DYNAMIC RESPONSE SENSITIVITY TO A HYDRODYNAMIC BEARING WEAR MODEL

Tiago Henrique Machado
Ricardo Ugliara Mendes
Katia Lucchesi Cavalca
Laboratory of Rotating Machinery, Faculty of Mechanical Engineering, University of Campinas
P.O.Box 6122, 200 Mendeleiev Street, 13083-970 Campinas, SP, Brazil
tiagomh@fem.unicamp.br; rumqld@gmail.com; katia@fem.unicamp.br

Abstract. *This work aims to analyze the influence of a wear model in cylindrical hydrodynamic bearings on the Frequency Response Function (FRF) of a rotor-bearing system. As noted in the past, the geometric discontinuities present in worn bearings causes an asymmetry in the dynamic properties and its influence can be noticed in the directional FRF of the rotating system. Previous works evaluated the unbalance response in different rotational speeds; therefore, this study analyzes the FRF in order to represent a more realistic case, when it is not possible to operate the rotor near the critical speeds. In this case, the FRF simulate an experimental condition where a magnetic actuator could be used to excite the rotor and the measurement is made in the bearing position. In this paper the analysis will be done through the evaluation of the vibration amplitude in the natural and whirl frequencies. The dynamic coefficients of the bearings are obtained with a spring-damper concept, in order to represent the inherent flexibility and damping of the oil film. In this case, the coefficients are evaluated by the classical perturbation theory applied to the displacements and velocities of the shaft center inside the bearings.*

Keywords: *Hydrodynamic Bearing, Wear Model, Linearized Dynamic Coefficients, Frequency Response Function (FRF)*

1. INTRODUCTION

The dynamic analysis of rotating machines involves many parameters and, therefore, this analysis should take into account the interaction among the components, such as bearings interactions. Failures associated with hydrodynamic bearing are one of the most common causes of forced shutdowns in rotating systems. Consequently, a critical issue in rotor dynamics is the evaluation of the bearings conditions, which directly affects the dynamic behavior and stability of the rotating system. In this context, the wear of the bearing material, affects the bearing clearance, which results in changes in the dynamic characteristics of the bearing and consequently, of the rotating system.

The first studies related to the emergence and development of wear in hydrodynamic bearings during numerous start/stop cycles of rotating machines date back to the experiments performed by Mokhtar *et al.* (1977). In this study, the wear was easily noticeable, and changes in diametrical clearance, surface finishing and roundness of the bearing were measured after different operation cycles have been completed.

Dufrane *et al.* (1983) investigated the wear of bearings in steam turbines to estimate the extension and the nature of this wear. Two models were established to the wear with non-circular geometry. The first model is based on the concept that the shaft makes a 'print' in the bearing and the second model is based on the assumption of an abrasive wear with an arc larger than the bearing diameter. Hashimoto *et al.* (1986) theoretically and experimentally investigated the effects of geometric changes due to wear in both laminar and turbulent hydrodynamic lubrication regimes, using the model proposed by Dufrane *et al.* (1983).

Recently, Bouyer *et al.* (2007) investigated the behavior of two lobes bearings subjected to numerous start/stop cycles of the rotating system. The experimental data for a lobular bearing was obtained and then a comparison between measured data and numerical results was made. Papadopoulos *et al.* (2008) presented a theoretical identification method for the wear in hydrodynamic bearings by means of measurements of the rotor response at a given point (usually the rotor midpoint). Gertzos *et al.* (2011) developed a practical methodology to identify the wear depth in hydrodynamic bearings. The method is based on measurements of the basic bearing characteristics, and thereby to obtain a real time (online) wear identification. Finally, Chasalevris *et al.* (2013) presented an investigation on the emergence of additional harmonic components in the transient response of a continuous rotor mounted on worn hydrodynamic bearings.

Machado (2014) noted that the changes, mainly, in the backward components of the unbalance response is an important indicator of bearing wear. The modal analysis used to distinguish between forward and backward modes was firsts presented by Lee (1991). Mesquita et al. (2002) compared the traditional frequency response function (FRF) with the directional frequency response function (dFRF) in the rotordynamic analysis. The identification of natural and operational modes using complex coordinates was the focus of the work of Kessler (1999). Cavalca and Okabe (2010) also used the dFRF in the study of the foundation effect on the rotor behavior.

Machado (2014) developed an identification method based in a bearings/rotor model using the unbalance response. The inconvenience of this method is that the rotor must operate near the critical speed, when the evidence of wear becomes detectable. Therefore, the next step of the research is to apply this concept to the directional frequency response function (dFRF). A magnetic actuator can be used to apply the excitation force and the measurements are taken in the bearings position.

In this work, the preliminary results of the new identification approach are presented. The influence of different degrees of bearing wear in operating conditions were simulated and compared.

2. THEORY

The pressure distribution, and consequently, the hydrodynamic forces generated in the oil film, are evaluated and properly adapted for situations where the oil film is discontinuous (Machado and Cavalca (2011)); as presented in more details in section 2.1. The wear model, section 2.2, is based on Dufrane *et al.* (1983). In the modeling of the rotor-bearings system, the rotor is represented by finite element method and the bearings dynamic coefficients are approximated by a spring-damper concept in order to represent the inherent flexibility and the damping of the oil film, respecting the linearity degree of these coefficients in the respective rotor operation range. The finite element model of both configurations are evaluated in this survey are presented in section 2.3. Finally, Lee's modal complex theory (1991) is briefly presented in section 2.4 in order to elucidate the conversion from the FRF measured to the dFRF.

2.1 Hydrodynamic bearings with geometric discontinuities

The basis of the hydrodynamic lubrication theory is the Reynolds equation, which gives the pressure distribution in the lubricating fluid. This pressure field is the information required to estimate the hydrodynamic supporting forces of the rotor, obtained by integration of the pressure field.

Thereby, after the hydrodynamic forces evaluation, it is possible to numerically calculate the equivalent stiffness and damping coefficients of the bearing (Fig. 1), to be inserted in the stiffness and damping matrices of the complete system.

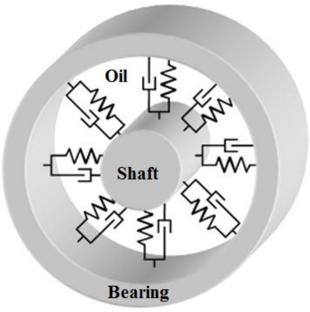


Figure 1. Model for the hydrodynamic bearing

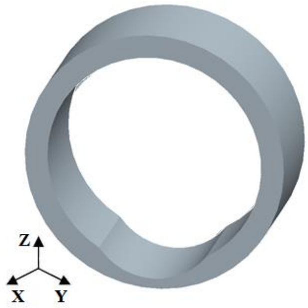


Figure 2. Bearing with discontinuity in the radial clearance

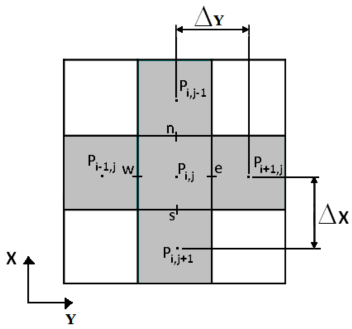


Figure 3. Finite volume mesh

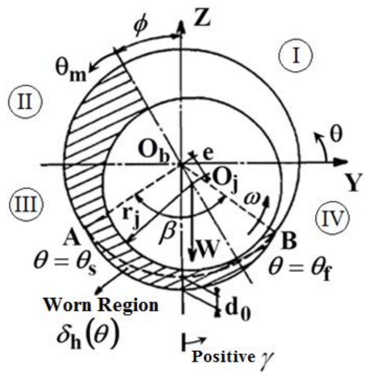


Figure 4. Worn bearing geometry

In order to analyze the wear effect, a geometric discontinuity is modeled in the radial bearing clearance, and consequently, in the oil film (Fig. 2). The procedure for the numerical solution of the Reynolds equation in the specific case in which the oil film thickness is discontinuous was introduced by Arghir et al. (2002) in one dimension, and then expanded to two dimensions in Machado and Cavalca (2011). The approach takes into account the mesh discretization by finite volumes (Fig. 3), using a uniform mesh.

In the close vicinity of the discontinuous film thickness, the pressure has an abrupt variation. The usual practice, Arghir *et al.* (2002), to describe this effect in the theoretical model is to write the generalized Bernoulli equation, immediately before and after the discontinuity, i.e., the pressure variation is attributed to a fluid velocity variation.

Regarding the boundary conditions, atmospheric pressure is considered at the axial edges of the bearing. Near the location of the minimum film thickness, the Sommerfeld boundary condition is applied along with a relaxation process. This process makes the negative pressures null after each iteration, assuming no cavitation inside the bearing.

The pressure distribution is numerically integrated to obtain the hydrodynamic forces components F_y and F_z. To calculate the bearing dynamic coefficients, the nonlinear forces are expanded in Taylor series and the resulting differential expressions are approximated by finite differences, using the centered formulation, as shown in Eq. (1), for the cross coupled stiffness coefficient (K) and a damping coefficient (R):

$$K_{yz} = \frac{\partial F_y}{\partial z} = \frac{\Delta F_y}{\Delta \bar{z}} \qquad\qquad R_{zy} = \frac{\partial F_z}{\partial \dot{y}} = \frac{\Delta F_z}{\Delta \dot{\bar{y}}} \tag{1}$$

where $\Delta\bar{z}$, $\Delta\bar{y}$ and $\Delta\dot{\bar{y}}$, $\Delta\dot{\bar{z}}$ are, respectively, small perturbations in the shaft equilibrium position for displacement and velocity.

2.2 Wear model

The wear model is based on the geometry proposed by Dufrane *et al.* (1983), due to its experimental validation presented in Hashimoto *et al.* (1986). Therefore, the hypothesis of abrasive wear is assumed, with some specific contributions to the model originally proposed by Dufrane *et al.* (1983), namely, the angular position of the wear is a variable parameter, as well as, the numerical model of the wear, using the finite volume method for the solution of the Reynolds equation. A sketch of a worn bearing is shown in Fig. 4. In this case, the wear pattern introduces an additional oil layer of depth $\delta_h(\theta)$ in the angular span from θ_s to θ_f.

According to Dufrane *et al.* (1983), the wear thickness is uniform in the axial direction and it is symmetrically located in the lower half of the bearing, i.e., symmetrically about the Z axis in Fig. 4. Practical cases of wear in bearings during the operation of real machines pointed to the possibility of the wear not to be centered about the Z axis. In several practical cases the wear is shifted to the fourth quadrant (where the shaft center, O_j, is located in Fig. 4). In general, this is the quadrant of the equilibrium position of the shaft within the bearing, for a rotation in the counterclockwise direction. Therefore, the model of Dufrane *et al.* (1983) was modified to take into account this additional parameter, which means that the center of the angular span of the wear can be displaced by an angular amount γ regarding the vertical axis.

The thickness of the lubricant film in the presence of wear, then, becomes:

$$h(\theta) = h_0(\theta) + \delta_h(\theta) \tag{2}$$

In the local reference system, denoted by θ_m in Fig. 4:

$$\begin{aligned} h_0(\theta) &= Cr + e\cdot\cos(\theta_m) \\ \delta_h(\theta) &= d_0 - Cr\cdot(1+\cos(\theta_m+\phi)) \end{aligned} \tag{3}$$

where Cr is the radial clearance, e is the eccentricity of the shaft and d_0 is the maximum wear depth.
Using e_y and e_z, the eccentricities components in Y and Z coordinates, Eq. (5) can be rewrite in the inertial reference system (Y, Z) as:

$$\begin{aligned} h_0(\theta) &= Cr - e_z\cdot\cos(\theta-\pi/2) + e_y\cdot\sin(\theta-\pi/2) \\ \delta_h(\theta) &= d_0 - Cr\cdot(1+\cos(\theta-\pi/2)) \end{aligned} \tag{4}$$

To the edges of the wear, between $\theta = \theta_s$ and $\theta = \theta_f$, the depth $\delta_h(\theta)$ is zero. Consequently, the extreme points of the wear are given by:

$$\theta_s = \pi/2 + \cos^{-1}(d_0/Cr - 1) + \gamma$$
$$\theta_f = \pi/2 - \cos^{-1}(d_0/Cr - 1) + \gamma \tag{5}$$

The film thickness can be rewritten as follows:

$$h(\theta) = \begin{cases} h_0, & 0 \le \theta \le \theta_s, \quad \theta_f \le \theta \le 2\pi \\ h_0 + \delta_h, & \theta_s < \theta < \theta_f \end{cases} \tag{6}$$

Equation (6) for the oil film thickness model was then inserted into the Reynolds equation to be solved by finite volumes method.

2.3 Finite Element Model

The Finite Element Model, as proposed by Nelson and Mac Vaugh (1976) was used to the rotor mechanical parts (Fig. 5) of the steel SAE 1030 shaft of 12 mm diameter and 600 mm length, which is split into 16 elements (represented by the dots and the numbers). The model considers the magnetic actuator journal placed at 200 mm from the first bearing, with 80 mm length, 40 mm diameter and made of steel SAE 1020. The system is supported by two hydrodynamic bearings (nodes 4 and 14) placed 387 mm from each other, where the first bearing is at 143 mm from the shaft end, both with 90 µm clearance, 20 mm width and 30 mm inner diameter. The lubricant used in this mechanical system is the oil AWS 32 from Castrol (ISO VG 32).

The equation of the motion is given by in Eq. (7), where *Ω* is the rotational speed, *M* is the mass matrix, *G* the gyroscopic effect matrix, *K* the stiffness matrix and *C* is the proportional structural damping matrix to *K* by a factor of $1{,}5\text{x}10^{-4}$ (according to Santana et al., 2010). *F* is the external forces due to the unbalanced mass and the actuator force; *q* is the generalized coordinate vector, with two translations and two rotations for each node.

$$[M]\{\ddot{q}(t)\} + ([C] + \Omega[G])\{\dot{q}(t)\} + [K]\{q(t)\} = \{f(t)\} \tag{7}$$

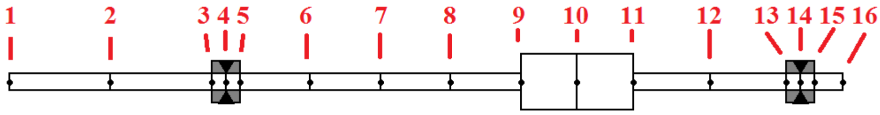


Figure 5. Finite element model of the rotor with bearings at nodes 4 and 14, and load at node 10.

In order to make the bearings operate in a higher eccentricity condition, the magnetic actuator (node 10) applies a constant downward force of 40 N.

The FRF (matrix $[H]^{-1}$) of the FEM model can be obtained from the system response to a harmonic excitation of frequency ω, as shown in Eqs. (8) and (9).

$$[H]\cdot\{q(t)\}e^{j\omega t} = \{f(t)\}e^{j\omega t} \Rightarrow \{q(t)\} = [H]^{-1}\{f(t)\} \tag{8}$$

$$[H]^{-1} = [-\omega^2[M] + j\omega([C] + \Omega[G]) + [K]]^{-1} \tag{9}$$

2.4 Directional Frequency Response Function

In order to perform the directional analysis, the physical response of the rotor (plane yz, perpendicular to the shaft) must be related to its forward and backward components. The transformation to complex co-ordinates used in this work is presented by Cavalca and Okabe (2010). Thus, for an external excitation force of frequency ω, the cartesian coordinates can be written in the complex plane as rotating vectors (Eq. (10)).

$$\{y\} = \{y_{re} + jy_{im}\}e^{j\omega t} = \{(y_{re}\cos\omega t - y_{im} sen\omega t) + j(y_{im}\cos\omega t + y_{re} sen\omega t)\}$$
$$\{z\} = \{z_{re} + jz_{im}\}e^{j\omega t} = \{(z_{re}\cos\omega t - z_{im} sen\omega t) + j(z_{im}\cos\omega t + z_{re} sen\omega t)\} \tag{10}$$

It is important to note that the imaginary parts in Eq. (10) are related to the rotation of the vectors {y} and {z}, and the real parts correspond to the physical coordinates which can be measured. Therefore, the system response in complex coordinates {q} can be written from the real parts, as shown in Eq. (11).

$$\{q\} = \left(y_{re}\cos\omega t - y_{im} sen\omega t\right) + j\left(z_{re}\cos\omega t - z_{im} sen\omega t\right) \tag{11}$$

The system response can also be described by two rotating vectors, being one forward and one backward, as in Eq. (12).

$$\begin{aligned} \{f\} &= \{f_{re} + jf_{im}\}e^{j\omega t} = \{(f_{re}\cos\omega t - f_{im} sen\omega t) + j(f_{im}\cos\omega t + f_{re} sen\omega t)\} \\ \{b\} &= \{b_{re} + jb_{im}\}e^{-j\omega t} = \{(b_{re}\cos\omega t + b_{im} sen\omega t) + j(b_{im}\cos\omega t - z_{re} sen\omega t)\} \end{aligned} \tag{12}$$

Comparing the both representations (Eqs. (11) and (12)), as in Eq.(13), the transformation matrix presented in Eq. (11) can be obtained.

$$\{p\} = \{f\} + \{b\} = \{q\} \tag{13}$$

$$\begin{Bmatrix} f \\ \bar{b} \end{Bmatrix} = [A]^{-1}\begin{Bmatrix} y \\ z \end{Bmatrix} \; ; \quad [A]^{-1} = \begin{bmatrix} 1/2 & j/2 \\ 1/2 & -j/2 \end{bmatrix} \tag{14}$$

Therefore, the FRF measured in physical coordinates (Eq. (9)) can be converted into directional coordinates (dFRF) by means of the transformation matrix given by Eq. (14). The dFRF $[dH]^{-1}$, resulting from the transformation, is presented in Eq. (15).

$$[dH]^{-1} = [A]^{-1}[H]^{-1}[A] \tag{15}$$

3. RESULTS AND DISCUSION

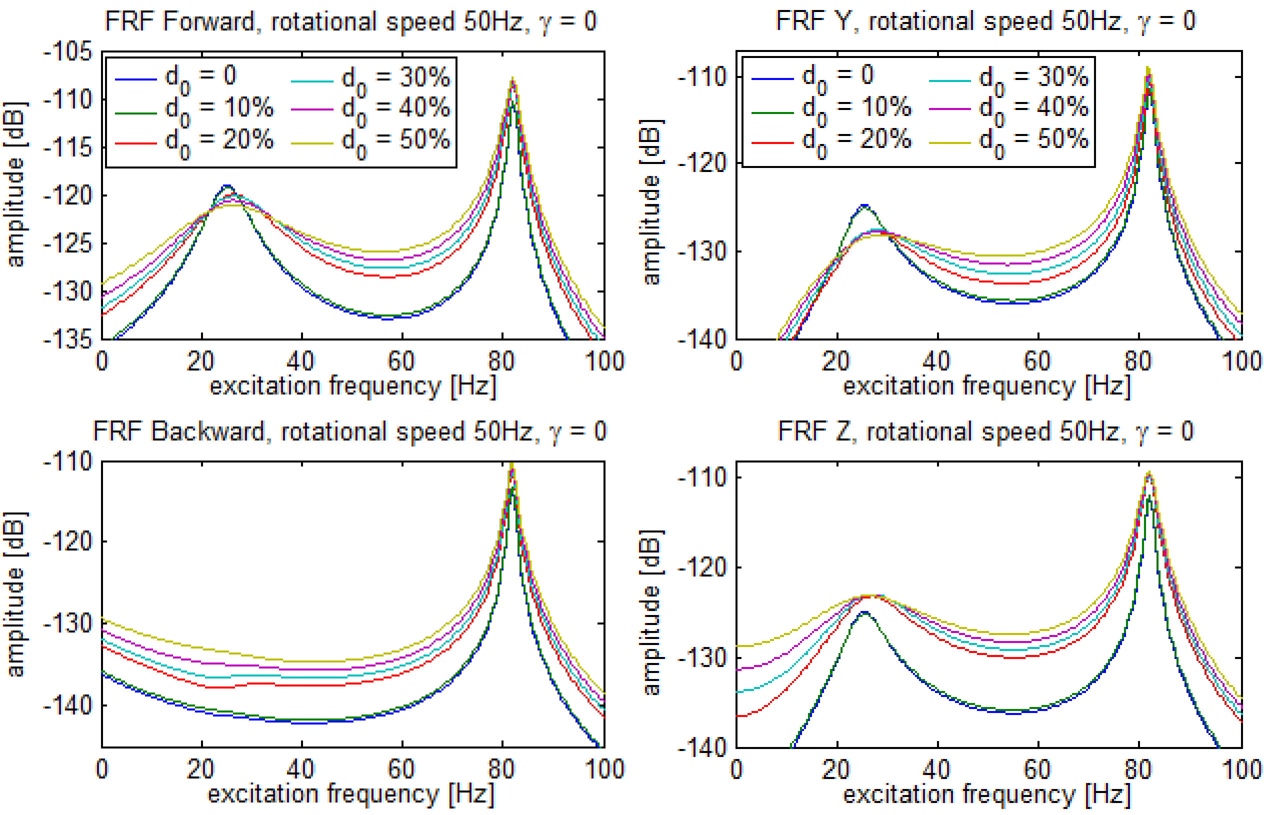
This section presents the results obtained in this work in order to analyze the influence of the wear model on the Frequency Response Function (FRF) of the rotor-bearing system shown in Fig. 5. In all cases, the system had only one worn bearing, the bearing at node 14. The FRFs (Eq. (9)) and dFRFs (Eq. (15)) analyzed were obtained considering a harmonic excitation at the actuator position (node 10) and the displacement at the worn bearing position (node 14).

Fig. 6 shows the influence of wear depth for six different configurations, where the wear depth is given by a percentage of the radial clearance, from 0% (for a bearing without any wear) to 50% (for a bearing with a maximum wear depth of 50% of the radial clearance). In the six cases, the rotational speed is 50 Hz and the wear angular position is fixed in 0° (symmetrical with respect to the vertical axis).

From Fig.6a, it can be noted that the introduction of the wear causes a significant rising of the backward component of the system response. This increase in the backward component shows that wear is responsible to introduce a greater degree of anisotropy in the bearings, and therefore, in the entire rotating system. Regarding the forward component, the difference is greater in the frequency range between the oil whirl frequency and the first natural frequency; an increase in the damping of the oil-whirl peak is also noted as the worn depth increases. In the FRF, Fig. 6b, an increase in the response of both directions can be noted as the worn depth increases, and the change in the oil-whirl peak can be noted as well.

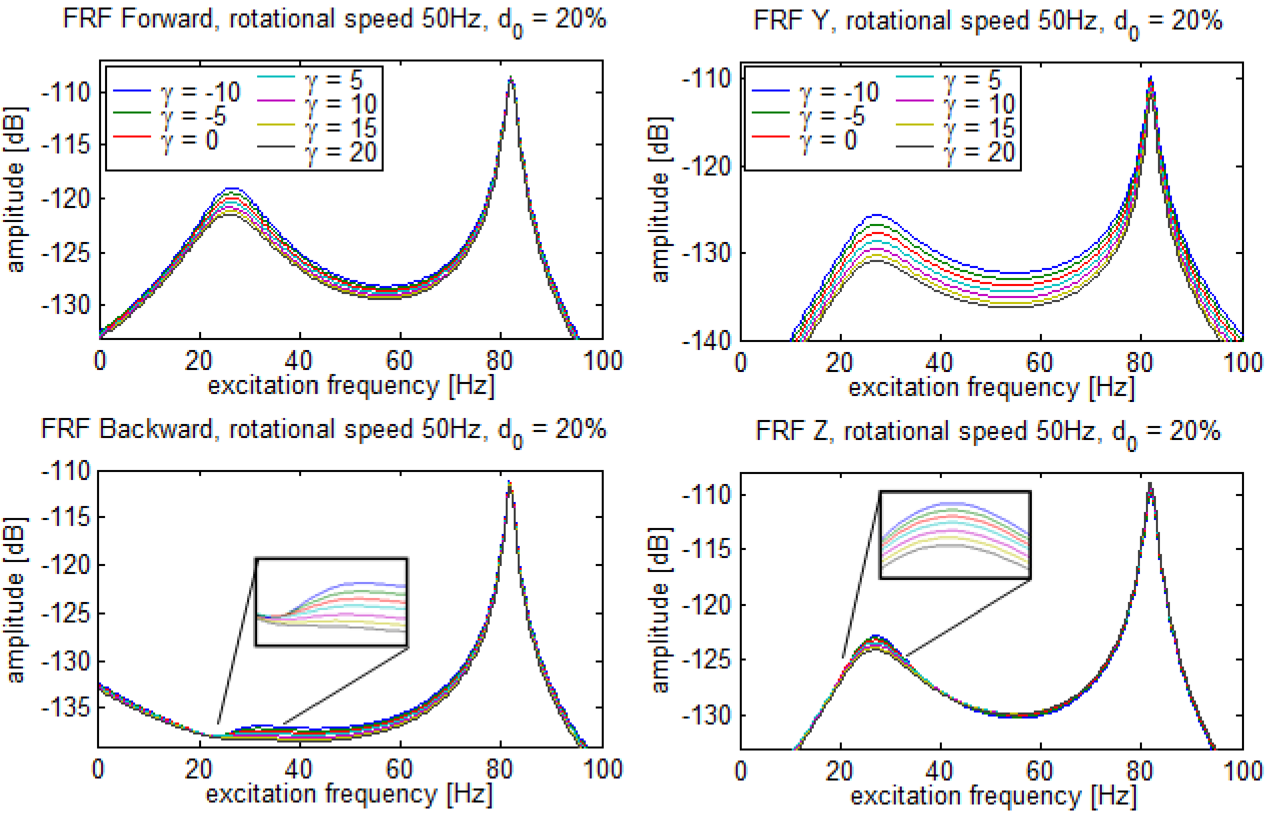
Fig. 7 shows the influence of wear angular position for six different configurations, from -10° (for a wear located on the third quadrant of Fig. 4) to 20° (for a wear located on the forth quadrant of Fig. 4). In the six cases, the rotational speed is 50Hz and the wear depth is fixed in 20% of the radial clearance.

Regarding Fig. 7a when the value of γ moved the wear center in the clockwise direction (negative value of γ) the value of the backward and forward components are higher than when the γ is moved in the counterclockwise direction (positive value of γ); and the change in the oil-whirl (forward) peak is more representative. This can be explained by the higher anisotropy of the dynamic stiffness coefficients of the bearing when $\gamma < 0$, as can be seen in Machado (2014). In physical coordinates (Fig. 8b), the amplitude changes are more significant in the vertical direction in the oil-whirl peak region.



(a) (b)

Figure 6. Influence of the wear depth: (a) directional FRF; (b) FRF in physical coordinates.



(a) (b)

Figure 7 Influence of wear for different angular positions: (a) directional FRF; (b) FRF in physical coordinates.

4. CONCLUSIONS

The results presented in this work are of interest for both researchers and rotating machines users. A numerical model to represent the main parameters of a worn bearing was used to analyze the influence of the wear in cylindrical hydrodynamic bearings on the Frequency Response Function (FRF) of a rotor-bearing system. An algorithm for calculating the directional frequency response of the entire rotating system, with the insertion of the wear model, was implemented and tested with different wear configurations, varying its depth and angular position.

The influence of wear depth can be seen in both directions Y and Z, of the FRF. This influence appears as an increase of magnitude and a change in the peak shape of the oil-whirl frequency. The same effects can be seen in the dFRF (direction coordinates); however, as oil-whirl is a forward response, it appears only in the forward direction of the dFRF.

The change in the wear angle presents significant influence in the amplitude of the oil-whirl peak in Y direction of the FRF, and the same influence can be observed in the oil-whirl peak in the forward direction of the dFRF.

The results showed that both FRF and dFRF are sensitive to the bearing wear. The changes in oil-whirl peak response seems to be the best indicator of bearing wear, once its peak shape and amplitude change as depth and angle of the wear change. For this reason, the authors consider that the dFRF is more suitable for bearing wear analysis once oil-whirl is a pure forward motion, appearing separated in the forward coordinate and not distributed in both directions (Y and Z) as in the FRF.

Therefore, a future identification procedure based on the directional Frequency Response Function (dFRF) seemed to be promising for the parameters of this type of failure.

5. ACKNOWLEDGEMENTS

The authors thank FAPESP, CAPES and CNPq for the financial support of this research.

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