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ACTIVE CONTROL SYSTEM FOR SUBSEA INSTALLATIONS

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Abstract. *Nowadays the subsea factory is a main topic in the offshore oil industry. However, subsea equipment are still positioned and installed by an operator that displaces the platform considering the images produced by a Remotely Operated Vehicle (ROV). This operation is totally dependent on the experience and skill of the operator. Another constraint is that this operation can only happen with good condition of sea waves and underwater visibility. The present article presents the development of an active control system in order to propose a trajectory control strategy to the subsea equipment positioning. This allows determining the precise platform motion that is required to obtain the desired position of the subsea equipment (displacement and stabilization). Thus, the subsea installation can be done without the direct intervention of the operator. This automated process increases efficiency and reliability of the operation, reducing the operating time and the risks of equipment damage. The governing equation of the physical model is obtained from the Euler-Bernoulli beam equation under external forces from the fluid in which it is submerged. An analytical solution is developed for the physical model. The accuracy of the proposed solution is verified through numerical simulations of the system with the finite difference method. An open-loop trajectory is calculated and a comparison is made between a prescribed trajectory of the subsea equipment and the response of the subsea equipment. As main result, this new technology proposes a faster and safer way to position and install subsea equipment, increasing the efficiency of the operation and the range of weather conditions in which the operation can be performed.*

Keywords: *Subsea Installation, Motion Planning, Tracking Planning*

1. INTRODUCTION

Nowadays, as the offshore exploration and production of oil and gas are advancing into deeper water (2000 m and more), the subsea factory becomes a main topic in the offshore oil industry. Most subsea equipment are built onshore and transported to the offshore installation site. One of the challenges of the subsea installation at great depths is the precise positioning of the equipment in the desired location on the seabed and this process is still done by an operator. The platform position is controlled by a Dynamic Positioning (DP) system, however, is still required an operator to determine the position that the platform should take to allow the subsea installation. The operator displaces the platform considering the images produced by a ROV to position and install the subsea equipment. Thus, this operation is totally dependent on the experience and skill of the operator. Another constraint is that this operation can only happen with good condition of sea waves and underwater visibility.

The accurate positioning of the equipment becomes even more difficult in deeper water due to the long distance between the platform and the subsea equipment to be positioned on the seabed. Relatively small environmental disturbances, such as ocean currents, waves and wind, can cause large gaps between the expected and the actual position of the subsea equipment. Rough waves and bad weather can force the operation wait several days until it becomes possible. The development of new technologies is necessary for a faster and safer way to position and install subsea equipment, increasing the efficiency of the operation and the range of weather conditions in which the operation can be performed.

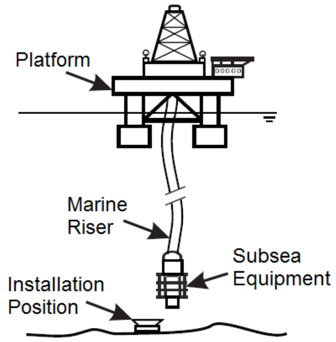

Figure 1. Subsea installation operation, adapted from (Fortaleza *et al.*, 2012)

A typical subsea installation is shown in Fig. 1, which basically consists of a platform, a marine riser and a subsea equipment. The platform is equipped with a DP system, which allows modifying its position using propellers and thrusters. The top-end of the riser is connected to the platform and the bottom-end is connected to the subsea equipment. A ROV monitors the position of the subsea equipment.

The system can be represented by a master/slave architecture (Fig. 2). The slave loop refers to the DP system and the platform, and its dynamics is much faster compared to the dynamics of the master loop. In turn, the master loop refers to the riser, the submarine and the operator. The dynamic of the master loop is influenced mainly by the mechanical wave propagation along the riser. In this article, the analysis will focus only on the master loop and it is assumed that the DP system positions the platform in the desired location instantly.

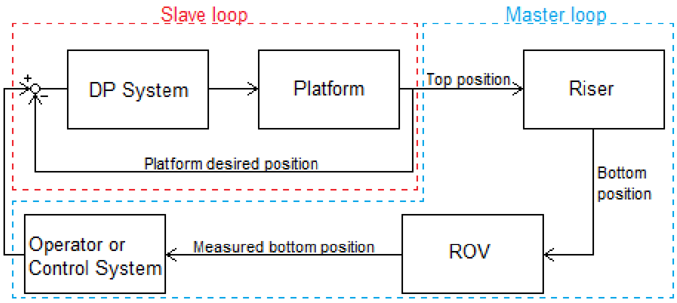

Figure 2. Block diagram of the system (master/slave architecture)

This paper presents the development of an active control system to replace the human operator in the loop. The control system will be able to determine the platform position that will allow the subsea installation. It is therefore a trajectory control strategy for the placement of the subsea equipment. The strategy is to position the subsea equipment at the desired position on the seabed, dynamically modifying the riser top-end, whose displacement is determined by the platform position. This allows determining the precise platform motion that is required to obtain the desired position of the subsea equipment (displacement and stabilization). The position of the subsea equipment is monitored by images produced in real time by a ROV, serving as input data to the control system.

The main idea is to design an open-loop trajectory to move the subsea equipment from its initial position to the desired installation location. Thus, the subsea installation can be done without the direct intervention of the operator. This automated process increases efficiency and reliability of the operation, reducing the operating time and the risks of equipment damage.

2. GOVERNING EQUATIONS

Typically, structures used in the installation of subsea equipment, such as risers, are slender and their transverse displacements are small compared to their length. In the case of a riser with constant cross section and under traction, the dynamic behavior of an infinitesimal element of the riser toward its horizontal displacement can be modeled by the Euler-Bernoulli beam equation Eq. (1) under external forces from the fluid in which it is submerged (Fard, 2001).

$$m_s \frac{\partial^2 \Upsilon}{\partial t^2}(z,t) = -EJ\frac{\partial^4 \Upsilon}{\partial z^4}(z,t) + \frac{\partial}{\partial z}\left(T(z)\frac{\partial \Upsilon}{\partial z}(z,t)\right) + F_h(z,t) \tag{1}$$

where Υ is the position of the riser in the horizontal direction and it is function of two parameters: the time t and the height z from the seabed. The other variables are: m_s the riser linear mass, $T(z)$ the function that describes the normal tension force along the structure's length, E the Young's modulus, J the second moment of area and F_h the external forces in the horizontal direction per unit length.

The external forces acting in the structure, except on its top and bottom-ends, where external forces are present due to the boundary conditions, can be approximated by the modified Morison's equation (Fard, 2001). Equation (2) represents the hydrodynamic forces associated with the displacement of a submerged body and do not include side forces resulting from the oscillatory vortex shedding around the structure.

$$F_h(z,t) = -m_F\frac{\partial^2 \Upsilon}{\partial t^2}(z,t) - \mu\frac{\partial \Upsilon}{\partial t}(z,t)\left|\frac{\partial \Upsilon}{\partial t}(z,t)\right| \tag{2}$$

where m_F is the additional fluid mass per unit length and μ is the drag per unit length. The additional fluid mass is defined by $m_F = \rho C_m S$, where ρ is the fluid density, C_m is the added mass coefficient and S is the cross-sectional area of the riser. The drag is defined by $\mu = \frac{1}{2}\rho C_d D$, where C_d is the drag coefficient and D is the outer diameter of the riser.

Denoting $m = m_s + m_F$ and considering the hydrodynamic forces Eq. (2) as the only external forces, Eq. (1) becomes:

$$\frac{\partial^2 \Upsilon}{\partial t^2}(z,t) = -\frac{EJ}{m}\frac{\partial^4 \Upsilon}{\partial z^4}(z,t) + \frac{\partial}{\partial z}\left(\frac{T(z)}{m}\frac{\partial \Upsilon}{\partial z}(z,t)\right) - \frac{\mu}{m}\frac{\partial \Upsilon}{\partial t}(z,t)\left|\frac{\partial \Upsilon}{\partial t}(z,t)\right| \tag{3}$$

The dissipative drag term can be linearized by replacing $\frac{\mu}{m}\left|\frac{\partial \Upsilon}{\partial t}(z,t)\right|$ by the constant 2α, which is calculated on the basis of $\frac{\mu}{m}$ and the average of $\frac{\partial \Upsilon}{\partial t}(z,t)$ along the structure. Thus, Eq. (4) represents the system governing equation for the linearized damping.

$$\frac{\partial^2 \Upsilon}{\partial t^2}(z,t) = -\frac{EJ}{m}\frac{\partial^4 \Upsilon}{\partial z^4}(z,t) + \frac{\partial}{\partial z}\left(\frac{T(z)}{m}\frac{\partial \Upsilon}{\partial z}(z,t)\right) - 2\alpha\frac{\partial \Upsilon}{\partial t}(z,t) \tag{4}$$

For the installation of subsea equipment in water depths of 2000 meters and greater, the weight of the structure and the weight of the subsea equipment have similar orders of magnitude and, thus, can neither be neglected. The normal tension $T(z)$ is formulated from the difference between the total weight and the buoyancy. Therefore, $T(z) = (m_s - \rho S)gz + T_e$, where T_e is the normal tension associated to the subsea equipment attached to the bottom-end: $T_e = (m_e - \rho V_e)g$, being m_e the mass of subsea equipment, V_e its volume and g the acceleration due to gravity. As risers for that water depth are very long structures, the span-depth ratio $\frac{D}{L}$ is on the order of 10^{-4} or lower. Thus, the beam therm $EJ\frac{\partial^4 \Upsilon}{\partial z^4}$ associated to the bending resistance may be neglected, as $\frac{\partial}{\partial z}\left(T(z)\frac{\partial \Upsilon}{\partial z}\right) \gg -EJ\frac{\partial^4 \Upsilon}{\partial z^4}$ (Fortaleza, 2009). Therefore, considering this simplification, the system equation becomes:

$$\frac{\partial^2 \Upsilon}{\partial t^2}(z,t) = \frac{\partial}{\partial z}\left[\left(\frac{(m_s - \rho S)\, gz}{m} + \frac{T_e}{m}\right)\frac{\partial \Upsilon}{\partial z}(z,t)\right] - 2\alpha\frac{\partial \Upsilon}{\partial t}(z,t) \tag{5}$$

To simplify the notation, the constant term $\frac{(m_s - \rho S)g}{m}$ can be replaced by the effective gravity g' and the constant term $\frac{T_e}{m}$ can be replaced by v^2, where v is known to be the wave propagation speed on the riser, so the Eq. (5) becomes:

$$\frac{\partial^2 \Upsilon}{\partial t^2}(z,t) = g'\frac{\partial \Upsilon}{\partial z}(z,t) + \left(g'z + v^2\right)\frac{\partial^2 \Upsilon}{\partial z^2}(z,t) - 2\alpha\frac{\partial \Upsilon}{\partial t}(z,t) \tag{6}$$

3. MOTION PLANNING

In this section, an analytical solution of Eq. (6) in the Laplace domain is presented, following the idea proposed by (Petit and Rouchon, 2001) for an undamped cable and adapted to a damped cable without the subsea equipment in papers (Fortaleza *et al.*, 2008) and (Fortaleza *et al.*, 2011). In a recent work (Fortaleza *et al.*, 2012) adapted to a damped cable with the subsea equipment, but the normal tension along the structure's length was considered to be constant, due only the heavy subsea equipment, neglecting the cable's own weight. The first step is to apply the t-Laplace transform on Eq. (6) and considering the system at rest at $t = 0$, the partial differential equation (PDE) can be rewritten as the following ordinary differential equation (ODE) regarding spacial derivatives only:

$$\left(g'z + v^2\right)\frac{\partial^2 \widehat{\Upsilon}}{\partial z^2}(z,s) + g'\frac{\partial \widehat{\Upsilon}}{\partial z}(z,s) - \left(s^2 + 2\alpha s\right)\widehat{\Upsilon}(z,s) = 0 \tag{7}$$

where $\widehat{\Upsilon}$ is the Laplace transform of Υ.

The solution of Eq. (7) with respect to z is:

$$\widehat{\Upsilon}(z,s) = \frac{\sqrt{2}}{\sqrt{g'}}\widehat{C}_1(s)I_0\left(2\sqrt{\frac{(s^2 + 2\alpha s)\,(v^2 + g'z)}{g'^2}}\right) + \frac{\sqrt{2}}{\sqrt{g'}}\widehat{C}_2(s)K_0\left(2\sqrt{\frac{(s^2 + 2\alpha s)\,(v^2 + g'z)}{g'^2}}\right) \tag{8}$$

where $\widehat{C}_1(s)$ and $\widehat{C}_2(s)$ are the functions to be determined by the system's boundary conditions. I_ν and K_ν are the modified Bessel functions of the first and second kind of order ν, respectively (Abramowitz and Stegun, 1972).

To determine $\widehat{C}_1(s)$ and $\widehat{C}_2(s)$ it is necessary two boundary conditions. The two boundary conditions are the reference trajectory of the bottom-end $\Upsilon(0,t)$ and its spacial derivative $\frac{\partial \Upsilon}{\partial z}(0,t)$, that represents the riser slope at its bottom-end. In this article, $\Upsilon(0,t)$ is set as a polynomial trajectory that smoothly migrates from an initial position to a final position in a specified stabilization time. The second boundary condition is obtained from the dynamics of the subsea equipment. Equation (9) is obtained for the subsea equipment from the Newton's second law of motion.

$$m_e\frac{\partial^2 \Upsilon}{\partial t^2}(0,t) = F_e(t) + F_h(0,t) \tag{9}$$

where F_e is the riser's force on the subsea equipment and F_h is the hydrodynamic forces also defined by Eq. (2). F_e is proportional to the riser's slope and defined as $F_e(t) = T_e\frac{\partial \Upsilon}{\partial z}(0,t)$. Also considering the linearized dissipative drag and rearranging Eq. (9), the second boundary condition is:

$$\frac{\partial \Upsilon}{\partial z}(0,t) = \frac{1}{T_e}\left(m_{te}\frac{\partial^2 \Upsilon}{\partial t^2}(0,t) + 2\alpha_e\frac{\partial \Upsilon}{\partial t}(0,t)\right) \tag{10}$$

where m_{te} is the subsea equipment mass added to its additional fluid mass and α_e is its linearized drag.

Differentiating Eq. (8) in relation to the coordinate z and assuming $z = 0$, the following system is obtained:

$$\begin{cases} \widehat{\Upsilon}(0,s) = \dfrac{\sqrt{2}}{\sqrt{g'}}\widehat{C}_1(s) I_0\left(\dfrac{2v}{g'}\sqrt{s^2+2\alpha s}\right) + \dfrac{\sqrt{2}}{\sqrt{g'}}\widehat{C}_2(s) K_0\left(\dfrac{2v}{g'}\sqrt{s^2+2\alpha s}\right) \\ \dfrac{\partial\widehat{\Upsilon}}{\partial z}(0,s) = \dfrac{\sqrt{2}}{\sqrt{g'}}\dfrac{\sqrt{s^2+2\alpha s}}{v}\widehat{C}_1(s) I_1\left(\dfrac{2v}{g'}\sqrt{s^2+2\alpha s}\right) - \dfrac{\sqrt{2}}{\sqrt{g'}}\dfrac{\sqrt{s^2+2\alpha s}}{v}\widehat{C}_2(s) K_1\left(\dfrac{2v}{g'}\sqrt{s^2+2\alpha s}\right) \end{cases} \tag{11}$$

Solving the system, the functions $\widehat{C}_1(s)$ and $\widehat{C}_2(s)$ are determined:

$$\begin{aligned} \widehat{C}_1(s) &= \frac{\sqrt{g'}}{\sqrt{2}}\left(\frac{K_1(x)}{I_0(x)K_1(x)+I_1(x)K_0(x)}\right)\widehat{\Upsilon}(0,s) \\ &+\frac{\sqrt{g'}}{\sqrt{2}}\frac{v}{\sqrt{s^2+2\alpha s}}\left(\frac{K_0(x)}{I_0(x)K_1(x)+I_1(x)K_0(x)}\right)\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s) \end{aligned} \tag{12}$$

$$\begin{aligned} \widehat{C}_2(s) &= \frac{\sqrt{g'}}{\sqrt{2}}\left(\frac{I_1(x)}{I_0(x)K_1(x)+I_1(x)K_0(x)}\right)\widehat{\Upsilon}(0,s) \\ &-\frac{\sqrt{g'}}{\sqrt{2}}\frac{v}{\sqrt{s^2+2\alpha s}}\left(\frac{I_0(x)}{I_0(x)K_1(x)+I_1(x)K_0(x)}\right)\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s) \end{aligned} \tag{13}$$

where $x = \frac{2v}{g'}\sqrt{s^2+2\alpha s}$.

The Wronskian between two modified Bessel functions is: $W\{K_\nu(x), I_\nu(x)\} = I_\nu(x)K_{\nu+1}(x) + I_{\nu+1}(x)K_\nu(x) = \frac{1}{x}$ (Abramowitz and Stegun, 1972). Therefore, Eq. (12) and Eq. (13) can be rewritten:

$$\widehat{C}_1(s) = v\frac{\sqrt{2}}{\sqrt{g'}}\sqrt{s^2+2\alpha s}\,K_1\left(\frac{2v}{g'}\sqrt{s^2+2\alpha s}\right)\widehat{\Upsilon}(0,s) + v^2\frac{\sqrt{2}}{\sqrt{g'}}\sqrt{s^2+2\alpha s}\,K_0\left(\frac{2v}{g'}\sqrt{s^2+2\alpha s}\right)\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s) \tag{14}$$

$$\widehat{C}_2(s) = v\frac{\sqrt{2}}{\sqrt{g'}}\sqrt{s^2+2\alpha s}\,I_1\left(\frac{2v}{g'}\sqrt{s^2+2\alpha s}\right)\widehat{\Upsilon}(0,s) - v^2\frac{\sqrt{2}}{\sqrt{g'}}\sqrt{s^2+2\alpha s}\,I_0\left(\frac{2v}{g'}\sqrt{s^2+2\alpha s}\right)\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s) \tag{15}$$

Another way to represent the modified Bessel function is by its integral representation (Abramowitz and Stegun, 1972). Thus, Eq. (14) and Eq. (15) becomes:

$$\widehat{C}_1(s) = v^2\frac{2\sqrt{2}}{g'\sqrt{g'}}\widehat{K}(s)\int_1^\infty \mathrm{e}^{-\frac{2v}{g'}\sqrt{s^2+2\alpha s}\,k}\sqrt{k^2-1}\,\mathrm{d}k + v^2\frac{\sqrt{2}}{\sqrt{g'}}\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s)\int_1^\infty\frac{\mathrm{e}^{-\frac{2v}{g'}\sqrt{s^2+2\alpha s}\,k}}{\sqrt{k^2-1}}\,\mathrm{d}k \tag{16}$$

$$\widehat{C}_2(s) = v^2\frac{2\sqrt{2}}{\pi g'\sqrt{g'}}\widehat{K}(s)\int_{-1}^1 \mathrm{e}^{-\frac{2v}{g'}\sqrt{s^2+2\alpha s}\,k}\sqrt{1-k^2}\,\mathrm{d}k - v^2\frac{\sqrt{2}}{\pi\sqrt{g'}}\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s)\int_{-1}^1\frac{\mathrm{e}^{-\frac{2v}{g'}\sqrt{s^2+2\alpha s}\,k}}{\sqrt{1-k^2}}\,\mathrm{d}k \tag{17}$$

where $\widehat{K}(s) = \left(s^2+2\alpha s\right)\widehat{\Upsilon}(0,s)$.

To return to the time-domain, the inverse Laplace transformation should be applied. As $\widehat{K}(s)$ and $\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s)$ are only functions of s, they can be placed in the integrand of Eq. (16) and Eq. (17). The identity presented by Eq. (18) can be used to inverse the Laplace transform (Mikusinski, 2014):

$$\mathrm{e}^{-x\sqrt{s^2+2\alpha s}} = \mathrm{e}^{-\alpha x}\mathrm{e}^{-xs} - \{f(x,t)\}\,\mathrm{e}^{-xs} \tag{18}$$

where $f(x,t) = \frac{x}{\sqrt{t^2+2xt}}\mathrm{e}^{-\alpha(x+t)}i\alpha J_1\left(i\alpha\sqrt{t^2+2xt}\right)$ and J_ν is the Bessel function of the first kind of order ν (Abramowitz and Stegun, 1972).

To simplify the notation: $x_1(k) = \frac{2v}{g'}k$. Applying Eq. (18) in Eq. (16) and Eq. (17):

$$\begin{aligned} \widehat{C}_1(s) &= v^2\frac{2\sqrt{2}}{g'\sqrt{g'}}\int_1^\infty\left[\widehat{K}(s)\,\mathrm{e}^{-\alpha x_1(k)} - \widehat{g}_1(x_1(k),s)\right]\mathrm{e}^{-x_1(k)s}\sqrt{k^2-1}\,\mathrm{d}k \\ &+v^2\frac{\sqrt{2}}{\sqrt{g'}}\int_1^\infty\left[\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s)\,\mathrm{e}^{-\alpha x_1(k)} - \widehat{g}_2(x_1(k),s)\right]\frac{\mathrm{e}^{-x_1(k)s}}{\sqrt{k^2-1}}\,\mathrm{d}k \end{aligned} \tag{19}$$

$$\begin{aligned} \widehat{C}_2(s) &= v^2\frac{2\sqrt{2}}{\pi g'\sqrt{g'}}\int_{-1}^1\left[\widehat{K}(s)\,\mathrm{e}^{-\alpha x_1(k)} - \widehat{g}_1(x_1(k),s)\right]\mathrm{e}^{-x_1(k)s}\sqrt{1-k^2}\,\mathrm{d}k \\ &-v^2\frac{\sqrt{2}}{\pi\sqrt{g'}}\int_{-1}^1\left[\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s)\,\mathrm{e}^{-\alpha x_1(k)} - \widehat{g}_2(x_1(k),s)\right]\frac{\mathrm{e}^{-x_1(k)s}}{\sqrt{1-k^2}}\,\mathrm{d}k \end{aligned} \tag{20}$$

where $\widehat{g}_1(x_1(k),s) = \widehat{f}(x_1(k),s)\,\widehat{K}(s)$ and $\widehat{g}_2(x_1(k),s) = \widehat{f}(x_1(k),s)\,\frac{\partial\widehat{\Upsilon}}{\partial z}(0,s)$.

The inverse Laplace transformation of the product of two functions can be calculated by the convolution formula (Mikusinski, 2014). Thus, the functions $g_1(x_1(k),t)$ and $g_2(x_1(k),t)$ are obtained:

$$g_1(x_1(k),t) = \int_0^t f(x_1(k),\tau)\,K(t-\tau)\,\mathrm{d}\tau \tag{21}$$

$$g_2(x_1(k),t) = \int_0^t f(x_1(k),\tau)\,\frac{\partial\Upsilon}{\partial z}(0,t-\tau)\,\mathrm{d}\tau \tag{22}$$

where $K(t) = u_0(t)\left(2\alpha\frac{\partial\Upsilon}{\partial t}(0,t) + \frac{\partial^2\Upsilon}{\partial t^2}(0,t)\right)$, also obtained by the convolution of $\left(s^2+2\alpha s\right)$ and $\widehat{\Upsilon}(0,s)$. The variable u_0 is the unit step function.

Now, applying the shifting theorem: $f(t-x)u_x(t) = \mathrm{e}^{-xs}\widehat{f}(s)$, both Eq. (19) and Eq. (20) can be rewritten in the time-domain:

$$\begin{aligned} C_1(t) \;&=\; v^2\frac{2\sqrt{2}}{g'\sqrt{g'}}\int_1^\infty u_{x_1(k)}(t)\left[K(t-x_1(k))\,\mathrm{e}^{-\alpha x_1(k)} - g_1(x_1(k),t-x_1(k))\right]\sqrt{k^2-1}\,\mathrm{d}k \\ &\quad + v^2\frac{\sqrt{2}}{\sqrt{g'}}\int_1^\infty u_{x_1(k)}(t)\left[\frac{\partial\Upsilon}{\partial z}(0,t-x_1(k))\,\mathrm{e}^{-\alpha x_1(k)} - g_2(x_1(k),t-x_1(k))\right]\frac{1}{\sqrt{k^2-1}}\,\mathrm{d}k \end{aligned} \tag{23}$$

$$\begin{aligned} C_2(t) \;&=\; v^2\frac{2\sqrt{2}}{\pi g'\sqrt{g'}}\int_{-1}^1 u_{x_1(k)}(t)\left[K(t-x_1(k))\,\mathrm{e}^{-\alpha x_1(k)} - g_1(x_1(k),t-x_1(k))\right]\sqrt{1-k^2}\,\mathrm{d}k \\ &\quad - v^2\frac{\sqrt{2}}{\pi\sqrt{g'}}\int_{-1}^1 u_{x_1(k)}(t)\left[\frac{\partial\Upsilon}{\partial z}(0,t-x_1(k))\,\mathrm{e}^{-\alpha x_1(k)} - g_2(x_1(k),t-x_1(k))\right]\frac{1}{\sqrt{1-k^2}}\,\mathrm{d}k \end{aligned} \tag{24}$$

Once both functions $C_1(t)$ and $C_2(t)$ are determined, one can apply the inverse Laplace transform on Eq. (8) to obtain the displacement of the riser in the time-domain $\Upsilon(z,t)$. First, representing the modified Bessel functions by theirs integral representation, Eq. (8) becomes:

$$\widehat{\Upsilon}(z,s) = \frac{\sqrt{2}}{\pi\sqrt{g'}}\widehat{C}_1(s)\int_{-1}^1 \frac{\mathrm{e}^{-2\sqrt{\frac{(v^2+g'z)}{g'^2}}\sqrt{s^2+2\alpha s}\,k}}{\sqrt{1-k^2}}\,\mathrm{d}k + \frac{\sqrt{2}}{\sqrt{g'}}\widehat{C}_2(s)\int_1^\infty \frac{\mathrm{e}^{-2\sqrt{\frac{(v^2+g'z)}{g'^2}}\sqrt{s^2+2\alpha s}\,k}}{\sqrt{k^2-1}}\,\mathrm{d}k \tag{25}$$

As $\widehat{C}_1(s)$ and $\widehat{C}_2(s)$ are only functions of s, they can be placed in the integrand of Eq. (25). Applying the identity presented in Eq. (18) and assuming $x_2(z,k) = 2\sqrt{\frac{(v^2+g'z)}{g'^2}}\,k$, Eq. (25) becomes:

$$\begin{aligned} \widehat{\Upsilon}(z,s) \;&=\; \frac{\sqrt{2}}{\pi\sqrt{g'}}\int_{-1}^1 \left[\widehat{C}_1(s)\mathrm{e}^{-\alpha x_2(z,k)} - \widehat{g}_3(x_2(z,k),s)\right]\frac{\mathrm{e}^{-x_2(z,k)s}}{\sqrt{1-k^2}}\,\mathrm{d}k \\ &\quad + \frac{\sqrt{2}}{\sqrt{g'}}\int_1^\infty \left[\widehat{C}_2(s)\mathrm{e}^{-\alpha x_2(z,k)} - \widehat{g}_4(x_2(z,k),s)\right]\frac{\mathrm{e}^{-x_2(z,k)s}}{\sqrt{1-k^2}}\,\mathrm{d}k \end{aligned} \tag{26}$$

where $\widehat{g}_3(x_2(z,k),s) = \widehat{f}(x_2(z,k),s)\,\widehat{C}_1(s)$ and $\widehat{g}_4(x_2(z,k),s) = \widehat{f}(x_2(z,k),s)\,\widehat{C}_2(s)$. The functions $g_3(x_2(z,k),t)$ and $g_4(x_2(z,k),t)$ are also obtained by the convolution formula:

$$g_3(x_2(z,k),t) = \int_0^t f(x_2(z,k),\tau)\,C_1(t-\tau)\,\mathrm{d}\tau \tag{27}$$

$$g_4(x_2(z,k),t) = \int_0^t f(x_2(z,k),\tau)\,C_2(t-\tau)\,\mathrm{d}\tau \tag{28}$$

Applying the shifting theorem, Eq. (29) is obtained, which represents the displacement of the riser in the time-domain.

$$\begin{aligned} \Upsilon(z,t) \;&=\; \frac{\sqrt{2}}{\pi\sqrt{g'}}\int_{-1}^1 u_{x_2(z,k)}(t)\left[C_1(t-x_2(z,k))\,\mathrm{e}^{-\alpha x_2(z,k)} - g_3(x_2(z,k),t-x_2(z,k))\right]\frac{1}{\sqrt{1-k^2}}\,\mathrm{d}k \\ &\quad + \frac{\sqrt{2}}{\sqrt{g'}}\int_1^\infty u_{x_2(z,k)}(t)\left[C_2(t-x_2(z,k))\,\mathrm{e}^{-\alpha x_2(z,k)} - g_4(x_2(z,k),t-x_2(z,k))\right]\frac{1}{\sqrt{1-k^2}}\,\mathrm{d}k \end{aligned} \tag{29}$$

The open-loop solution is found integrating numerically Eq. (29). Thus, assuming $z = L$, being L the riser length, the control trajectory $\Upsilon_o(L,t)$ for the DP system is found.

4. NUMERICAL SIMULATION

In order to validate the analytical solution, a numerical simulation by finite difference method is proposed, following the idea presented in (Fortaleza, 2009). This method discretizes the continuous function $\Upsilon(z,t)$ and defines $Y(t)$ as a vector of the structure displacement for N equidistant points along the structure. Thus, the governing PDE (Eq. (3)) is approximated by a finite set of coupled ODEs (Eq. (30)), valid for all elements not affected by the boundary conditions.

$$\begin{aligned}\frac{\mathrm{d}^2Y_n}{\mathrm{d}t^2} &= -\frac{EJ}{m}\frac{(Y_{n-2}-4Y_{n-1}+6Y_n-4Y_{n+1}+Y_{n+2})}{l^4}+\frac{(T_e+mg'nl)}{m}\frac{(Y_{n-1}-2Y_n+Y_{n+1})}{l^2}\\ &\quad +g'\frac{(-Y_{n-1}+Y_{n+1})}{2l}-2\alpha\frac{\mathrm{d}Y_n}{\mathrm{d}t}\end{aligned} \tag{30}$$

where $n \in (1,\ldots,N)$ is the element position, l is the distance between two adjacent points ($l = L/N$).

Being the subsea equipment as the first element ($n = 1$) and the riser top-end as the last element ($n = N$), and considering $U(t)$ as the platform position, defined by the DP system, the boundary conditions are: at the subsea equipment the equation of motion is given by Eq. (9) and $\frac{\partial^2\Upsilon}{\partial z^2}(0,t) = 0$; and at the top-end $\Upsilon(L,t) = U(t)$ and $\frac{\partial\Upsilon}{\partial z}(L,t) = 0$, as the riser is attached to the platform by a fixed support. Thus, the ODEs for the first and last elements are:

$$\frac{\mathrm{d}^2Y_1}{\mathrm{d}t^2} = -\frac{EJ}{m_{te}}\frac{(Y_1-2Y_2+Y_3)}{l^4}+\frac{T_e}{m_{te}}\frac{(-Y_1+Y_2)}{l}-2\alpha_e\frac{\mathrm{d}Y_1}{\mathrm{d}t} \tag{31}$$

$$\begin{aligned}\frac{\mathrm{d}^2Y_N}{\mathrm{d}t^2} &= -\frac{EJ}{m}\frac{(Y_{N-2}-4Y_{N-1}+6Y_N-3U(t))}{l^4}+\frac{(T_0+mg'Nl)}{m}\frac{(Y_{N-1}-2Y_N+U(t))}{l^2}\\ &\quad +g'\frac{(-Y_{N-1}+U(t))}{2l}-2\alpha\frac{\mathrm{d}Y_N}{\mathrm{d}t}\end{aligned} \tag{32}$$

Representing the discrete system in the state space form:

$$\begin{cases}\dot{\mathbf{X}} = \mathbf{AX}+\mathbf{B}U\\ y = \mathbf{CX}\end{cases} \tag{33}$$

where $\mathbf{A}$ is the state matrix, $\mathbf{B}$ is the input vector and contains the acceleration of the structure associated to the riser top-end displacement: $\mathbf{B} = \left[0,\cdots,0,-\frac{EJ}{ml^4},3\frac{EJ}{ml^4}+\frac{T_e+mg'lN}{ml^2}+\frac{g'}{2l}\right]^T$, $\mathbf{C}$ is the output row-matrix with only one non-zero entry to obtain the subsea equipment displacement: $\mathbf{C} = [1,0,\cdots,0]$, U is the control trajectory $\Upsilon_o(L,t)$, y is the system output $\Upsilon(0,t)$, $\mathbf{X}$ is the state vector: $\mathbf{X} = \left[Y_1,\cdots,Y_N,\frac{\mathrm{d}Y_1}{\mathrm{d}t},\cdots,\frac{\mathrm{d}Y_N}{\mathrm{d}t}\right]^T$, and $\dot{\mathbf{X}}$ is its time derivative.

The state matrix describes the system behavior:

$$\mathbf{A} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ \mathbf{A}' & -\mathbf{K}\end{bmatrix}_{2N\times 2N} \tag{34}$$

where $\mathbf{0}$ is a square matrix of order N, $\mathbf{I}$ is an identity matrix of order N, $\mathbf{K}$ is a diagonal matrix containing the accelerations associated to the hydrodynamic damping and $\mathbf{A}'$ is the stiffness matrix, which is a symmetric matrix containing the structure internal accelerations according to the discrete model presented.

Table 1. Parameters of the marine riser installation system

Parameter	Value	Unit
Riser length	2000	m
Riser outer diameter	0.55	m
Riser inner diameter	0.50	m
Riser drag coefficient	1.2	
Riser added mass coefficient	2	
Subsea equipment mass	4×10^5	kg
Subsea equipment diameter	4.6	m
Subsea equipment drag coefficient	0.4	
Subsea equipment added mass coefficient	0.5	
Steel Young's modulus	210×10^9	Pa
Steel density	7.860×10^3	kg/m^3
Sea water density	1.025×10^3	kg/m^3

5. RESULTS

The parameters assumed in all simulations are presented in Tab. 1. The subsea equipment is assumed to be a sphere of steel, the same material as the riser. The riser is assumed to be fulfilled of sea water. The system was considered to be stationary at initial condition ($t = 0$), with the riser and the subsea equipment completely aligned with the vertical axes.

The open-loop solution is found integrating numerically Eq. (29). Thus, assuming $z = L$, being L the riser length, the control trajectory $\Upsilon_o(L, t)$ for the DP system is found. To verify the accuracy of the analytical solution proposed (Eq. (29)) with all its assumptions and simplifications, the control trajectory $\Upsilon_o(L, t)$ is used as the system input for numerical simulation of the discrete system (Eq. (33)). During all numerical simulations it was considered a system of order 600 and no environmental disturbances was considered (waves and currents). Figure 3 shows the numerical simulation response in blue for the subsea equipment trajectory $\Upsilon_N(0, t)$. Also is presented in red the reference subsea equipment trajectory $\Upsilon_R(0, t)$ and in black the control trajectory $\Upsilon_o(L, t)$. $\Upsilon_R(0, t)$ is a polynomial trajectory that smoothly moves the subsea equipment to a final position 1 m away from the initial position, as presented in the Motion Planning section as a boundary condition of the system. This first numerical simulation also considered the linear damping assumption.

Regarding Fig. 3, one can note the numerical simulation response $\Upsilon_N(0, t)$ has a reduced overshoot of 15 mm, then oscillates around the final position. Despite the overshoot and oscillation, the numerical simulation response $\Upsilon_N(0, t)$ is close enough to the reference subsea equipment trajectory $\Upsilon_R(0, t)$. As expected, the assumption of modeling the riser as a cable by neglecting the bending resistance is valid and did not impact the system response.

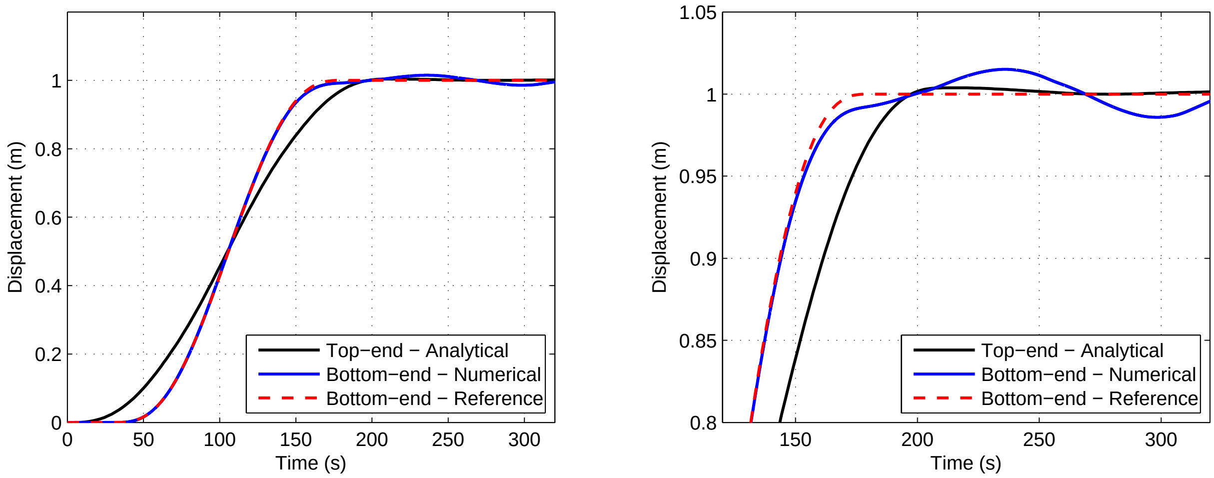

Figure 3. Subsea equipment trajectory response considering linear damping, control trajectory and reference subsea equipment trajectory

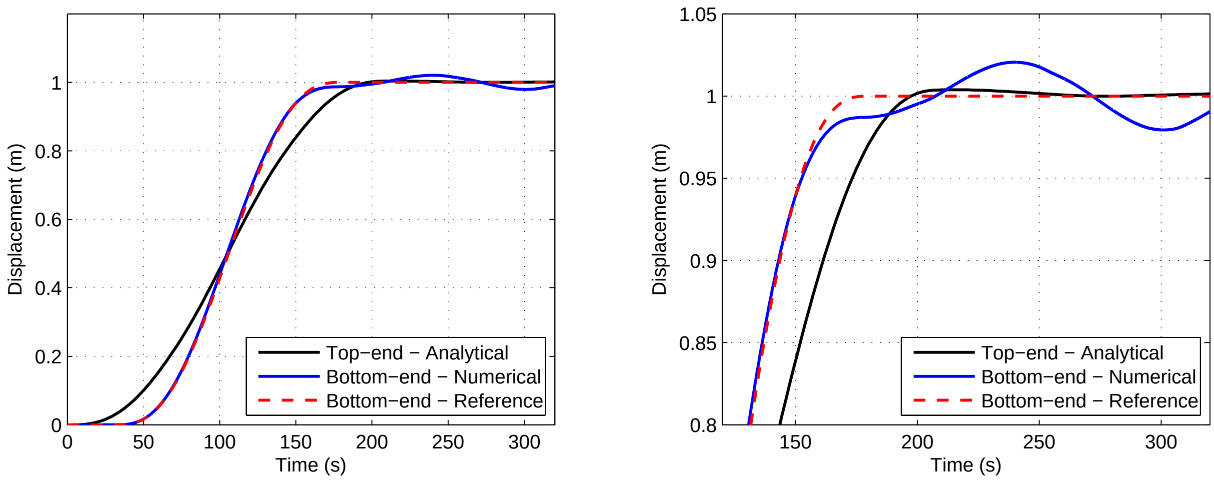

Figure 4. Subsea equipment trajectory response considering non-linear damping, control trajectory and reference subsea equipment trajectory

The numerical method can also be used to simulate the system considering the non-linear dissipative drag term of the Eq. (3). The linear dissipative drag 2α of the discrete system is replaced by the non-linear damping $\frac{\mu}{m}\left|\frac{\partial \Upsilon}{\partial t}(z,t)\right|$. Again, the control trajectory $\Upsilon_o(L,t)$ is used as the system input for numerical simulation. Figure 4 shows in blue the numerical simulation response $\Upsilon_N(0,t)$, in red the reference subsea equipment trajectory $\Upsilon_R(0,t)$ and in black the control trajectory $\Upsilon_o(L,t)$.

Regarding Fig. 4, is also observed a small overshoot of the numerical simulation response $\Upsilon_N(0,t)$ and a oscillation around the final position. In this case, the overshoot was of 20 mm. Despite that, the numerical simulation response $\Upsilon_N(0,t)$ is close enough to the reference subsea equipment trajectory $\Upsilon_R(0,t)$. It is important to note that the numerical response of the system considering the non-linear damping is close to that obtained considering the linear damping case, as shown in Fig. 3. Thus, the linearization of the damping is valid and did not have a significant impact on system response.

6. CONCLUSION

The present article presents the development of an active control system in order to propose a trajectory control strategy to the subsea equipment positioning. The control trajectory for the DP system was obtained by an analytical solution of the governing equation for a damped tensioned riser with its top-end fixed to a platform and with a subsea equipment attached at its bottom-end. The riser is damped by the external hydrodynamics forces from the fluid in which the riser is displaced.

The analytical solution provides the displacement of the riser in the time-domain for a specified reference trajectory of the subsea equipment. This allows determining the precise movement of the platform that is required to obtain the desired position of the subsea equipment. The accuracy of the analytical solution proposed, as well its assumptions, were verified through numerical simulations with the finite difference method.

The overshoot observed for the non-linear case was of 20 mm, which is totally acceptable considering the accuracy required for the installation of subsea equipment. Thus, the analytical solution could be validated by numerical simulation. Taking the displacement of the riser top-end, it is determined the open-loop control trajectory for the DP system. For futures works, a close-loop should be presented to compensate the unknown environmental disturbances and perturbations that tends to take the subsea equipment away from the reference trajectory.

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8. RESPONSIBILITY NOTICE