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NON-LINEAR ENERGY HARVESTING SYSTEM CHAOS EXCLUSION VIA ACTIVE CONTROL

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Abstract. *To enhance energy harvesting systems efficiency the non-linear projects highlighted as a promised solution. Nevertheless the non-linear schemes result in chaotic system behavior which instead increases the amount of mechanical energy transfer from vibration source to vibration system is the cause of non-prediction system comportment. The system chaotic behavior can be measured by Lyapunov Exponents. Applying an Optimal Linear Control as active controller to the energy harvesting system subjected to a periodic excitation is possible to remove the chaotic behavior without losses in mechanical energy transfer. This research is part of PhD thesis related to energy harvesting system enhancement in a deep discussion about chaotic behavior exclusion via active control.*

Keywords: *Chaos, Lyapunov Exponent, Optimal Linear Control, Energy Harvesting, Efficiency*

1. INTRODUCTION

The demand of electric energy is growing faster in the last decade (Vullers et al., 2009 and Lee et al., 2013) and the necessity for long term electric energy sources is presenting as a real necessity. In the other hand the environment was damaged for petroleum as carbon emission and other sources as hydroelectric power plant has also a significant environment impact. A promised source of clean electric energy can be the energy harvesting systems (Thomas et al., 2006; Raghunathan et al., 2005; Vullers et al., 2009; Howells, 2009; Lee et al., 2013).

The harvesting system are those that harnessing energy from small energy sources as vibration, solar radiation and small gradient of temperature (Cepnik et al., 2011; Miller et al., 2011). There are a productive research in energy harvesting systems in the last few years and mostly of works are focusing in improving the transduction efficiency.

Considering the vibration as an energy source to be transduced to electric energy the prominent researches are dealing to conduct the system behavior to the resonance effect (Wu et al., 2006; Sari et al., 2008; Eichhorn et al., 2009; Blarigan et al., 2012; Tang et al., 2013) however the resonance occurs only in a few frequency band and the ambient vibration normally have a large frequency band (Harne and Wang, 2013; Tang et al., 2013). To enhance the energy harvesting system efficiency taking in consideration the resonance and the extensive bandwidth some studies are developing solutions for control the system behavior as mechanic, magnetic and piezoelectric control (Blarigan et al., 2012; Park and Park, 2013; Eichhorn et al., 2009; Challa et al., 2008; Wang and Inman, 2012; Tang et al., 2013). Among the controlled systems to increase energy harvesting system efficiency the piezoelectric solution is highlighted as the most promised (Tang et al., 2013).

The piezoelectric control can be classified as active, semi active and passive (Wang and Inman, 2012; Tang et al., 2013) considering the necessity of external electric source. The active control does not need an external electrical energy source and uses the harvested energy to conduct the system behavior.

This paper consider an active control based on Optimal Linear Control project (Rafikov and Balthazar, 2008; Chavarette et al., 2011; Chavarette, 2013a; Chavarette, 2013b; Tusset et al., 2012) to enhance a non-linear energy harvesting system efficiency subject to a periodic excitation source. The proposed controller system was already developed (Ferreira et al., 2014). On the referred paper the author considered the chaos exclusion only for a singular frequency rate and do not extended the study to a larger bandwidth. The present research expands the study to a frequency rate from 0.4 to 1.0 and analyzes the chaos exclusion for the considered band.

2. ENERGY HARVESTING SYSTEM MODEL

The energy harvesting system for vibration mechanical energy source to be transduced to electrical energy is the one studied by Erturk and Inman (2011) as shown in Fig. 1.

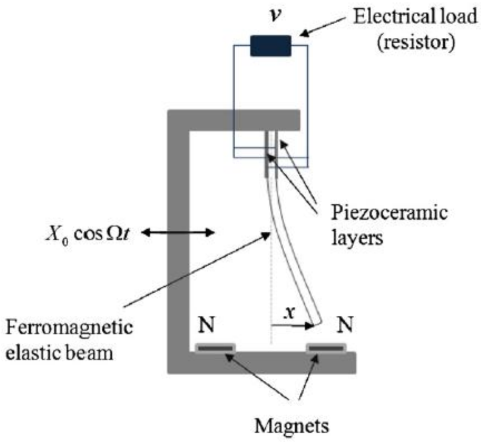

Figure 1. Non-linear energy harvesting system (Erturk and Inman, 2011)

A mathematical model of the non-linear energy harvesting constituted of a cantilever beam with a piezoelectric film of PZT attached and coupled to an electric load is given.

$$\begin{aligned} &\ddot{x} + 2\zeta\dot{x} - \frac{1}{2}x(1 - x^2) - \chi v = f cos(\Omega t) \\ &\dot{v} + \Lambda v + \kappa\dot{x} = 0 \end{aligned} \quad (1)$$

The equation is dimensionless defined according resonance were ζ is damping factor, χ is piezoelectric mechanical coupling coefficient, Λ is reciprocal of time constant for capacitive load and κ is piezoelectric electric coupling coefficient.

Isolating $\ddot{x}$ and $\dot{v}$ in equation (1):

$$\begin{aligned} &\ddot{x} = \frac{1}{2}x(1 - x^2) - 2\zeta\dot{x} + \chi v + f cos\Omega t \\ &\dot{v} = -\Lambda v - \kappa\dot{x} \end{aligned} \quad (2)$$

Adopting $y_1 = x$, $y_2 = \dot{x}$ and $y_3 = v$, the equation is given in space-state form as:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \frac{1}{2} y_1 (1 - {y_1}^2) - 2\zeta y_2 + \chi y_3 + f \cos \Omega t \\ \dot{y}_3 &= -\kappa y_2 - \Lambda y_3 \end{aligned} \tag{3}$$

Applying Runge-Kutta forth order in equation (3) for the parameters $\zeta = 0.01$, $\Omega = 0.8$, $\chi = 0.05$, $\kappa = 0.5$, $\Lambda = 0.05$ and $f = 0.083$ and considering initial conditions $x(0) = 1$, $\dot{x}(0) = 0$ and $v(0) = 0$ from samples from 0 to 500 (smaller energy orbity) and initial samples $x(0) = 1$, $\dot{x}(0) = 1.3$ and $v(0) = 0$ from samples from 500 to 2,500 (higher energy orbit) it is determined the voltage rate, time history and phase portrait considering samples in interval of 0.1 totalizing 25,000 time samples according Figures 2a, 2b and 2c.

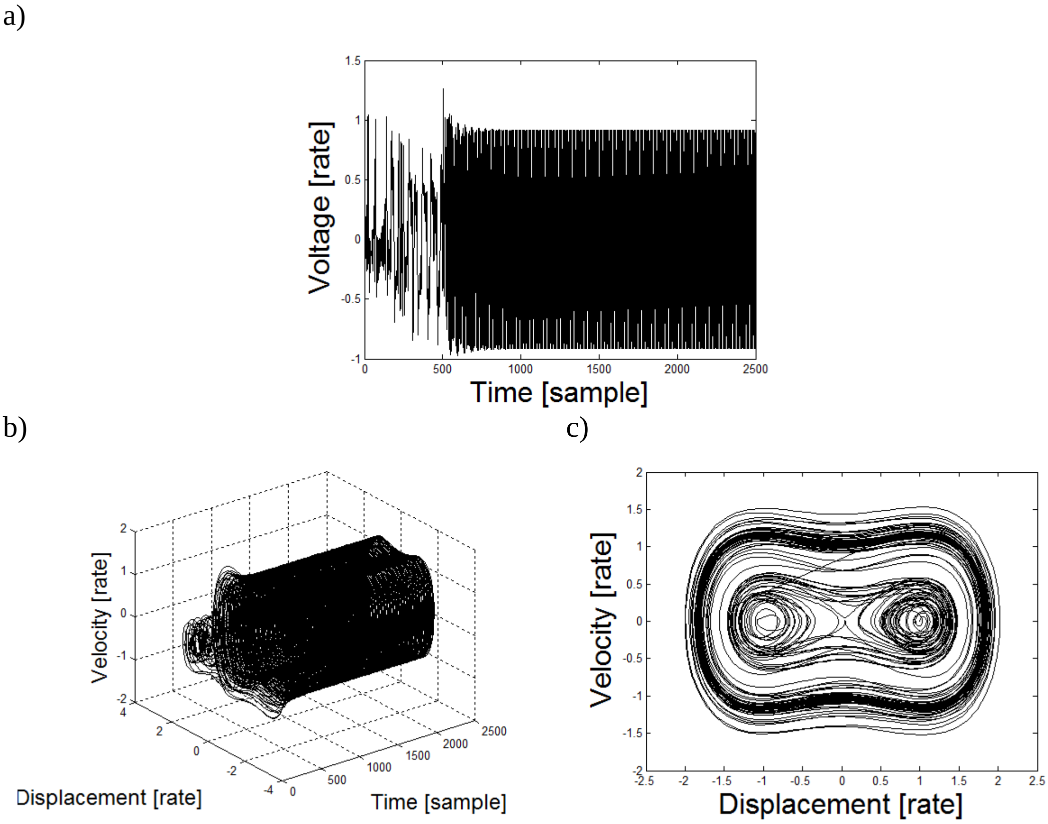

Figure 2. (a) voltage rate, (b) time history, (c) phase portrait.

3. CONTROL OF CHAOTIC SYSTEMS

The controller used in this research is Optimal Linear Control (OLC) which guarantees the linear control application in non-linear systems. Considering a controlled non-linear system given by equation 4 (Rafikov and Balthazar, 2008; Chavarette et al., 2011; Chavarette, 2013a; Chavarette, 2013b; Tusset et al., 2012).

$$\dot{y} = A(t)y + h(y) + Bu, \qquad y(0) = y_0 \tag{4}$$

Were $y \in \Re^n$ is a state vector, $A(t) \in \Re^{nxn}$ is a boundary conditions matrix (parameters) which elements are time depending, $B \in \Re^{nxm}$ is a matrix of constants, $u \in \Re^m$ is a control vector and $h(y) \in \Re^n$ is a vector which the elements are continuous non-linear functions, $h(0) = 0$. It is highlighted that the chosen of $A(t)$ is not the only influence of controller efficiency. For a finite time interval and A, B, Q e R been matrixes of constants elements, the positive defined matrix P is the solution of the algebric non-linear Riccati equation, dada given by:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{5}$$

4. CHAOS EXCLUSION

Using an Optimal Linear Control to enhance energy harvesting efficiency was already explored by Ferreira et al. (2014) and also was found chaos exclusion for the considered system as shown in Figure 3a (non-controlled chaotic system) and 3b (non-chaotic system with OLC controller). The chaotic behavior of the non-controlled system was confirmed by Lyapunov Exponents. However the referred study considered only the external excitation frequency for Ω = 0.8 which characterizes the resonant system behavior.

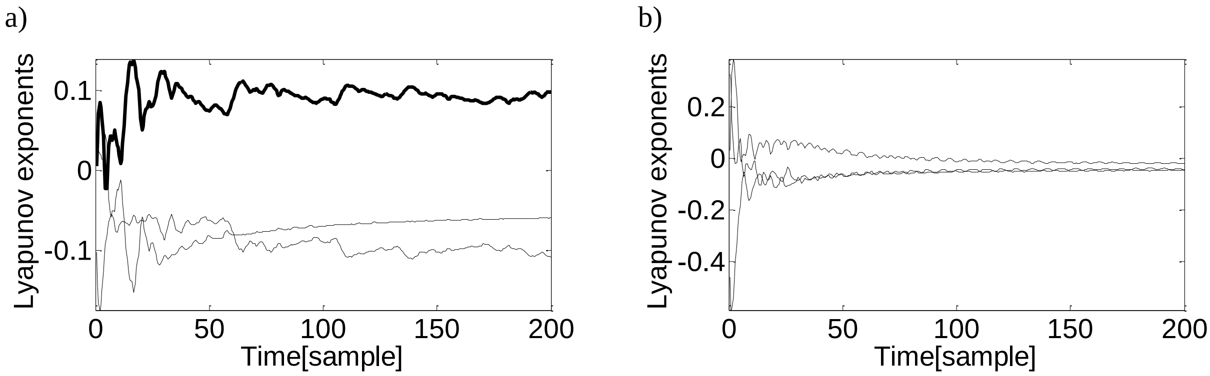

Figure 3. (a) non-controlled system and (b) controlled system via OLC. (Ω = 0.8)

Extrapolating the excitation frequency for all the studied bandwidth from Ω = 0.4 to Ω = 1.0 there were the following Lyapunov Exponents found.

Table 1. Lyapunov Exponents for Ω = 0.4 to Ω = 1.0.

Ω	λ_1	λ_2	λ_3
0.4	-0.0099686	-0.010902	-0.049129
0.5	-0.0061835	-0.014537	-0.049279
0.6	-0.0059287	-0.014954	-0.049117
0.7	-0.0012167	-0.020041	-0.048742
0.8	0.098421(1)	-0.05965	-0.10877
0.9	0.061115(1)	-0.054327	-0.076789
1.0	0.096093(1)	-0.049164	-0.11693

(1) Positive Lyapunov Exponent indicating chaotic behavior

For Ω = 0.8 to Ω = 1.0 it is found a chaotic behavior. The study for Ω = 0.8 is already presented. Considering Ω = 0.9 the behavior is shown in Figure 4a without control and 4b for OLC controlled system.

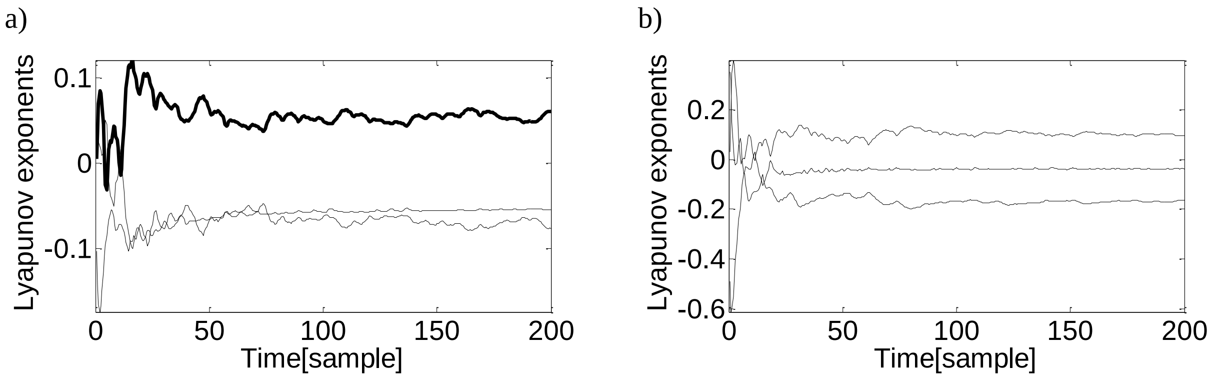

Figure 4. (a) non-controlled system and (b) controlled system via OLC. (Ω = 0.9)

For time samples from 0 to 200 it is not possible to affirm that the OLC controller could eliminate the chaotic behavior. Extending the study to more time samples it is found an asymptotically behavior to exclude the chaos as shown in Figure 5.

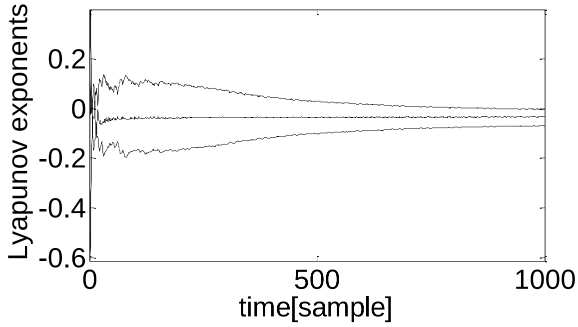

Figure 5. Lyapunov Exponent for OLC controlled system. (Ω = 0.9)

Considering Ω = 1.0 the chaotic behavior is given by Figure 6.

a) b)

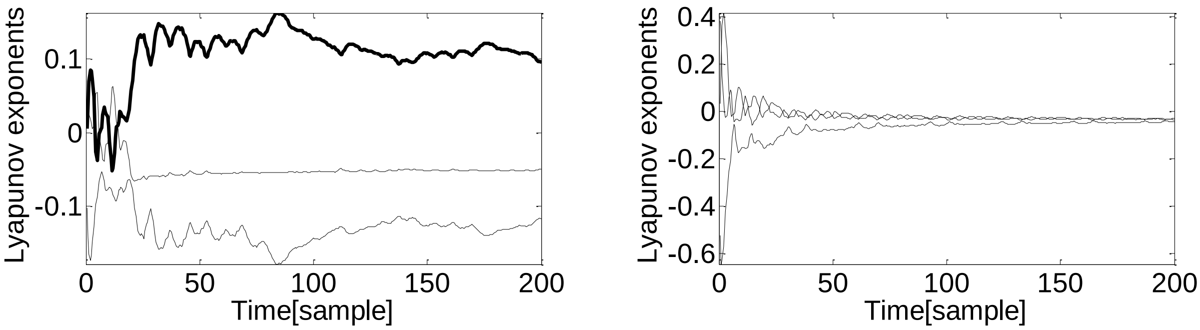

Figure 6. (a) without control and (b) OLC control. (Ω = 1.0)

For Ω = 1.0 it is also found that the OLC controller excluded the chaotic system behavior. It is important to be notice that the system energy for controlled system grows for all studied frequency rate and significantly for the cases were the chaotic behavior is excluded.

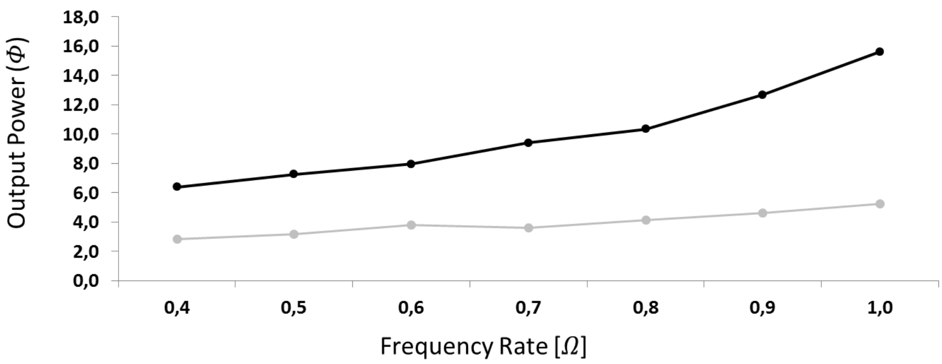

Figure 7. Net output power according frequency rate Ω. Grey Line: without control. Black Line: OLC controller (Ferreira et al., 2014)

5. CONCLUSIONS

The present work analyze a mathematical model of the non-linear energy harvesting constituted of a cantilever beam with a piezoelectric film of PZT attached and coupled to an electric load to frequency bandwidth from $\Omega = 0.4$ to $\Omega = 1.0$. As shown in Figures 3, 5 and 6 the OLC controller excluded the chaotic system behavior. It is also highlighted that when the chaos is eliminated the system became even more efficient to transduce vibration to electric power as shown in Figure 7. All the matrixes were derived to evaluate the chaotic behavior according to frequency rate.

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