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NUMERICAL AND EXPERIMENTAL ENDENTATION ANALYSIS OF PLANE HYPERELASTIC MEMBRANES

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Abstract. *The study of membranes behavior when subjected to pressures and tensions from all kinds, as well as their mechanical properties, is receiving a noticeable attention over the last years due to several applications on various engineering fields, including, actuators, robotics, sensors, airspace, bioengineering and inflatable structures for civil engineering. This work presents a numerical and experimental study of a membrane subjected to an indentation and penetration transversally to the its undeformed plane. Friction is considered on the interface of the membrane and indenter, which makes this study a pioneer on this kind of analysis. The numerical studies were performed on Abaqus CAE finite element software. The experimental tests are intended to validate the numerical results. Experimental apparatus were developed especially to perform the tests done in this research.*

Keywords: *elasticity, membranes, friction, rubber, large deformation*

1. INTRODUCTION

The study of membranes behavior and their properties has received a remarkable attention in the last years because of the wide applicability of membranes in several areas of engineering, including miniature actuators, robotics, sensors, aerospace, filters, bioengineering and inflatable structures for civil engineering.

The membrane theory has a wide application in the biomedical area, especially biological tissues modeling, that was published in numerous important works in recent decades.

The problem of membrane indentation has already been addressed in other recent studies, however, none of these consider the effects of friction on the interaction between the structures comprising the problem. This study aims to present a numerical and experimental analysis that considers the effects of friction. The continuous indentation analysis, and finally the membrane rupture has great importance in various sectors of industry. In recent years several surveys and products were developed with membranes. These have gotten more and more applications, especially for use of inflatable and tensioned temporary structures. The membrane structures combine fast assemblage, portability and low weight compared with fully rigid structures.

The analysis of structures subjected to large deformations has its beginning with the pioneer work of Rivlin (1948) in nonlinear elasticity. This work makes use of notation and analysis proposed by Green and Adkins (1960).

More recent advances in the field of finite elasticity can be found in Libai and Simmonds (1998) so as in Fu and Ogden (2001).

Choosing the right energy functional for each material is essential in their characterization for the analysis using the finite deformations elasticity theory. The simplest and thus more limited functional is Neo Hookean, which is a variation of the Mooney-Rivlin model (1940), and the Mooney-Rivlin model itself.

Ogden (1972), Yeoh (1993), and Arruda and Boyce (1993), among others, have proposed models of higher order and therefore more complex for the characterization of hyperelastic materials, rubbers and elastomers. Selvadurai (2006) as well as Saccomandi and Ogden (2004) present a critical analysis of various models for hyperelastic materials.

The work of Garcia, Ruiz and Lopez (2005) presents an interesting analysis of the characterization of hyperelastic materials and their peculiarities. They present in this paper studies of the relationship between the characterization made from the strain stress results obtained by uniaxial and biaxial tests.

Pamplona (2014) has numerous works on the analysis of large deformations in hyperelastic membranes.

2. NUMERICAL ANALYSIS

The constitutive laws used on the numerical analysis of the membrane problem are written based on the strain invariants. Being known the strain parameters and the material constitutive laws it is possible to obtain the deformation energy density function w.

The elastic deformation energy E, is calculated by the integration on the non-deformed volume, of the function w, which is the elastic potential measured by unity of volume on the non-deformed material as seen on Eq. 1, where V is the membrane non-deformed volume.

$$E = \int_v w \, dV \tag{1}$$

Considering the material as uncompressible, the deformation energy function depends only of the first and second deformation invariants.

For each constitutive law applied to the material there is a different function for w as can be seen on Tab. 1.

Table 1 - Constitutive equations for the materials

Neo–Hookean	$w = C_1(I_1 - 3)$
Mooney Rivlin	$w = C_1(I_1 - 3) + C_2(I_2 - 3)$
Ogden	$w = \sum_{i=1}^{n} \mu_n \frac{\left(\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3\right)}{\alpha_n}$

For a Neo-Hookean material only $C_1$is needed to describe the material, for a Mooney Rivlin $C_1$and C_2, and for Ogden material $C_{1,\ldots n}$ and $\alpha_{1,\ldots n}$ where n is the order of the equation. An Ogden material can be from order 1 to n, but on this work only first and second order were studied.

The energy formulations for rubber used in this work are based on the strain energy density function w, a scalar function which relates the deformation energy of a material with the strain gradient.

The function w is commonly used to describe a hyperelastic material from the principle that the material stress can be obtained from the derivative of the power density function with respect to deformation of the material.

For a hyperelastic and isotropic material this function relates the elastic potential energy stored just by deformation of the material, not taking into account the effects of hysteresis and heat dissipation.

2.1 Neo-Hookean Model

The Neo-Hookean model was originally proposed in 1948 by Ronald Rivlin to characterize the stress-strain relationship of hyperelastic materials whose behavior are nonlinear. This model is based on the characteristics of the polymer chains present in the rubber. These chains can move in relation to the other within the body as a result of applying a tension. After a value of approximately 20% elongation, this displacement capacity reaches the limit and cross strings begin to restrict movement, what causes a significant increase in deformation resistance of the material. This feature is not shaped by Neo-Hookean model, which is why it starts to deviate from the experimental values in the case of large deformations. This model is also limited to describe bi-axial stresses and is not suitable for the analysis of the problem in question, which are expected to have large deformations and axial symmetry, tensions in two directions.

The strain energy density for an incompressible material is:

$$w = C_1(I_1 - 3) \tag{2}$$

Where C_1 is the material constant parameter and I_1 is the first invariant of the deformation tensor, which is:

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \tag{3}$$

Where $\lambda_1^2,\ \lambda_2^2\ e\ \lambda_3^2$ are the principal extensions.

2.2 Mooney-Rivlin Model

The Mooney-Rivlin model was originally proposed in 1940 by Melvin Mooney and subsequently expressed in terms invariant by Ronald Rivlin in 1948. It is also a model that characterizes the stress-strain relationship in hyperelastic materials where the strain energy density is described as an invariant linear combination of two, namely:

$$w = C_1(I_1 - 3) + C_2(I_2 - 3) \quad (4)$$

Were $C_1$and $C_2$are the material constants measured experimentally and $I_1$and $I_2$are the first and second Cauchy-Green invariants and $\lambda_{1,2,3}$ are the extensions of the membrane on the axis.

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad (5)$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 \quad (6)$$

2.3 Ogden Model

The Ogden model to describe the behavior of hyperelastic materials is the most complete of the three models discussed in this work. It was described by Ray W. Ogden 1972. This model describes the material as isotropic incompressible with the power density function on Eq. (7).

$$w = \sum_{p=1}^{N} \frac{\mu_p}{\alpha_p}(\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3) \quad (7)$$

Were N, μ_p and α_p are the material constants.
If the material being described is incompressible we can rewrite the equation as:

$$w = \sum_{p=1}^{N} \frac{\mu_p}{\alpha_p}(\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_1^{-\alpha_p}\lambda_2^{-\alpha_p} - 3) \quad (8)$$

By comparison we see that the Ogden model approaches the Neo-Hookean model when $N = 1$ and $\alpha = 2$ and from Mooney Rivlin model when $N = 2, \alpha_1 = 2, \alpha_2 = -2$ assuming by incompressibility that $\lambda_1^2\lambda_2^2\lambda_3^2 = 1$.

3. FINITE ELEMENT ANALYSIS

The numerical analysis and the results of the membrane behavior in the process of continuous indentation will be analyzed experimentally and by the use of finite element analysis (FEA). For simplification purposes the problem of an initially plane membrane being transversely indented by a rigid indenter was divided in two parts, indentation with and without lateral contact of the membrane and the indenter i.e. when the membrane geometry has vertical tangent at the top side of the cone as can be seen on Fig 1.

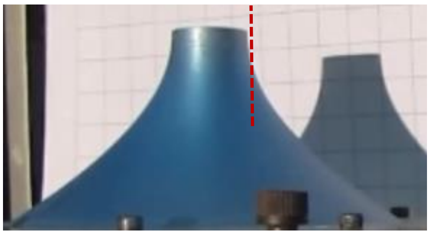

Figure 1 – Indented membrane

Analyses were made using *Abaqus* CAE v6.11 software for different materials, different coefficients of friction on contact. It conducted a parametric study of different diameters and thicknesses indenter membrane.

An element of SAX2 type, quadratic axisymmetric, peel and quadratic relationship was used. The discretization of the membrane in its entirety was not homogeneous, i.e., elements of different areas have different sizes from 0.001mm to 1mm depending on the region of the membrane. The model used has 722 nodes and 361 elements, number found after conducting a convergence analysis.

The use of continuous linear axisymmetric type membrane elements (CAX4H) and quadratic relationship membrane elements (MAX2) were tested. The results of the two simulations showed no significant differences. It was used SAX2 shell elements for the analysis as they don`t need initial tensions to obtain convergence.

The friction was modeled as Coulomb model and the friction coefficients were measured experimentally. The FEA was made for four different friction coefficients (µ): 0, 0.2, 0.46 and 0.8 and two steps of the indentation process were analyzed, the point of lateral contact as described and the maximum displacement of the indenter of 90mm.

Neo-Hookean, Mooney Rivlin and Ogden of first and second order models were simulated by FEA. Figures 2 to 5 show the visual results for the Neo-Hookean model analysis.

For µ=0 the results can be seen on Fig 2. The two on the left show the beginning of lateral contact at 50mm and at the right the results for 90mm displacement where is possible to see much higher tensions. It is also clear that there are no differences on the tension at the top of the indenter and on the annular region on the top of the side due to the lack of friction.

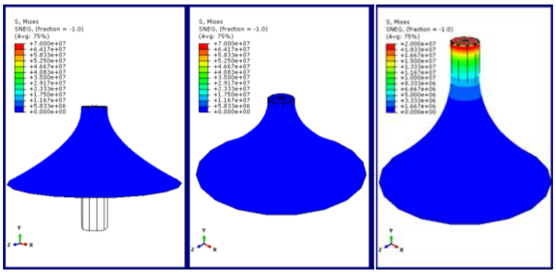

Figure 2 - FEA for µ=0

The results for µ=0.2 are shown on Fig 3. The two on the left show the beginning of lateral contact at 52mm and at the right the results for 90mm. There is around 50% difference between the tensions shown on the top and on the side of the indenter due to the presence of friction on this case.

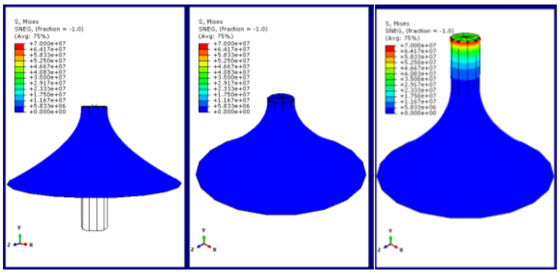

Figure 3 - FEA for µ=0.2

Figure 4 shows the results for µ=0.46 where is possible to see the results for the beginning of lateral contact at 70mm displacement and at 90mm.

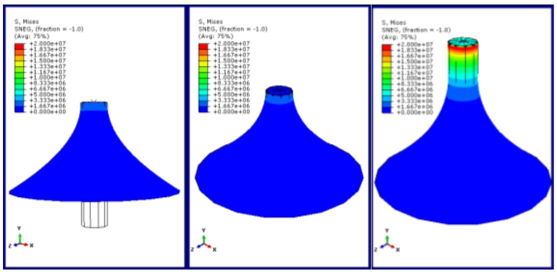

Figure 4 -FEA for µ=0.46

On Fig. 5 is shown the results of the FEA for µ=0.80. It becomes clear that the presence of a big friction coefficient reduces the movement of the membrane on the top of the indenter and by so reduces the deformation on this region that can be seen by a darker color on the image.

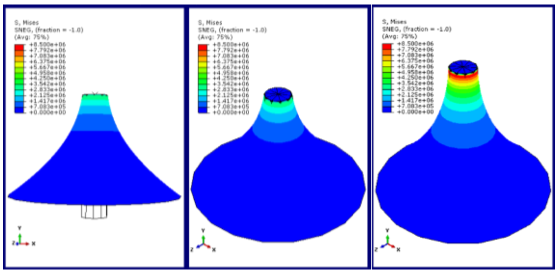

Figure 5 - FEA for μ=0.8

Similar results on the respect to the distribution of the stresses on the membrane were observed to all the other models studied although different normal reaction forces on the indenter the end of indentation and the maximum tension at the end of indentation and at 90mm were observed. This results are compared on section 5 of this work.

4. EXPERIMENTAL ANALYSIS

The first part of the experimental analysis was to characterize the material on its mechanical properties by elongation tests and friction tests, both made on an *Instron* traction machine using appropriate accessories. The membrane used on all tests was a dental dam that had its thickness measured. Only three friction coefficients were obtained experimentally, μ=0 was obtained using water based lube on the membrane interaction area. μ=0.46 was the friction on the dry membrane in contact with the aluminum indenter, and μ=0.8 mas obtained by adding pieces of 600 grain sandpaper to the surface of the indenter. Figure 6 shows the indentation process with sandpaper at 30, 60, 65 and 90mm.

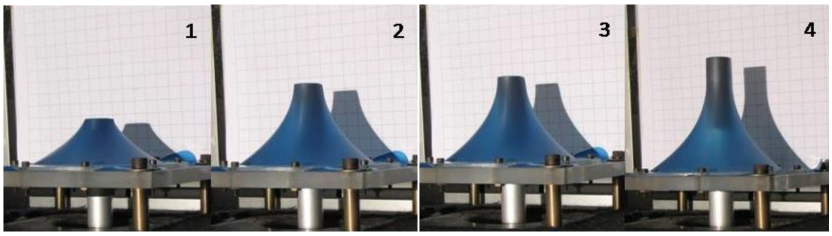

Figure 6 - Different steps of the indentation process

5. RESULTS AND CONCLUSION

In Fig. 7 we see a comparison chart of results Force vs. Displacement for all models and the experimental result for μ=0 on the left and μ=0.8 on the right. In this graph, circular marks on the lines indicate the point of the beginning of lateral contact of the membrane on the indenter. It can be observed on the graph on the left that for displacements below 40mm functional Ogden 1st order and 2nd order describe the behavior of the material, but not for higher values. Such behavior is explained by the fact that at high tensions the lubricant film layer in the experimental test is pushed out of the contact area, making the actual friction is greater than zero, which implies an increase of the reaction force recorded. In the case of μ=0.8 2nd Order Ogden and Mooney Rivlin are the ones that fit the best the experimental results.

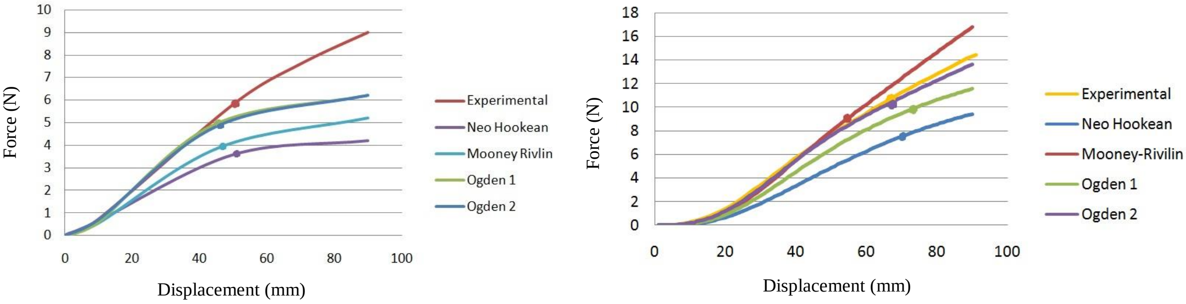

Figure 7 - Results for all models and the experimental result for μ=0 and μ=0.8

It was concluded that the characterization of a hyperelastic latex membrane through uniaxial tensile testing has differences with the results of a bi-axial tensile test, which is the case of the indentation test.

The characterization of the material of the membrane by the use of a bi-axial test is difficult to be implemented experimentally and it was not attempted in this study, although the literature (Garcia et. al., 2005) indicate that this type of test is the most suitable for hyperelastic characterization of a membrane.

For the numerical analysis of the indentation problem of the membrane was observed that the choice of the model that characterize the material influences the force vs. deformation curve and therefore must be analyzed for each test that you want to compare.

As for the numerical-experimental comparison one can see that for the case of lubricated indenter there are greater differences between the results of vertical displacement and reaction force at the end of the indentation. The big difference is explained by the difficulty in applying the lubrication in each indentation test similarly, and also the fact that the lubricant film layer is continuously expelled from intender-membrane interface as the test is done.

It follows that the models that best describe the behavior of these membranes studied in general, are Ogden 1st and 2nd order, with satisfactory results in terms of vertical reaction force and the vertical displacement of the indenter, which are the parameters that were measured experimentally. For simplicity of the equations the 1st order Ogden model is suggested as the most appropriate to describe the material used in this work.

6. ACKNOWLEDGEMENTS

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