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ROBUST CAMPBELL DIAGRAM OF A SIMPLE 2DOF ROTORDYNAMIC SYSTEM

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Abstract.
The aim of the present article is to propose a strategy to define the safe operation region of a simple 2-DOF rotordynamics system. Some parameters of the system, such as stiffness and damping, are modeled as random variables, and the propagation of the uncertainties is performed by means of Monte Carlo simulations. The Campbell diagram is constructed for each simulation, generating probabilistic envelopes, instead of only two lines (backward and forward whirling). From these probabilistic envelopes it is possible to determine the safe operation region of the machine, for a given confidence level.

Keywords: *Campbell Diagram, Random Variables, Uncertainties Quantification, Monte Carlo Simulations*

1. INTRODUCTION

The dynamic analysis of rotor is relevant to gain insight into the different phenomena that arise in rotating machines. This knowledge can be applied both for design and maintenance. Rotordynamics is a branch of mechanics that studies the operation and behavior of rotating machines. This behavior encompasses a wide variety of physical phenomena that may interfere with the proper functioning of the machine and may lead to failures if not identified or corrected. In Childs (1993), Lalanne and Ferraris (1990), Yamamoto and Ishida (2001) the authors present the most important subjects in rotordynamics, such as vibration, dynamic model, lubrication, bearings, seals, among others.

Uncertainties is a critical issue in structural systems. Its identification is a current challenge that allows predicting failures and establishing safety operations margins. Uncertainties are present due to manufacturing on account of manufacturing process, material properties, degradation of the structure, etc. Dealing with uncertainties basically consists of quantification and propagation to the variable of interest. In the former, the uncertain variable or parameter is usually model as a probability function. In the latter, some methods are used to propagate the variability from input to output. in Sarrouy *et al.* (2012), Schueller and Pradlwarter (2009), Ritto *et al.* (2011) the authors propose three main methods for solving this type of problems. Firstly, uncertainties are characterized by means of Taylor Series Expansion on a set of random variables. In the second method, uncertainties are model by Hermite multivariate polynomials that are normally distributed. The third method is the Monte Carlo Simulation, which consist in a direct computation of the response from a set of values randomly distributed.

In this paper, a Monte Carlo simulation will be used in order to analyze the propagation of uncertain parameters in a rotating system. Given a set of parameters, described by probability density functions, stochastic Campbell diagrams will be created, showing the behavior of the system eigenvalues. These diagrams will show a region, bounded by an upper and lower envelope, that represents the 98% of each eigenvalue occurrences. Also, a mean eigenvalue curve is showed in the diagram to determine the distribution skewness. For the purpose of obtaining a generalized analysis, a stochastic Campbell diagram is created with dimensionless parameters. This paper is organized as follows. First, a deterministic model is presented, showing the classical Campbell diagram. Then, a stochastic model is presented, showing the stochastic Campbell diagram. Finally, results and discussions are presented.

2. Rotating System

The rotating system is comprised of a flexible shaft, a rigid disk and 2 bearings as shown in Figure 1a. The disk is restricted to rotate in the axial and radial directions, with no translation of its center of mass, as shown in Figure 1b. The deterministic model of the system is describe in Equation 1.

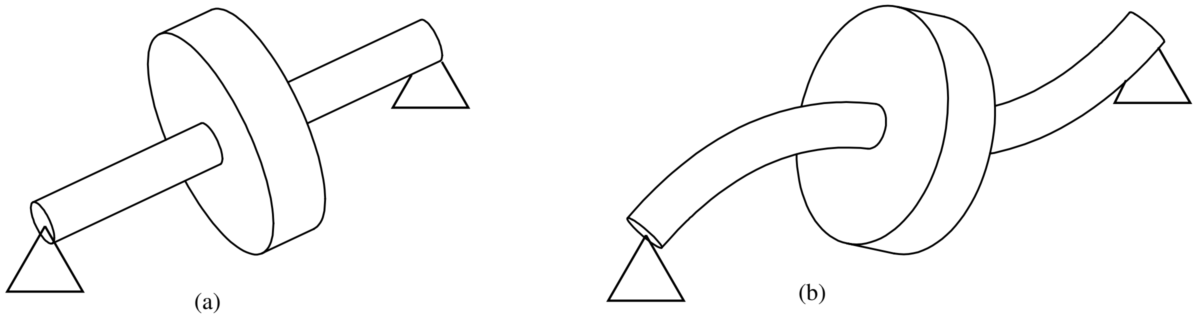

Figure 1: Rotating system

$$\begin{aligned} I_t\ddot{\alpha} - 2I_t\Omega\dot{\beta} + c\dot{\alpha} + k\alpha = 0 \\ I_t\ddot{\beta} + 2I_t\Omega\dot{\alpha} + c\dot{\beta} + k\beta = 0 \end{aligned} \tag{1}$$

where, α and β are the tilting angles of the disk, Ω is the angular speed of the shaft, I_t is the transverse moment of inertia of the disk, k is the shaft stiffness coefficient and c is the shaft damping coefficient. Note that the polar moment of inertia I_p does not appears in Equation 1 because the geometry of the rotordynamic system will be chosen such that $I_t \approx I_p$. In matrix notation, Equation 1 can be rewritten as follows.

$$\mathbf{M}\ddot{\mathbf{\Theta}} + \mathbf{G}(\Omega)\dot{\mathbf{\Theta}} + \mathbf{C}\dot{\mathbf{\Theta}} + \mathbf{K}\mathbf{\Theta} = \mathbf{0} \tag{2}$$

where,

$$\mathbf{M} = \begin{bmatrix} I_t & 0 \\ 0 & I_t \end{bmatrix}, \mathbf{G}(\Omega) = \begin{bmatrix} 0 & -2I_t\Omega \\ 2I_t\Omega & 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}, \mathbf{\Theta} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{3}$$

$\mathbf{\Theta}$ is the vector containing the system states and $\dot{\mathbf{\Theta}}$ and $\ddot{\mathbf{\Theta}}$ are its first and second time derivatives. $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ represent the Inertia Matrix, Stiffness Matrix and Damping Matrix. $\mathbf{G}(\Omega)$ is the frequency-dependent Gyroscopic Matrix that represents the effects of the system mass distribution rotating about the longitudinal axis.

Equation 2 can be rewritten in terms of first order derivatives as follows.

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} \tag{4}$$

where,

$$\boldsymbol{x} = \begin{bmatrix} \mathbf{\Theta} \\ \dot{\mathbf{\Theta}} \end{bmatrix}, \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}(\mathbf{C} + \mathbf{G}(\Omega)) \end{bmatrix} \tag{5}$$

In order to determine the eigenvalues and eigenvectors of the previous equation, the following eigenvalue problem has to be solved.

$$\mathbf{A}\boldsymbol{u}_k = \lambda_k \boldsymbol{u}_k \tag{6}$$

The eigenvectors $\boldsymbol{u}_k$ represent the mode shapes of the system. The eigenvalues λ_k are complex numbers whose imaginary part represents the vibration frequency each mode shape. Its real part has to do with system stability. If it is negative, the vibration response will decay over time. If it is positive the vibration amplitude will increase over time. When the real part is zero, the system response will oscillate with no damping. Due to the frequency dependence of the gyroscopic matrix $\mathbf{G}(\Omega)$, the mode shapes changes as the shaft angular speed Ω varies.

2.1 Stochastic Model

The Equation 2 determines the system behavior assuming it is deterministic. Nevertheless, if the system parameters are uncertain, a stochastic model can be created in order to quantify the input uncertainties and to propagate them to the output. The input uncertainties are model as random variables as follows.

$$\begin{aligned}\tilde{I}_t &= \mathcal{N}\mathcal{T}(\bar{I}_t, \sigma_i)\\ \tilde{C} &= \mathcal{N}\mathcal{T}(\bar{C}, \sigma_c)\\ \tilde{K} &= \mathcal{N}\mathcal{T}(\bar{K}, \sigma_k)\end{aligned} \tag{7}$$

where, $\bar{I}_t$ $\bar{C}$ and $\bar{K}$ are the mean values of the parameters and $\mathcal{N}\mathcal{T}$ is the truncated normal distribution. Considering Equation 7 e 2, the stochastic system equation is defined as follows.

$$\tilde{\mathbf{M}}\ddot{\mathbf{\Theta}} + \tilde{\mathbf{G}}(\Omega)\dot{\mathbf{\Theta}} + \tilde{\mathbf{C}}\dot{\mathbf{\Theta}} + \tilde{\mathbf{K}}\mathbf{\Theta} = \mathbf{0} \tag{8}$$

where $\tilde{\mathbf{M}}$, $\tilde{\mathbf{G}}$, $\tilde{\mathbf{C}}$ e $\tilde{\mathbf{K}}$ are matrices containing the random parameters. Similarly, the stochastic eigenvalue problem is defined as:

$$\tilde{\mathbf{A}}\tilde{\boldsymbol{u}}_k = \tilde{\lambda}_k \tilde{\boldsymbol{u}}_k \tag{9}$$

As can be seen in Equation 9, the new eigenvalue problem has stochastic eigenvectors and eigenvalues. The parameters uncertainty propagates through the system equation and affects the mode shapes and their frequencies. Similarly to the deterministic model, in the stochastic model the eigenvalues changes as the shaft angular speed varies. However, for a given speed, each mode shape frequency has a random distribution. In this paper, each mode shape frequency distribution is determined from a Monte Carlo Simulation (MS).

2.2 Stochastic Model with Dimensionless Parameters

In this section a dimensionless stochastic Campbell diagram for Equation 2 is presented. Firstly, a dimensionless variable τ is defined as follows.

$$\tau = t\,\omega_n \tag{10}$$

where, w_n is the system natural frequency. Using Equation 10 in Equation 2, the dimensionless system equations are as follows.

$$\begin{aligned}\ddot{\alpha}_a - \Pi_1\dot{\beta}_a + \Pi_2\dot{\alpha}_a + \alpha_a &= 0\\ \ddot{\beta}_a + \Pi_1\dot{\alpha}_a + \Pi_2\dot{\beta}_a + \beta_a &= 0\end{aligned} \tag{11}$$

where, α_a and β_a are the dimensionless system states and

$$\begin{aligned}\Pi_1 &= \frac{2\Omega}{\omega_n}\\ \Pi_2 &= \frac{c}{I_t\omega_n}\end{aligned} \tag{12}$$

are the dimensionless system parameters. Equation 11 can be rewritten in matrix form as follows.

$$\mathbf{M}_a\mathbf{\Theta}_a + \mathbf{G}_a(\Omega)\dot{\mathbf{\Theta}}_a + \mathbf{C}_a\dot{\mathbf{\Theta}}_a + \mathbf{K}_a\mathbf{\Theta}_a = \mathbf{0} \tag{13}$$

where,

$$\mathbf{M}_a = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \mathbf{G}_a = \begin{bmatrix}0 & -\Pi_1\\ \Pi_1 & 0\end{bmatrix}, \mathbf{C}_a = \begin{bmatrix}\Pi_2 & 0\\ 0 & \Pi_2\end{bmatrix}, \mathbf{K}_a = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \mathbf{\Theta}_a = \begin{bmatrix}\alpha_a\\ \beta_a\end{bmatrix} \tag{14}$$

Once the system equations are dimensionless, a new stochastic model is created using the uncertain parameters from previous section. Thus, a dimensionless stochastic parameters $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ are obtained. Therewith, the following dimensionless stochastic system is obtained.

$$\tilde{\mathbf{M}}_a\ddot{\mathbf{\Theta}}_a + \tilde{\mathbf{G}}_a(\Omega)\dot{\mathbf{\Theta}}_a + \tilde{\mathbf{C}}_a\dot{\mathbf{\Theta}}_a + \tilde{\mathbf{K}}_a\mathbf{\Theta}_a = \mathbf{0} \tag{15}$$

As well as for was done in the previous section, the following equation is the dimensionless eigenvalue problem.

$$\tilde{\mathbf{A}}_a\tilde{\boldsymbol{u}}_{ak} = \tilde{\lambda}_{ak}\tilde{\boldsymbol{u}}_{ak} \tag{16}$$

This eigenvalue problem is similar to the previous one. The dimensionless eigenvalues and eigenvectors change as the shaft angular speed change and they are also represented by a random distribution computed via MS.

3. Results

The mathematical models presented in the previous section are now computationally-implemented in Matlab in order to quantify the mode shapes frequencies uncertainties. To get an understanding about the behavior of the system as the shaft angular speed varies, Campbell diagrams are developed.

3.1 Deterministic Campbell Diagram

As Equation 2 is frequency-dependent, due to the angular speed Ω that appears in the Gyroscopic Matrix, a Campbell diagram is created using Algorithm 1 to perform a frequency analysis of the system behavior. The parameters used are: disk radius $D = 0.2$ m, disk density $\rho = 7800$ kgm^{-3},, disk length $d = 0.01$, shaft stiffness $k = 1000$ Nm^{-1}, shaft damping $c = 2$ Ns/m. The CD in Figure 2 shows the frequency of the eigenmodes obtained from Equation 2. When the shaft is stationary, there is only one frequency: the system natural frequency. As the shaft angular speed increases, two eigenfrequency branches are created. The increasing branch is called the Forward Whirl Mode (FWM), while the decreasing branch is called the Backward Whirl Mode (BWM). The straight line $\Omega = \omega$ is plotted in the diagram to determine the system resonance. As can be seen in the CD, the intersection points between the straight line and the FWM and BWM curves yields the resonance points or critical speeds. From Figure 2, the natural frequency of the system is 962.8 RPM and the resonance speed is 561.6 RPM.

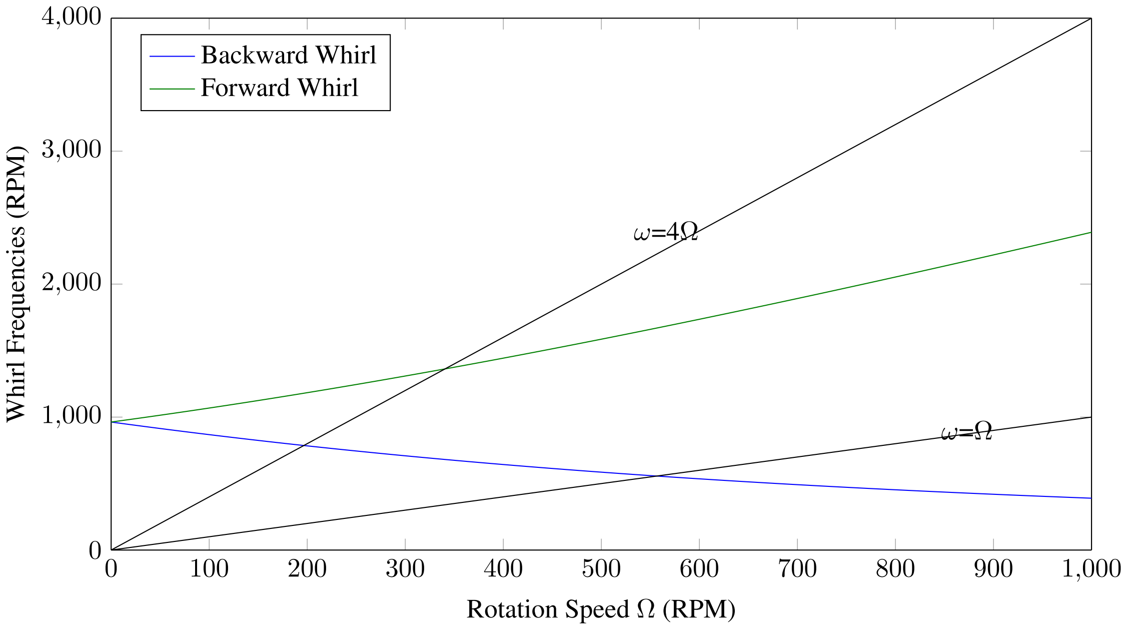

Figure 2: Campbell diagram

Algorithm 1: Deterministic Campbell Diagram

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begin DCD
- Initialize system parameters I, k and c
- Compute frequency-independent system matrices $\mathbf{I}$, $\mathbf{C}$, $\mathbf{K}$
- **for** $\Omega = 1 : \Omega_k$ **do**
  - Compute gyroscopic matrix $\mathbf{G}(\Omega)$
  - Compute eigenvalues ω
- Plot ω vs. Ω

3.2 Stochastic Campbell Diagram

Using Equation 9, the parameters uncertainty are propagated through the Equation 2 to determine the eigenvalues random distribution for each shaft angular speed. Thus, the FWM and BWM in the CD are not represented by curves, but by probabilistic regions in which they are likely to appear. To do so, a Monte Carlo Simulation (MCS) is implemented in Matlab and a Stochastic Campbell diagram (SCD) is created using Algorithm 2. the inertia, damping and stiffness standard deviation used in the simulation were $\sigma_i = 0.01$ kgm^2, $\sigma_c = 0.01$ Ns/m and $\sigma_k = 20$ Nm^{-1}. The random parameters are depicted as histograms in Figure 3. In Figure 4 the SCD is shown, using Equation 2 and 7. The branches that appeared in the classical CD are now comprised of 2 probabilistic envelopes that stand for the 1% (upper envelope) and 99 $\%$ (lower envelope) of the propagated whirl modes distribution in terms of the shaft angular speed. The curve inside the region is the mean frequency of each whirl mode. The envelopes in the SCD of Figure 2 show the dispersion of the whirls frequency in terms of the shaft angular speed. This dispersion does not change considerably over the frequency. Likewise, the mean curve remains in the middle of the envelopes which means the whirl modes distribution has no skewness. The intersection between the straight line $\omega = \Omega$ and each pair of envelopes yields the random interval where the system probably reaches the resonance frequency. In Figure 5 is shown the backward and forward whirl distributions for 560 RPM.

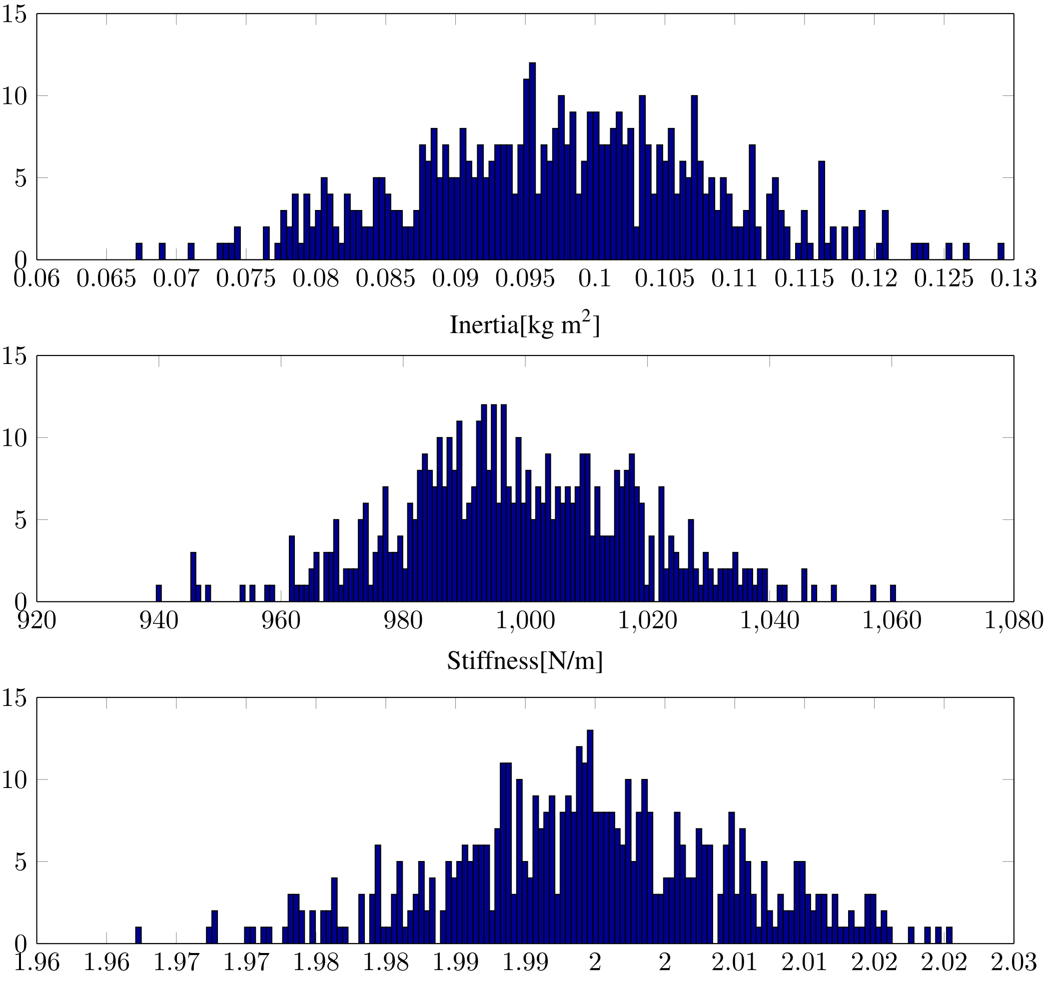

Figure 3: Histogram of the random parameters

Algorithm 2: Stochastic Campbell Diagram

begin SCD
- Initialize nominal values of system parameters I, k and c
- **for** $i = 1 : n$ **do**
  - Compute frequency-independent uncertain matrices $\tilde{\mathbf{I}}$, $\tilde{\mathbf{C}}$, $\tilde{\mathbf{K}}$
  - **for** $\Omega = 1 : \Omega_k$ **do**
    - Compute uncertain gyroscopic matrix $\tilde{\mathbf{G}}$
    - Compute uncertain eigenvalues ω
- Compute 1[st], 99[th] percentile and mean of ω distribution
- Plot ω vs. Ω with percentiles

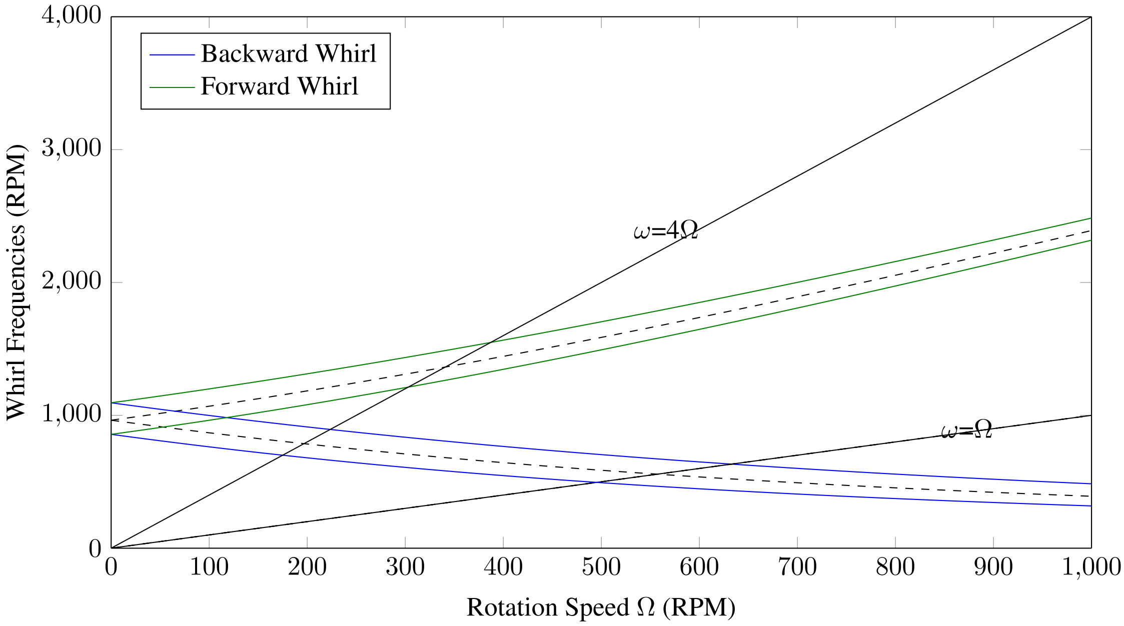

Figure 4: Stochastic Campbell diagram

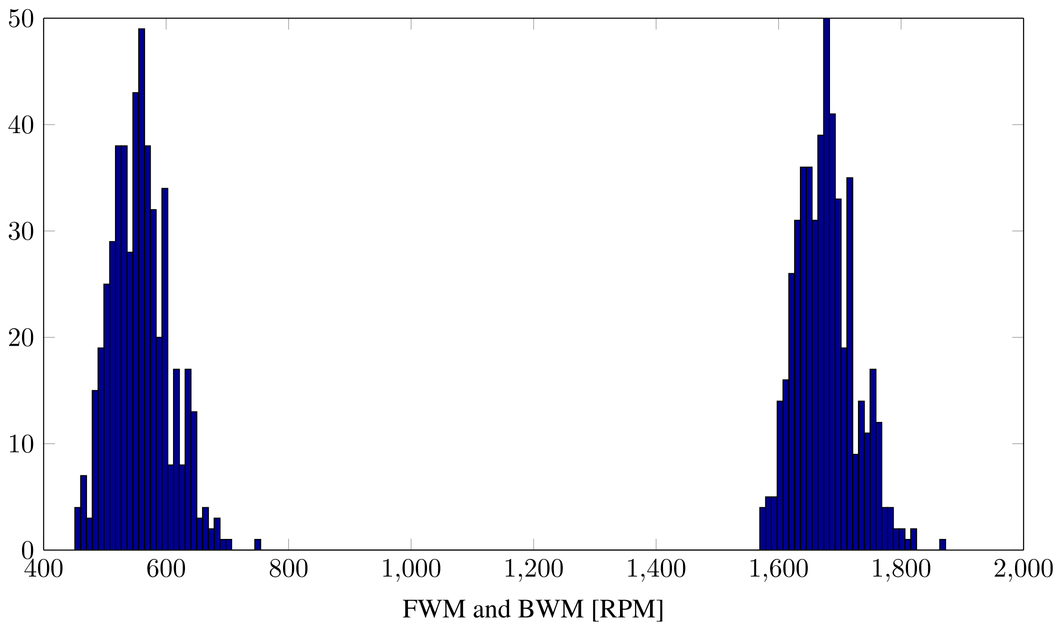

Figure 5: Histogram for the FWM and BWM at a shaft angular speed of 560 RPM.

3.3 Dimensionless Stochastic Campbell Diagram

Using Equation 15 and the same random parameters of previous section, a dimensionless stochastic Campbell diagram (DSCD) is created by means of Algorithm 3. In Figure 6 the DSCD is shown. From previous analysis we saw in the DCD that the bifurcation point of the eigenvalue bracnhes is the natural frequency. In this case, the branches start bifurcating at 1, the dimensionless natural frequency. The DSCD is independent of the natural frequency. In order to know the physical rotor speed or physical whirl frequencies of any value from the DSCD, it is necessary to multiply that value by the natural frequency.

Algorithm 3: Dimensionless Stochastic Campbell Diagram

```
begin DSCD
    Initialize nominal values of dimensionless system parameters Π_1, Π_2
    for i = 1 : n do
        Compute dimensionless frequency-independent uncertain matrices Ĩ_a, C̃_a, K̃_a
        for Ω = 1 : Ω_k do
            Compute dimensionless uncertain gyroscopic matrix G̃_a
            Compute uncertain eigenvalues ω_a
    Compute 1st, 99th percentile and mean of ω_a distribution
    Plot ω_a vs. Ω_a with percentiles
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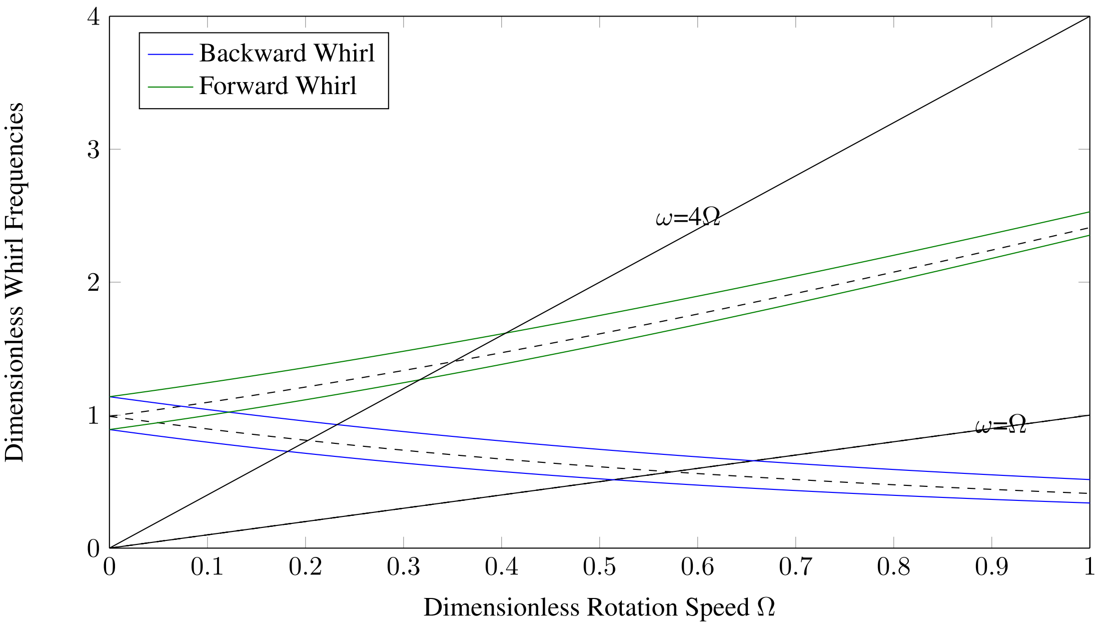

Figure 6: Dimensionless stochastic Campbell diagram

4. Conclusions

A stochastic Campbell diagram where created to quantify uncertainties of a simple rotating system. The system parameters where modeled as random variables and their impact on the frequency mode shapes where determined by creating stochastic Campbell diagrams via Monte Carlo simulations. The forward and backward whirls distributions, for a interval of shaft angular speed, where depicted as regions with an upper and lower envelope containing the 98% of the occurrences. This diagram allowed to get an understanding about the behavior of the whirls dispersion given a set of random parameters. Then, a dimensionless stochastic Campbell diagram was developed to analyze how the parameters uncertainties impact in the dimensionless system. Finally, a dimensionless stochastic Campbell diagram was created, showing the dimensionless eigenvalues distribution behavior.

5. References

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