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# NUMERICAL AND EXPERIMENTAL STUDY OF ACOUSTIC HORNS

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***Abstract.*** *Horns are acoustic elements specially designed for maximum transmission of sound pressure, for example, in sound systems (mainly external and warning equipment), musical instruments, cleaning apparatus, receivers and microwave transmitters. The focus of this work is to validate numerical models to simulate the acoustic behavior of horns. The acoustic behavior of horn is obtained by numerical simulation of mathematical models obtained by the finite element method and by experimental data. To obtain experimental data it was used the method of two microphones. The parameter used to compare the models was the reflection coefficient. The results obtained by comparing numerical and experimental data showed some divergences due to the numerical model limitation, but the results were good and show promise.*

***Keywords:*** *Acoustic Horn, FEM, Coefficient of Reflection, Sound Pressure*.

## 1. INTRODUCTION

According Udawalpola (2008) horn-like structures appear in devices for both acoustic and electromagnetic waves. Brass instruments, sirens, and outdoor loudspeakers contain horns that participate in the generation and distribution of sound waves. Horns are also found in microwave receivers and transmitters, used for satellite communications, for instance, and small horn-like devices even appear in optical devices.

The horns are used in situations to obtain the maximum power transmission, such as signaling devices, sound large environments, surface cleaning and filters. According to Kolbrek (2008), Post and Hixson (1994), acoustic horns are elements with two basic functions, transmission of sound energy and the ability to direct the sound waves, directivity.

In a sound source, often called the "driver" a lot of energy is lost in the transformation of electrical power into sound power, mechanical and electromagnetic inefficiencies, Udawalpola (2011). The use of a horn to sound wave transmitter avoid the loss of sound energy transmission. It is responsible for transmitting plane its geometry and discharges them into the environment with minimal losses. Such losses are the result of the geometry change that generates pressure waves contrary to the propagation. The author mentions that a tube can also be given as efficient in transmitting the waves, but since it adopted the hypothesis of "infinite tube." Another important point mentioned is the greatest loss of energy in low frequency than at high frequency due to the length of the waves. In the case of low frequency, the horn must have sufficient length and diameter to cope with longer wavelength, since the high frequency, short wavelength requires lower horns.

As important as the transmission of sound energy is the directivity. According Kolbrek (2008), the ability of a horn to concentrate the sound waves from a source prevents energy is wasted on unwanted areas. This fact makes the horns, essential instruments in acoustic projects in open environments, and it is fundamental the sound directivity effect providing listeners the same power regardless of the position occupied on the environment and also avoid transmitting sound waves to unwanted sites.

Noreland (2010) and Udawlpola (2010) built a physical model of a horn with geometries defined by optimization routines. The measurement made by Udawpola (2010) was performed using the device designated as impedance measurement head. This device is responsible for measuring the pressure contrary to the wave emitted and the comparative values of the reflection factor. The author also cites the importance of the test is conducted in a controlled environment, an anechoic chamber which is an element with directivity, can be affected by external noise.

This work is developed in two steps. The first is the obtainment of the numeric model of a commercial horn (used for car audio installation) through geometric measurement, mesh generation and assembly of the model in FEM. The second step is the experimental measurements of the reflection coefficient to be compared with the numeric values.

## 2. MATHEMATICAL MODEL

In this work is obtained a mathematical model to solve the wave equation for axisymmetric applications. This method can be applied in systems with different geometries such as horns, pipes, mufflers, etc., considering cylindrical symmetry.

For axisymmetric problems, it is assumed that the sound pressure does not depend on θ coordinated, therefore, does not varies radially about the longitudinal axis. Thus, the equation that governs the phenomenon is reduced to:

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}+\frac{\partial^2 p}{\partial z^2}-\frac{1}{c^2}\frac{\partial p}{\partial t}=0 \qquad (1)$$

Which can be conveniently rewritten in matrix form as:

$$\left[[\mathbf{K}]-k_0^2[\mathbf{M}]\right]\{p\}=\{f\} \qquad (2)$$

Where $k_0=\omega/c$; [**K**] is the stiffness matrix, [**M**] is the inertia matrix and {f} is the excitation vector. They can explicitly be written as:

$$[\mathbf{K}]=\int_{\Omega^*}\left[\frac{\partial\phi_i}{\partial r}\frac{\partial\phi_j}{\partial r}+\frac{\partial\phi_i}{\partial z}\frac{\partial\phi_j}{\partial z}\right]r\,dr\,dz \qquad (3)$$

$$[\mathbf{M}]=\int_{\Omega^*}\phi_i\phi_j\,r\,dr\,dz \qquad (4)$$

$$\{f\}=-j\rho_0\omega\int_{\Gamma}u_n\phi_j\,r\,d\Gamma \qquad (5)$$

## 3. THE HORN GEOMETRY AND EXPERIMENTAL BENCH

The most important parameter for comparison of the experimental model with the theoretical is the geometry, not only the form but the dimensional accuracy between models. For an experimental analysis was chosen commercial horn, easily found in the market and made of injected plastic.

In order to simplify the execution of the practical analysis we used a horn found in sound equipment stores. The CHC model of 14-50 trinyum Hinor manufacturer was chosen. It shares some important qualities: it is made of aluminum (highlighting the fact that the dimensional stability and less influence of one's own horn vibration during testing); its dimensions enabling be assessed in three-dimensional geometry measuring equipment and ease of adaptation to the instruments already present in the laboratory. The drawing (Figure 2) CHC 14-50 horn template implementation would be sufficient to generate an accurate mathematical model, but the manufacturing process employed does not guarantee the accuracy of the forms, mainly for the inner part. As the horn is manufactured using sand mold casting, which is a naturally imprecise process that after being fused, and the finishing is done by a paint layer, then the drawing is only a reference, and thus an experimental measurement is necessary to obtain the correct shape of the horn.

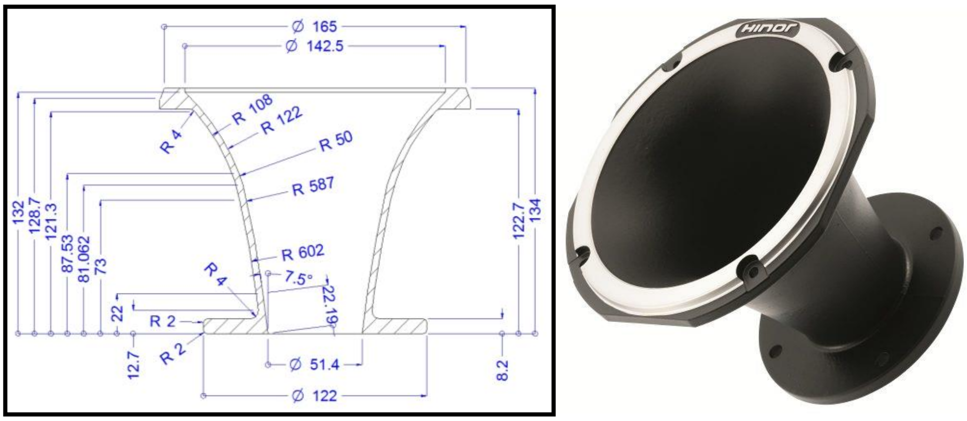


Figure 1: Sectional view of the horn CHC 14-50, drawing that is used for the production of foundry molds and isometric view on the right.

The geometry of the horn was obtained using a coordinate measuring machine, three-dimensional model Scirocco DEA with a capture software of the points PC-DMIS. After capturing some points in the outside part of the horn, which may be considered less important, a scan of the whole inner part is made. After establishing a reference axis, measurements were taken in the longitudinal direction, from the smaller diameter to the larger diameter. The distance from one point to another is automatically calculated by the acquisition software and an imaginary triangle is created between the last two measured points. If the angle between the hypotenuse and the adjacent cateto does not meet the established standards, the measuring of the last point is redone in another position. The process of capturing the points of the inner face is given automatically, after the programming performed by the operator.

The measured along the axis was repeated three more times, varying 90 ° in the measurement starting point. Based on the points collected by the coordinate measurement, it generated an IGS file with the points with two coordinates. As expected, differences were found between the theoretical and the physical models. The most significant differences were lack of parallelism between the initial and the end circles of the horn and the roundness of the geometry that should be circular. During the modeling and reconstruction of the part in 3D, Solid Works® software was used, and some considerations were made:

- The points captured by the coordinate measuring of the inner face of the horn were joined using the "spline" function 3D software;
- From the point cloud, the sketches that formed the basis for the construction of three-dimensional geometry were generated;
- To create the inner surface of the horn, circles were created parallel to the horn input, through at least three of the baselines;
- A surface was created from these circles using the tool “loft” do Solid Works®.

The experimental measurements showed some distortions relative to the drawing and especially the lack of longitudinal symmetry ie is non-axisymmetric. However, for theoretical analysis, it is assumed that the horn is symmetric along the longitudinal axis, so the problem will be solved in 2D. This fact is important because the 3D representation has not played the "roundness" found in measurements. To solve the problem using the finite element method, it was added to the model the intake tube and then it was defined the domain where the analysis will be done. Using the meshing system of the Algor® software, a linear triangular mesh was generated. With a computer software routine developed in Fortran®, the mesh was regularized, renumbered and a node was inserted in the middle of each side of the element by turning in a quadratic triangle element.

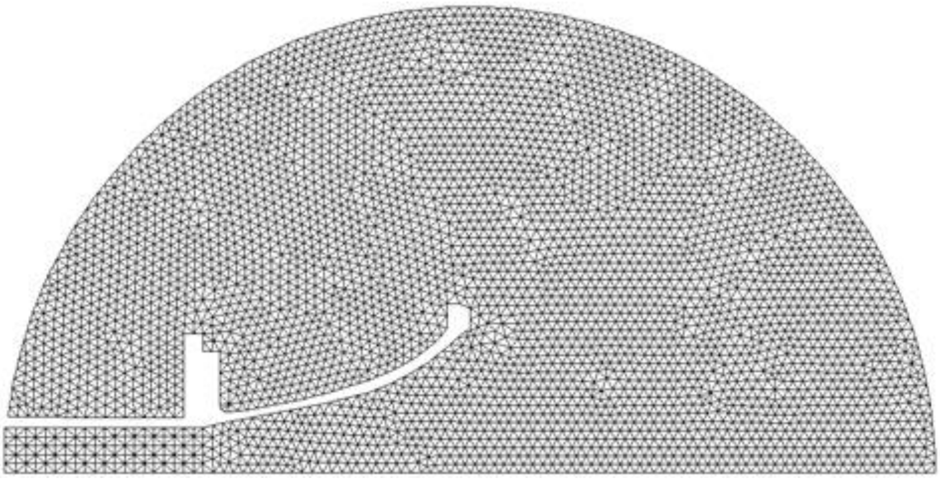

Figure 2: Finite Element Mesh, quadratic triangular elements with 16.279 nodes and 7956 elements.

### 3.1 Two microphone method

This measurement method is based on plane wave theory, Lima (2008), for propagation in ducts. For propagating plane waves, it defines actuating sound field in accordance with Figure 3, such as:

$$p(z,f) = p_i(f)e^{+jkz} + p_r(f)e^{-jkz} \tag{6}$$

where, $j = \sqrt{-1}$, p(z,f) is the Fourier transform of the sound pressure, $p_i$ is the sound pressure propagating in incident direction, $p_r$ is the sound pressure propagating in reflected direction, “f” is the excitation frequency and “k” is the wavenumber.

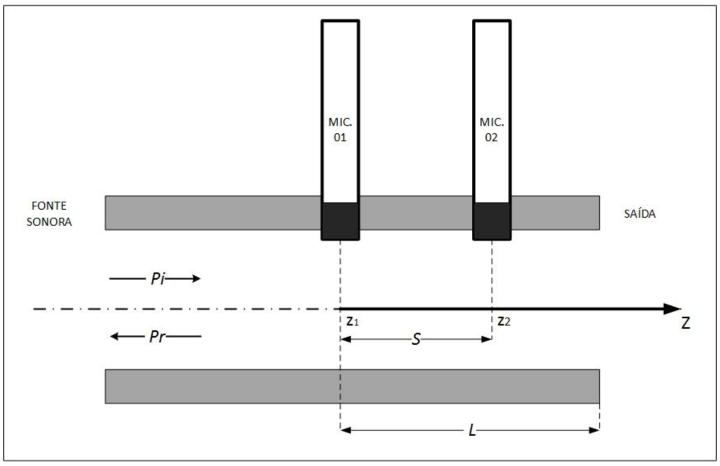

Figure 3: A sketch of an impedance tube configuration for two microphones method.

The complex sound pressure at the position of the microphones, can then be calculated as:

$$p_1(f) = p_i(f)e^{+jkz_1} + p_r(f)e^{-jkz_1} \tag{7}$$

And

$$p_2(f) = p_i(f)e^{+jkz_2} + p_r(f)e^{-jkz_2} \tag{8}$$

By solving these equations for the incident and reflected portion, we have:

$$p_i = \frac{1}{2j\mathrm{sen}[k(z_1 - z_2)]}\left(p_1 e^{-jkz_2} - p_2 e^{-jkz_1}\right) \tag{9}$$

And

$$p_r = \frac{1}{2j\mathrm{sen}[k(z_1 - z_2)]}\left(p_2 e^{+jkz_1} - p_1 e^{+jkz_2}\right) \tag{10}$$

The transfer function of the incident and reflecting waves between the microphone positions are defined as:

$$H_{12} = \frac{p_2(f)}{p_1(f)} = \frac{p_i(f)e^{+jkz_1} + p_r(f)e^{-jkz_1}}{p_i(f)e^{+jkz_2} + p_r(f)e^{-jkz_2}} \tag{11}$$

Dividing all terms by $p_i$:

$$H_{12} = \frac{p_2}{p_1} = \frac{e^{+jkz_1} + \frac{p_r(f)}{p_i(f)}e^{-jkz_1}}{e^{+jkz_2} + \frac{p_r(f)}{p_i(f)}e^{-jkz_2}} \tag{12}$$

If the tube is straight, it can be assumed that the passive acoustic system is linear, then there is a linear relationship between $p_i$ and $p_r$. Assuming that the direction of the z axis is positive in the passive duct direction, complex reflection coefficient for the sound pressure is defined as:

$$R(f) = \frac{p_r}{p_i} \tag{13}$$

and

$$H_{12} = \frac{p_2(f)}{p_1(f)} = \frac{e^{+jkz_1} + R(f)e^{-jkz_1}}{e^{+jkz_2} + R(f)e^{-jkz_2}} \tag{14}$$

Where:

$$|R| \leq 1 \tag{15}$$

The reflection coefficient, R, is defined as the ratio of the complex reflected and incident pressure and can be determined using the transfer function $H_{12}$ as:

$$R(f) = \left[\frac{H_{12}(f) - e^{+jk(z_2 - z_1)}}{e^{-jk(z_2 - z_1)} - H_{12}(f)}\right] e^{2jkz_1} \tag{16}$$

### 3.2 Experimental procedures

The experimental analyzes were done using the Two microphones method. For data acquisition it was used a Signal Analyzer HP 3566/67, which has a Source Programmable HP 35656B module and another non-programmable, 35653C module. The data were processed in a microcomputer connected to the analyzer through a specific interface.

The laboratory setup, consists of connecting the CHC 14-50 horn in a tube with internal diameter very close to its initial diameter, in this tube are allocated the Bruel & Kjaer 4635 microphones. The function of this tube and connect to the horn sound source is to make the pressure waves that are generated by the sound source plane, also the waves traveling in the opposite direction generated by the effects of the horn geometry variation. In summary, the tube must be able to ensure that the waves passing the area of the microphones is plane so that it is possible to apply the method of the two microphones.

The other end of the tube connects the sound source, in this case a loudspeaker type "mid range" installed in a box of sealed type. The signal for loudspeaker coil power comes from a programmable module HP 35656 which is completely controlled via software and allows control of frequency (pure tone or random excitation), excitation time at each frequency and the maximum voltage that is sent to coil the speaker. The signal acquired by the microphone and packed by HP 3566/67 analyzer is processed on the computer where a first analysis can be done, especially to verify the quality of the data collected, even the equipment being powered by stabilized source, noise may interfere with the results.

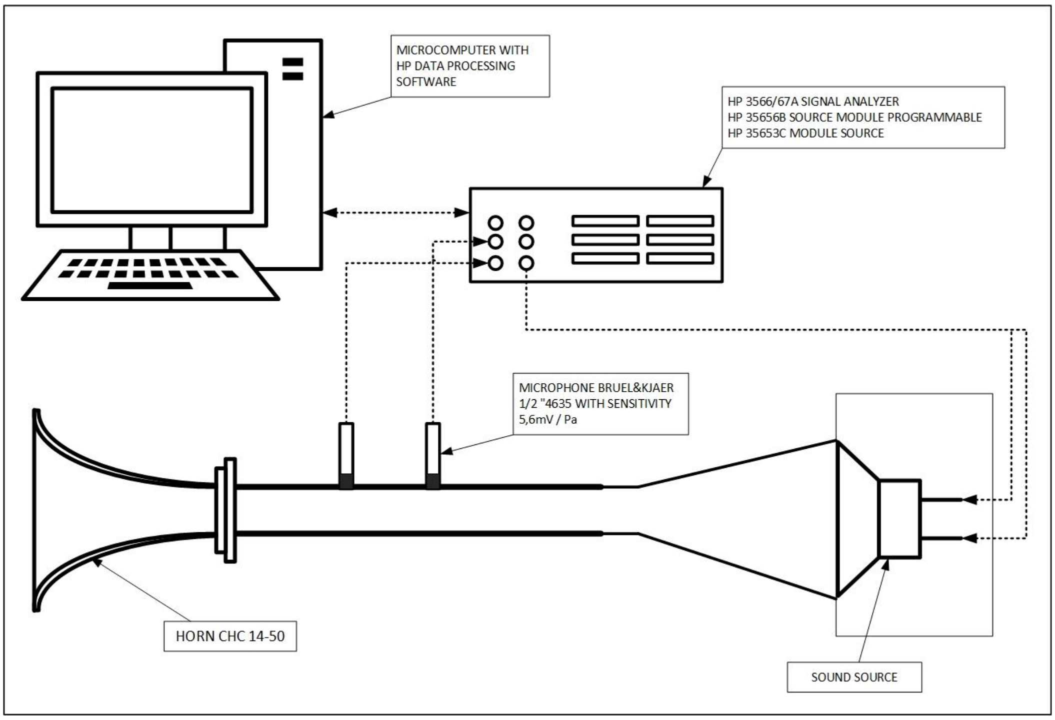


Figure 4: Mounting device of the test bench.

The acquired data allow further analysis in software as Matlab®. The tests were performed in the laboratory, a closed and a large room, but without acoustic treatment on the walls or ceiling, and the distance between the front face of the horn and the closest obstacle was approximately 4 meters. Due to the low power employed in the tests, this test condition was satisfactory for this work but a test on an anechoic chamber will bring best results.

## 4. NUMERIC AND EXPERIMENTAL RESULTS

The developed computational routine is capable of numerically analyzing the acoustic behavior over a wide range of prescribed frequencies. The data presented in the Figures 5 and 6 were chosen based on experiments that revealed some frequencies with the greatest interest because of the low reflection factor of these points.

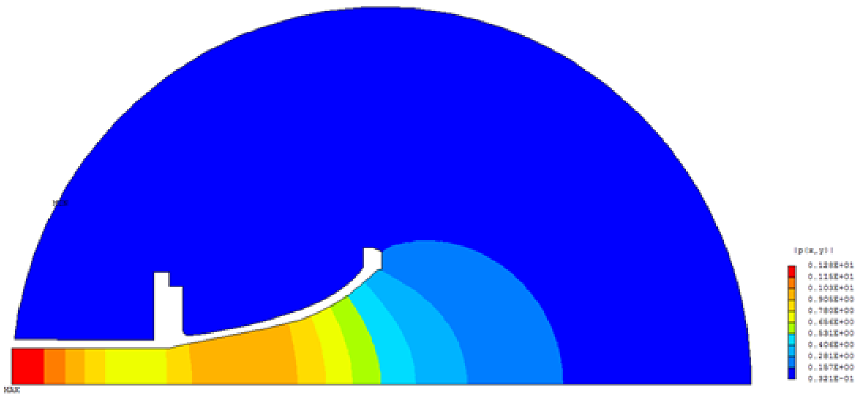

Figure 5: Magnitude of Sound Pressure for the frequency of 940 Hz.

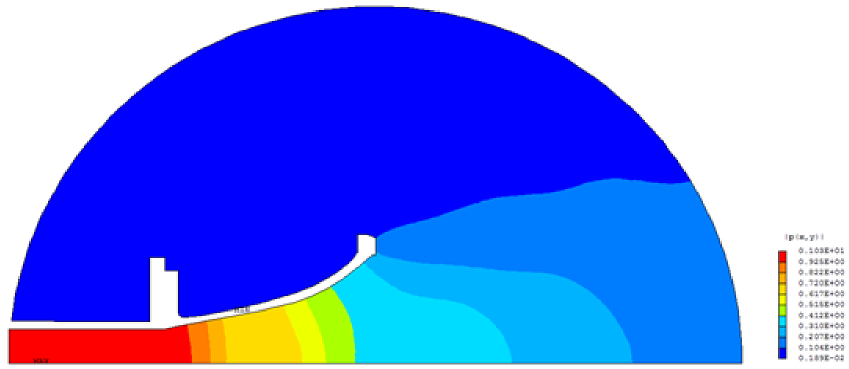

Figure 6: Magnitude of Sound Pressure for the frequency of 2870 Hz.

It is observed that the pressure zones indicate the propagation of plane waves along the initial tube, confirming what was reported in the literature. It is also possible to note the beginning of the formation of spherical waves when the geometry gradually increases its diameter. The reflection coefficient calculated through the FEM in the tube was calculated, more specifically at the beginning and at the central node of the mesh.

The experimental data can be confronted with the theoretical data obtained from the Finite Element Method. According to Lima (2008), to perform the measurement using the technique of two microphones some recommendations are important so that errors are minimized, the most important is to respect the measuring range imposed by Eq. (17) where it imposes recommended limits of frequency.

$$\frac{0{,}1c}{2s} < f < \frac{0{,}8c}{2s} \qquad (17)$$

Where "c" is the velocity of sound in meters per second and "s" the distance between the microphones. The tests was carried out using different distances between microphones. These limits are shown in Figure 7, to a temperature of 15 ° C and a distance of 30 mm microphones.

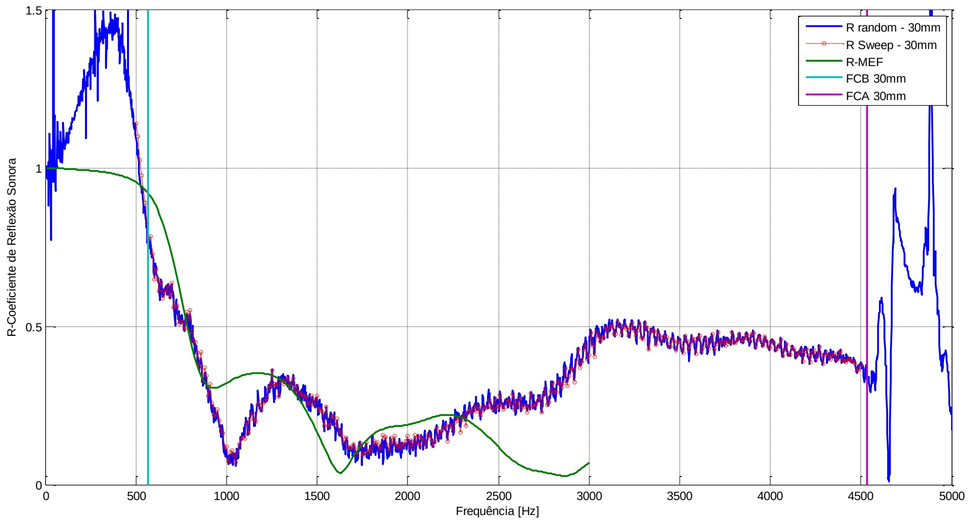


Figure 7: Results of measurements with 15 ° C; blue line represents the results with random excitation; the red line with marker shows the results of an sweep test and green line shows the theoretical results.

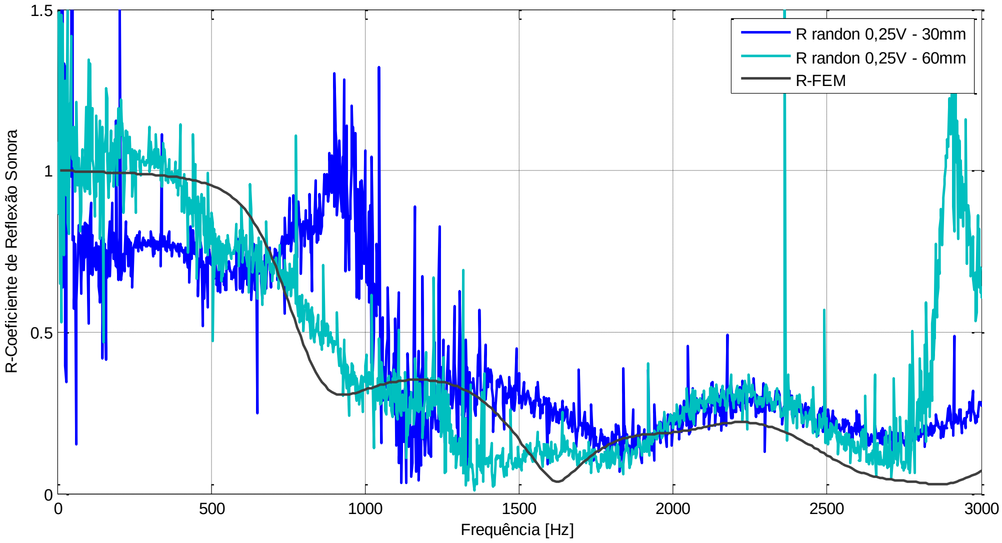


Figure 8: Results of the measurements with temperature of 20 ° C; the blue line shows the results of a random test with microphone distance of 30mm and cyan line represents the same random test with distance between microphones of 60mm.

In another measurement, the experimental data obtained using the distances between the microphones of 30 and 60mm did not show the same consistency of Figure 8. It is observed that the theoretical result was the same behavior of the experimental results to 60mm, apparently there a delay between the peaks and valleys of the curve.

Based on the theory of plane wave and the measuring method of the two microphones, it is clear that the horn working frequency (or any other element that wanted to analyze) during the tests must be compatible with the theoretical and practical evidence of the literature. So it is not possible to make measurements using the technique of the two microphones for frequency values above the first propagation mode (theory of plane wave), also references show that should be respected the measuring range defined by the distance of microphones. Remembering that both the first limiting frequency oscillation mode as the measuring range depends directly on the speed of sound, which depends on the temperature.

## 5. CONCLUSION

During the horn geometry measurement stage analyzed, deviations were observed in the horn geometry, such as roundness and slight asymmetry, problems that are inherent of components manufactured by sand casting mold. But the real impact of this is observed when comparing the theoretical results with the experimental, and reinforced when made optimization and noticed that a small variation in the geometry changed the reflection coefficient substantially. This makes clear the importance of producing geometries with the precision required to maintain the curve of the reflection coefficient within the desired. From the experimental point of view, conducting the measurement process of a finished part to obtain its geometry for subsequent construction of a theoretical model has proven to be not easy, requiring equipment with high precision and the possibility of evaluating piece by whole.

As noted in Figure 7 and Figure 8 there was a disparity between the calculated reflection coefficient and measured. This deviation can be attributed to failure in building the model (part drawing) that failed to represent important details on the project. In Figure 01, note that the horn has some details CHC 14-50 at its outer geometry that were not included in the theoretical model. Details are not influencing the main surface analysis, which remains axisymmetric, but may have influenced the measured result. The measuring apparatus was not mounted in an anechoic chamber, this can cause errors in pressure measurement, in particular the reflected wave which resulted in a distorted reflection coefficient.

For the experimental results when evaluating the reflection coefficient outside the range indicated in equation 17, they may have large distortion or physical inconsistencies, as a measured reflection coefficient greater than one.

Despite these previous considerations, the theoretical and the experimental results showed some agreement, mainly in the shape of the curve of the reflection coefficient, with a gap between the minimum points.

## 6. REFERENCES

Bängtsson, E., Noreland, D., Berggren, M., 2003. “Shape optimization of an acoustic horn”. Comput. Methods Appl. Mech. Engrg.

Barbieri, R., Barbieri, N., 2006. “Finite element acoustic simulation based shape optimization of a muffler”. Appied Acoustics.

Barbieri, R., Barbieri, N., 2013. “Acoustic horns optimization using finite elements and genetic algorithm”. Applied Acoustics.

Barbieri, R., Barbieri, N., 2012. “The technique of active/inactive finite elements for the analysis and Optimization of acoustical chambers”. Applied Acoustics.

Chandrupatla, T. R., Belegundo, A. D., 2002. *Introduction to Finite Elements in Engineering.* 3rd ed. Prentice-Hall.

Fahy, F. J., 1984. “Rapid method for the measurement of sample acoustic impedance in a standard wave tube”. Journal of Sound and Vibration, vol. 147, n. 3, pp. 371-394.

Freehafer, J. E., 1940. “The Acoustical Impedance of an Infinite Hyperbolic Horn”. The Journal of the Acoustical Society of America.

Ihlenburg, F., Babuska, I., 1997. “Solution of Helmholtz problems by knowledge-based FEM”. Computer Assisted Mechanics and Engineering Sciences.

Lima, K. F., 2008. *Metodologia de Avaliação de Filtros Acústicos Reativos.* Tese (Doutorado) – Universidade Federal de Santa Catarina.

Kausel, W., Mayer, A., Nachtmann, G., 2007. “Experimental demonstration of the effect of wall vibrations on the radiated sound of the horn and a search for possible explanations”. ISMA.

Kolbrek, B., 2008. “Horn Theory: An Introduction, Part 1”. AudioXpress.

Kolbrek, B., 2008. “Horn Theory: An Introduction, Part 2”. AudioXpress.

Kolbrek, B., 2013. “Modal sound propagation in curved horns of rectangular cross-section”. Norwegian University of Science and Technology.

Noreland, D., Udawalpola, R., Seoane, P., Wadbro, D., Berggren, M., 2010. “An efficient loudspeaker horn designed by numerical optimization: an experimental study” Report / UMINF, ISSN 0348-0542.

Munjal, M. L. , (1987). *Acoustics of ducts and mufflers.* Ed. John Wiley, 1 ed. New York, USA, 328 p.

Post, J., Hixson, E., 1994. “A Modeling and Measurement Study of Acoustic Horns”. University of Texas.

Udawlpola, R., 2010. “Shape optimization for acoustic wave propagation problems”. Acta Universitatis Upsaliensis. Digital Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology 709. 42 pp. Uppsala.

Wadbro, E., Udawlpola, R., Berggren, M., 2010. “Shape and Topology Optimization of an Acoustic Horn-Lens Combination”. J. Comput. Appl. Math., 234:1781-1787.

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