time discernible at $\tau = 0.2$. This effect increases with γ. In order to avoid it, the numerical data shown for γ = 10 and 15 was generated using N_T = 20 000 and 30 000, respectively. The results for γ = 2 and 5 were utilized N_T = 10 000. Nevertheless, no discernible differences can be observed between both model solutions for these latter three cases, where γ = 10 and 15.

For the hydrodynamic model, numerical simulations of thermoacoustic wave generation and transmission were performed using different values for the acoustic impedance for a unidimensional cavity filled with air, initially quiescent at standard pressure and temperature ($P_0 = 101.325Pa$ and $T_0 = 300K$). It was studied the effects of the pressure boundary condition implemented at the behavior of the pressure over time. This analysis was performed for values of Z between 0.9 and 1.0. The evolution of the pressure over time at the midpoint of the cavity is showed in the Fig. 5.

The results obtained and showed in Fig. 5 are consistent with the expected behavior. The acoustic impedance effect was the reduction of the pressure amplitude over time and the pressure returned to the original one P_0 for the steady state solution. Similar behavior is observed in experimental studies reported in the several reviews (Brown, 1992; Lin e Farouk, 2008)

The thermodynamic model has low computational coast specially when compared with the hydrodynamic model. Then, the thermodynamic model was used to perform long times simulations. The hybrid method of solution become feasible interesting analyses. The Fig. 6 shows the result for the same problem studied experimentally by Brown (1992). The result obtained for the maximal pressure amplitude ($179Pa$) was corrected predicted using a acoustic impedance $Z = 0.14$ as can be seen in Fig. 6.

The analyses showed in Fig. 6 was done trying to approximate a value for the acoustic impedance with the objective of obtain the same pressure inside the cavity observed at the experiment. For the experiment, the gas nitrogen was used at standard temperature and pressure. We considered in our simulation, sudden change in right wall temperature, which is impossible in practice. But once the circuit resistor-capacitor used at the experiment has an good efficiency and minimal lost, this approximation is acceptable (Brown, 1992).

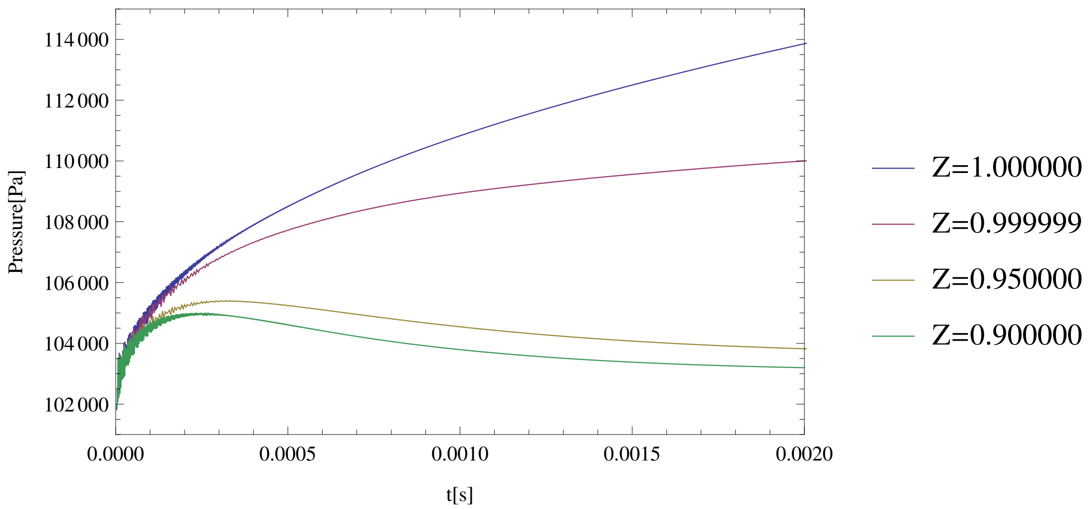

Figure 5. Evolution of the pressure over time at the midpoint ($x = 0.5$) in a cavity filled with air under a excitation temperature $T_1 = 400K$.

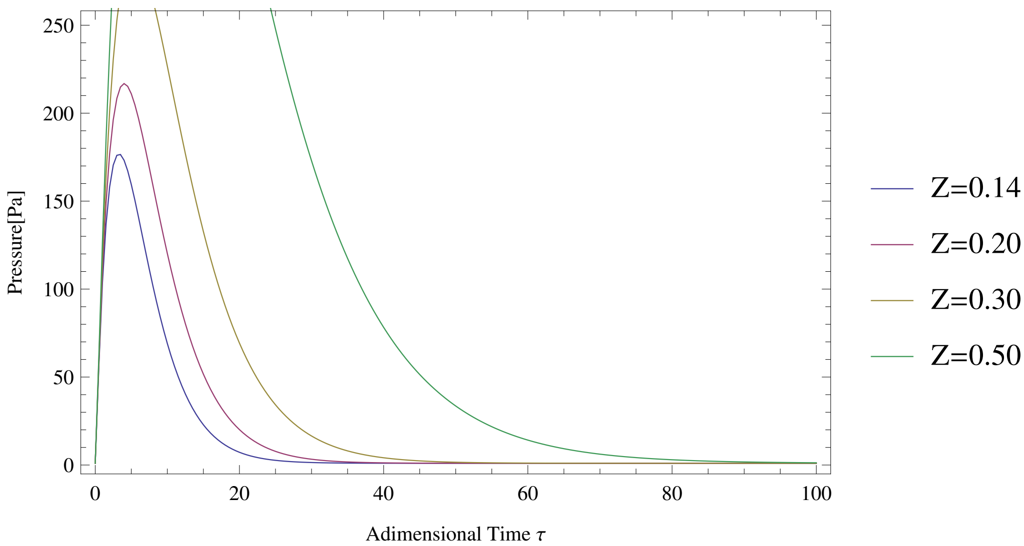

Figure 6. Evolution of the pressure over time at the midpoint in a cavity filled with nitrogen under a excitation temperature $T_1 = 550K$, same problem experimentally studied by Brown (1992).

5. CONCLUSIONS

In this work, it was performed simulations for supercritical fluid under zero gravity in order to capture the piston effect induced by heated walls and it was studied the effects of pressure boundary conditions on the behavior of the pressure over time inside a cavity filled with compressible fluids. The main conclusions are outlined below:

- The thermoacoustic heat transfer involves different scales of characteristic times, once the piston effect occurs in milliseconds, the diffusion phenomena occurs in 10^3 seconds. Then, to capture both effects it is necessary a thermoacoustic step time marching in time until long over times. As consequence, we have a considerable computational time using the hydrodynamic model. Months are the order of magnitude of this computational time for simple analyses as the performed for thermodynamic model showed in Fig.6. Our code was developed using parallel computation, even so, it was only possible for short times analysis using the hydrodynamic code.

- The value of the acoustic impedance Z in the present work, was approximated such that the result for the pressure amplitude was reached. It is necessary a deeper study of the real value of the physical property using information about the wall material.

- A natural improvement of the present work is to analyze the nonlinear effects in the fluid properties caused by the excitation temperature at the right wall. This study is being done by other member of the research group.

- When all fluid properties are considered constant, it is possible to derive an arbitrary precision analytical solution of the thermodynamic model. This is done by combing the Generalized Integral Transform Technique with the Matrix Exponential Method. Accuracy is controlled by a single parameter, which represents the number of terms employed in the convergent summation series solution.
- The selection of an equation of state is dependent on the imposed fluid properties. They must satisfy both Mayer's generalized relation and the sound speed definition. It is only in this scenario that thermodynamic and hydrodynamic models yield the same results. Otherwise, the entropy balance as well as the adiabatic compression mechanism are not equally accounted for by both models, leading to discrepancies between their respective results.

6. ACKNOWLEDGEMENTS

This optional section must be placed before the list of references.

7. REFERENCES

Alves, L.S de B, 2013 "An Integral Transform Solution for Unsteady Compressible Heat Transfer in Fluids near their Thermodynamics Critical Point". *Thermal Science*, Vol. 17, pp. 673–686.

Barmatz, M., Hahn, I., Lipa, J.A. and Duncan, R. V., 2007 "Critical Phenomena in Microgravity: Past, Present and Future". *Review of Modern Physics*, Vol. 79, pp. 1–52.

Boukari, H., Shaumeyer, J.N., Briggs, M.E. and Gammon, R.W., 1990. "Critical Speeding Up in Pure Fields". *Physical Review*, Vol. 41-A, pp. 2260–2263.

M. A. Brown. *Thermally Induced Pressure Wave in a Gas: Experimental Observation and Theoretical Prediction of Thermoacoustic Convection.* PhD thesis, University of Pennsylvania, Philadelphia, PA, 1992.

Carlès, P., 1974. "A Brief Review of the Thermophysical Properties of Supercritical Fluids". *The Journal of Supercritical Fluids*, Vol. 53, pp. 2–11.

Dahl, D. and Moldover, M.R., 1972. "Thermal Relaxation near the Critical Point". *Physical Review*, Vol. 6-A, pp. 1915–1920.

Farouk, B., Oran, E.S., and Fusegi, T., 2000. "Numerical Study of Thermoacoustic Waves in an Enclosure". *Physics of Fluids*, Vol. 12-5, pp 1052–1061.

L. Trilling. *On Thermally Induced Sound Fields*. Group report. M.I.T. Fluid Dynamics Research Group, 1954.

Lin, Y., and Farouk, B., 2008. "Experimental and Numerical Studies of Thermally Induced Acoustic Waves in an Enclosure". *Journal of Thermophysics and Heat Transfer*, Vol. 22-1, pp 105–114.

Murat K. Aktas. *Thermoacoustically Induced and Acoustically Driven Flows and Heat Transfer in Enclosures*. PhD thesis, Drextel University, 2004.

Nakano, A., 2007. "Studies on Piston and Soret Effect in a Binary Mixture Supercritical Fluid". *International Journal of Heat and Mass Transfer*, Vol. 50, pp. 4678–4687.

Nakano, A. and Shiraishi, M., 2005. "Piston Effect in Supercritical Nitrogen around the Pseudo-Critical Line". *International Communications in Heat and Mass Transfer*, Vol. 32, pp. 1152–1164.

Nitsche, K. and Straub, J., 1987. "The Critical Hump of Cv under Microgravity, Results from d-Spacelab Experiment". *Proceedings of the 6th European Symposium on Material Sciences under Microgravity Conditions*, ESA SP-256.

Onuki, H.H., and Ferrel, R.A., 1990. "Fast Adiabatic Equilibration in a Simple-Component Fluid near the Liquid-Vapor Critical Point". *Physical Review*, Vol. 41, pp 2264–2267.

Shen, B. and Zhang, P., 2013. "An Overview of Heat Transfer near the Liguid-Gas Critical Point under the Influence of the Piston Effect: Phenomena an Theory". *International Journal of Thermal Sciences*, Vol. 71, pp. 1–19.

Y. Huang e H. H. Bau. Thermoaocustic waves in a semi-infinite medium. *International Journal of Heat and Mass Transfer*, 38(8):1329–1345, 1995.

Y. Masuda, T. Aizawa, M. Kanakubo, N. Saito, Y. Ikushima, 2002 "One dimensional heat transfer on the thermal diffusion and piston effect of supercritical water". *International Journal of Heat and Mass Transfer*, Vol. 45, pp.3673-3677

Yiqiang Lin e Bakhtier Farouk. Experimental and numerical studies of thermally induced acoustic waves in an enclosure. *Journal of Thermophysics and Heat Transfer*, 22(1):105–114, January-March 2008.

Zappoli, B., Bailly, D., Garrabos Y., Neindre, B.L., Guenoun and Beysens, D., 1990. "Anomalous Heat Transport by the Piston Effect in Supercritical Fluids under Zero Gravity". *Physical Review*, Vol. 41-A, pp. 2264–2267.

Zappoli, B., 2003. "Near-Critical Fluid Hydrodynamics". *Comptes Rendus Mecanique*, Vol. 331, pp. 713–726.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.