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A MULTIVARIABLE FREQUENCY DOMAIN CONTROL DESIGN FOR INDUSTRIAL ELECTRICAL OVENS

José Maria Galvez
Universidade Federal de Minas Gerais, Departamento de Engenharia Mecânica
Av.Antônio Carlos, 6627, Pampulha, 31270-901, Belo Horizonte – Minas Gerais - Brasil
jmgalvez@ufmg.br

Abstract. *The control of multivariable systems has been a wide area for investigation of new methods and techniques. One of the primary objectives of the control design problem is to overcome the usually existent cross-coupling between inputs and outputs which obscures the effects of a specific loop controller on the system behavior. In the case of temperature control the problem is still more challenging due to the time delays and nonlinearities involved in the process. This paper considers the application of frequency domain techniques to the design of multivariable feedback controllers for nonlinear, multiple-heating-zones electrical oven systems. Multivariable frequency domain techniques are used to analyze the system, to improve the system decoupling and to validate a single-input single-output (SISO) linear control design approach. Simulation results are presented.*
Keywords: *multivariable frequency domain, control design, industrial electrical ovens.*

1. INTRODUCTION

Large industrial ovens usually include several heating zones with individual sensors and actuators to control the output temperature profile. However, due to the heat flow among these zones there is a cross iteration among sensors and actuators. In every case of multiple input – multiple output (MIMO) systems this iteration may become relevant and cause serious difficulties to the control system (Araújo, 1995 and Bristol, 1966). The case studied is a tubular-shaped electrical oven with six inputs and six outputs, and six meters long; the control objective is to regulate the temperature profile (Araújo, 1995). A major challenge in this case is that the construction specifications work against the control performance. To improve the smoothness of the temperature profile, the iteration among the heating zones must be increased which causes the undesirable cross-coupling among sensors and actuators.

2. THE MODELLING

The unidirectional and saturation characteristics of the power source and the two different time constants for cooling and heating make the system a nonlinear one. However, it can be shown that for some temperature profiles the system behaves linearly allowing linear analysis and design. Dynamic tests were performed to obtain and validate a linear model. Based on experimental results, a simplified tri-diagonal matrix transfer function model was built such that

$$Y(s) \quad = \quad G(s)\ U(s)$$

where

$$G(s)=\begin{bmatrix} G_{11}(s) & G_{12}(s) & 0 & 0 & 0 & 0 \\ G_{21}(s) & G_{22}(s) & G_{23}(s) & 0 & 0 & 0 \\ 0 & G_{32}(s) & G_{33}(s) & G_{34}(s) & 0 & 0 \\ 0 & 0 & G_{43}(s) & G_{44}(s) & G_{45}(s) & 0 \\ 0 & 0 & 0 & G_{54}(s) & G_{55}(s) & G_{56}(s) \\ 0 & 0 & 0 & 0 & G_{65}(s) & G_{66}(s) \end{bmatrix}$$

with

$$G_{ii}(s)=\frac{10.8\ e^{-\,\mathrm{t}}}{68.7\ s+1} \quad \& \quad G_{ij}(s)=\frac{5.25\ e^{-25\,\mathrm{t}}}{108.33\ s+1}$$

and

$$Y(s)=\begin{bmatrix} y_1(s) & y_2(s) & y_3(s) & y_4(s) & y_5(s) & y_6(s) \end{bmatrix}^T \; U(s)=\begin{bmatrix} u_1(s) & u_2(s) & u_3(s) & u_4(s) & u_5(s) & u_6(s) \end{bmatrix}^T$$

Figure 1 shows the system block diagram.

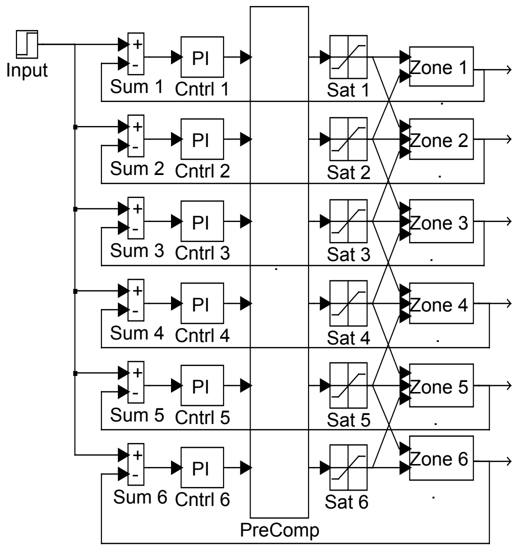

Figure 1. The system block diagram.

Multivariable frequency domain methods are just a sophisticated generalization of the classical SISO frequency domain techniques and so they are usually implemented following a trial and error procedure. On the other hand, the most important design techniques set the controller design problem as an optimization problem to determine the controller structure and its parameters. In these approaches, the practical aspects of the control problem are neglected in exchange for convenience in the theoretical formulation and numerical solution of the optimization problem. To validate the controller design, some of these techniques are used in this work following a more application-oriented approach. In the case studied, the steady state temperature profile was the main control objective. So, the system was required to fulfill two basic specifications, a fast transient response and a null steady state error.

Figure 2 shows the frequency response of the $G_{ii}(s)$ transfer function (main diagonal). Notice that the cross-over frequency (0 dB) is around 0.15 rd/min.

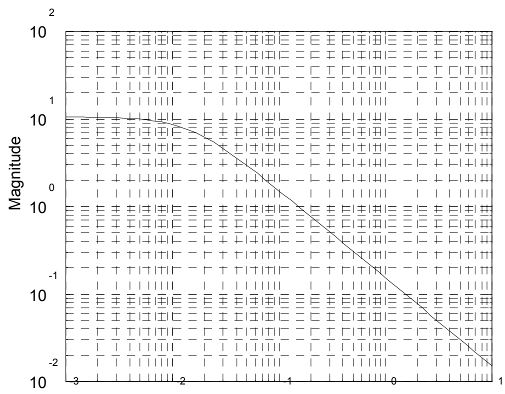

Figure 2. Frequency Response of $G_{ii}(s)$.

To deal with low frequency disturbances and to obtain good tracking of step inputs the plant is required to be a type-1 system. Thus, a performance weighting function was defined as:

$$W_p(s) \;=\; \left(\frac{50\ s+1}{100\ s}\right)\ [I] \;=\; 0.5\left(\frac{s+0.02}{s}\right)\ [I]$$

Figure 3 shows the performance weighting function $w_p(s)$.

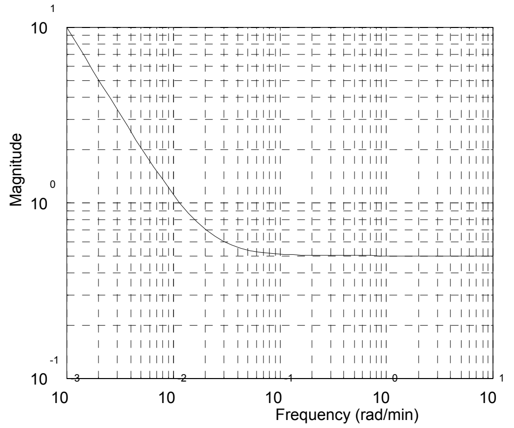

Figure 3. The performance weighting function, $w_p(s)$.

Considering the overall characteristics of the system, the maximum system gain drift is expected to be less than 15 %. Then, an uncertainty weighting function was defined as:

$$W_i(s) \;=\; 0.15 \left(\frac{s+1}{0.1\;s+1}\right)\;[I] \;=\; 1.5\left(\frac{s+1}{s+10}\right)\;[I]$$

Figures 4 shows the uncertainty weighting function $w_i(s)$.

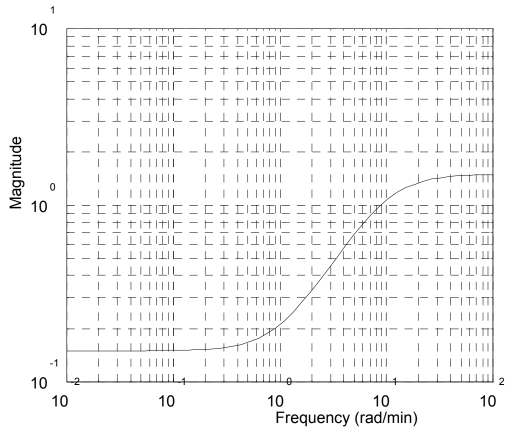

Figure 4. The uncertainty weighting function, $w_i(s)$.

To ensure a good steady state performance, the closed-loop system requires low frequency decoupling at 0 rd/s. Then, the decoupling pre-compensator can be computed as

$$K_I = G(0)^{-1} \cong \begin{bmatrix} +0.1467 & -0.1113 & +0.0822 & -0.0579 & +0.0368 & -0.0179 \\ -0.1113 & +0.2289 & -0.1692 & +0.1191 & -0.0758 & +0.0368 \\ +0.0822 & -0.1692 & +0.2658 & -0.1871 & +0.1191 & -0.0579 \\ -0.0579 & +0.1191 & -0.1871 & +0.2658 & -0.1692 & +0.0822 \\ +0.0368 & -0.0758 & +0.1191 & -0.1692 & +0.2289 & -0.1113 \\ -0.0179 & +0.0368 & -0.0579 & +0.0822 & -0.1113 & +0.1467 \end{bmatrix}$$

for this gain matrix, u^* is given by

$$u^* \cong \begin{bmatrix} 0.0787 & 0.0286 & 0.0529 & 0.0529 & 0.0286 & 0.0787 \end{bmatrix}^T$$

since the pre-compensated system has became diagonal dominant at low frequencies, then a PI controller can be designed to set the steady state error to zero. An acceptable solution in this case is given by

$$K_2 = K_p \left(1 + \frac{1}{T_i\ s}\right) [I];\ K_p = 3,\ T_i = 80$$

Thus, a multivariable controller can be built as

$$K(s) = K_1\ \ K_2(s)$$

and the open loop transfer function matrix becomes

$$G(s)\ K(s) = G(s)\ \ K_1\ \ K_2(s)$$

3. THE CONTROLLER VALIDATION

In general, MIMO systems are represented by a matrix transfer function of the form:

$$y(s) = T(s)\ P(s)\ r(s) + S(s)\ d(s) - T(s)\ m(s)$$

where r(s) is the reference input, d(s) represents the disturbances and m(s) is the measurement noise.

In this case, $S(s)$ is known as the sensitivity matrix and is given by:

$$S(s) = [I + G(s)K(s)]^{-1}$$

$T(s)$ is the system closed loop transfer function matrix (or complementary sensitivity function) and is given by:

$$T(s) = [I + G(s)K(s)]^{-1}\ G(s)\ K(s) = S(s)G(s)K(s)$$

also, the input sensitivity functions are defined as

$$S_i(s) = [I + K(s)G(s)]^{-1}$$
$$T_i(s) = [I + K(s)G(s)]^{-1}\ K(s)\ G(s) = K(s)G(s)S_i(s)$$

Considering a multiplicative model for the plant uncertainty one has:

$$G(s) = G_0(s)[I + W_i(s)]$$

Then, from the literature, the following criteria to evaluate the system performance and stability can be established:

- The criterion for nominal performance is defined by

$$\left\| S(s)\ \ W_p(s) \right\|_\infty < 1$$

where $W_p(s)$ is a performance weighting matrix.

In the case that $W_p(s)$ has the form

$$W_p(s) = w_p(s)\ \ [I]$$

the criterion becomes

$$\bar{\sigma}[S(s)] < \frac{1}{w_p(s)} \tag{C1}$$

where $\bar{\sigma}[.]$ is the greatest singular value of [.]

- The criterion for robust performance (non structured uncertainty) is given by

$$\gamma\,\bar{\sigma}\big(w_p(s)S_i(s)\big) + \bar{\sigma}\big(w_i(s)T_i(s)\big) \le 1 \tag{C2}$$

where γ = min (plant condition number, controller condition number).

- The criterion for robust stability (non structured uncertainty) is defined by

$$\left\| T(s)\ W_i(s) \right\|_\infty < 1$$

where $W_i(s)$ is an uncertainty weighting matrix.

In the case that $W_i(s)$ has the form

$$W_i(s) = w_i(s)\ [I]$$

the criterion becomes

$$\bar{\sigma}[T(s)] < \frac{1}{w_i(s)} \tag{C3}$$

- The robust performance condition for structured uncertainty [4], is given by

$$\mu\big(Q(s)\big) < 1 \qquad \forall\omega \tag{C4}$$

where the matrix $Q(s)$ is defined as

$$Q(s) = \begin{bmatrix} w_p(s)\ S_0(s) & w_p(s)\ S_0(s)\ G_0(s) \\ -w_i(s)K(s)S_0(s) & -w_i(s)K(s)S_0(s)G_0(s) \end{bmatrix}$$

with

$$S_0(s) = \big(I + G_0(s)\ K(s)\big)^{-1}$$

- The robust stability condition for structured uncertainty [4], is given by

$$\mu\big(Q_{22}(s)\big) < 1 \qquad \forall\omega \tag{C5}$$

where

$$Q_{22}(s) = -w_i(s)K(s)S_0(s)G_0(s)$$

Figure 5 shows partial Bode diagrams of the open loop system with no pre-compensation. Figure 6 presents partial Bode diagrams of the open loop pre-compensated system. It can be seen that the cross-coupling gains became neglectible at low frequencies and that the system band-pass was substantially reduced by the pre-compensation.

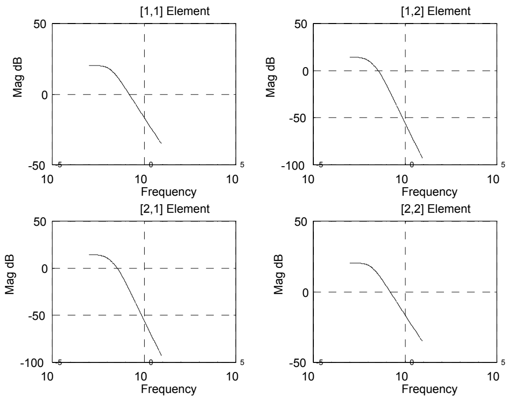

Figure 5. Bode diagrams of the open loop system.

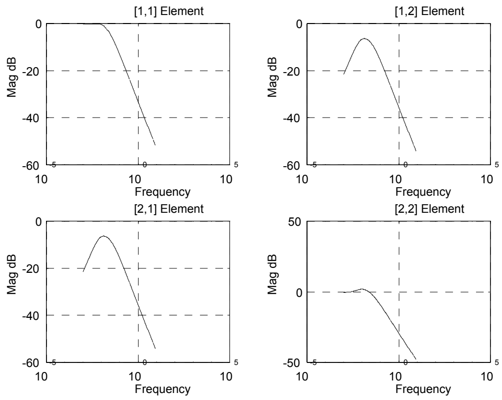

Figure 6. Bode diagrams of the pre-compensated open loop system.

Figure 7 shows the test functions for nominal performance ($1/w_p$) and robust stability ($1/w_i$). Figure 8 shows the principal gains of the open-loop system (controller + plant). Figure 9 shows the nominal performance criterion corresponding to Equation C1. Figure 10 shows the robust performance condition for non structured uncertainty given by Equation C2. Figure 11 shows the robust stability criterion for no structured uncertainty given by Equation C3. Figure 12 shows the μ (Q) value for robust performance (Equation C4). Figure 13 shows the μ (Q22) value for robust stability (Equation C5).

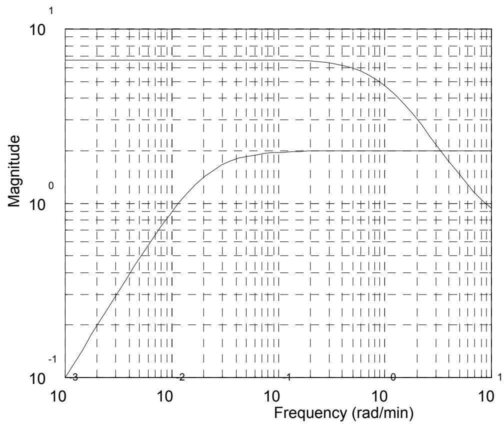

Figure 7. The test functions for nominal performance ($1/w_p$) and robust stability ($1/w_i$).

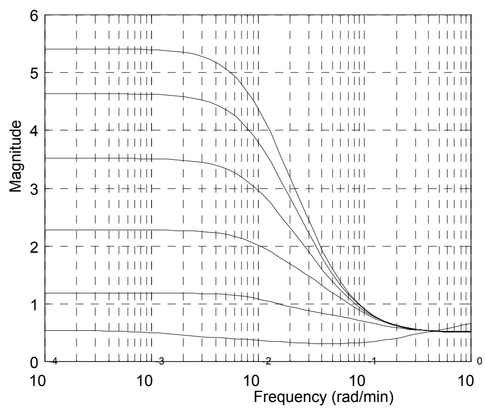

Figure 8. The principal gains of the open-loop system (controller + plant).

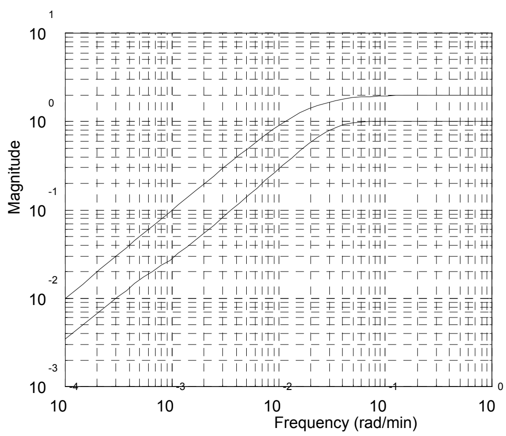

Figure 9. The nominal performance condition given by Equation C1 (NSU).

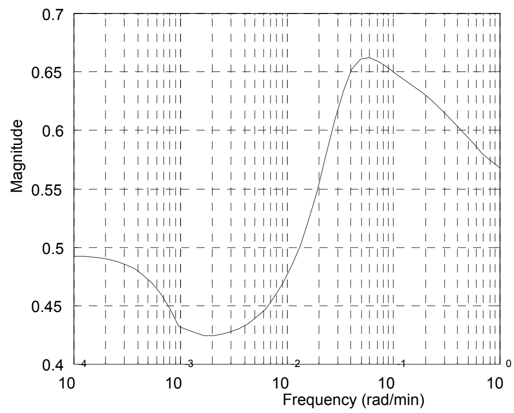

Figure 10. The robust performance condition given by Equation C2 (NSU).

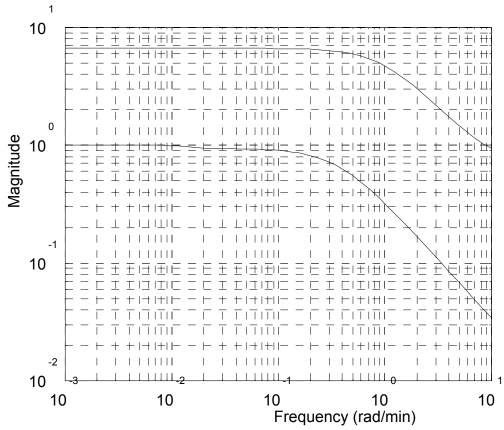

Figure 11. The robust stability condition given by Equation C3, (NSU).

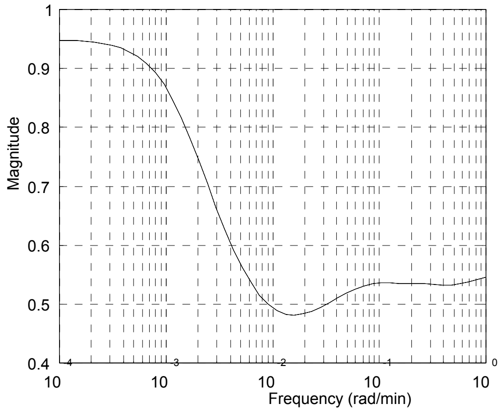

Figure 12. The μ(Q) condition for robust performance, Equation C4, (SU).

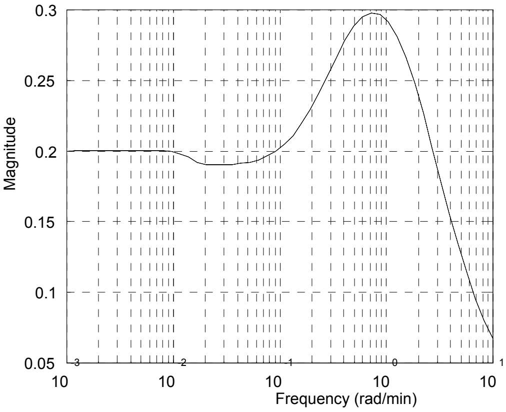

Figure 13. The μ(Q22) condition for robust stability, Equation C5, (SU).

4. SIMULATION RESULTS

Simulation experiments were performed to validate the multivariable control design. The tri-diagonal model used in the numerical simulations includes the non-linearity of the power source. Due to the unidirectional power source, negative power values cannot be applied in practice. This usually causes a high level of performance deterioration on the multivariable pre-compensator and controller which are designed based on a linear model of the plant. However, in this case, the system behaves linearly under the control design specifications. Figure 14 shows (from the bottom to the top) the open loop step response for Zone 2 with no cross coupling, one-side cross-coupling and two-side cross coupling effects. Figure 15 presents simulation results for the closed loop pre-compensated system with the multivariable PI controller.

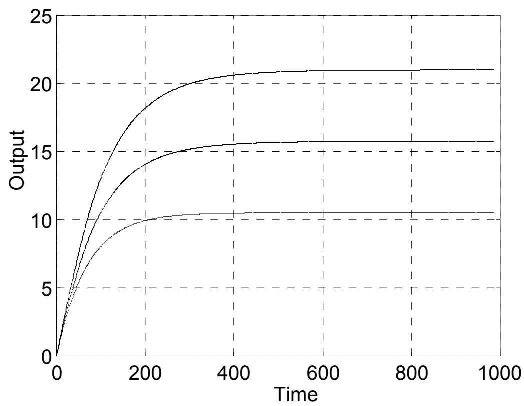

Figure 14. The open loop response (zone 2).

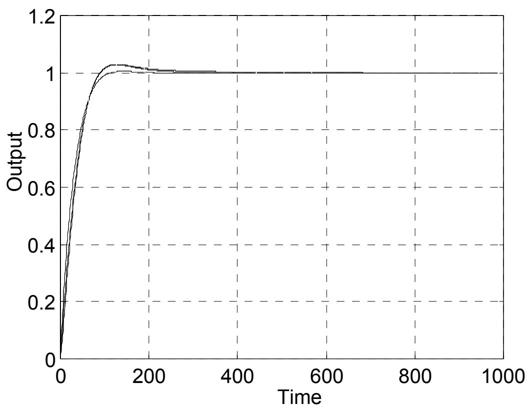

Figure 15. The pre-compensated closed loop system step response with PI controller.

5. FINAL COMMENTS AN CONCLUSIONS

An interesting application of multivariable frequency domain techniques to a nonlinear electrical oven with six heating zones has been presented in this work. Due to the oven construction, the heat flow rate among the zones was high and so it was the cross coupling between inputs and outputs.

It was verified that for some temperature profiles, the nonlinear characteristics of the power source can be avoided allowing a linear analysis and design.

The system decoupling was easily reached by using a pre-compensator tuned at 0 rad/min. Since the pre-compensated plant became clearly diagonal dominant at low frequencies, the system specification was reached using a diagonal PI controller.

The multivariable frequency domain techniques used to validate the controller shown to be very usefull, however, both conditions for robust performance under no structured and structured uncertainty (Equations C2 and C4) gave conservative results as it was verified in simulation and in practice.

Finally, a general procedure for control design can be suggested as follows:

- Find the optimal cross-coupling gains and modify, if possible, the plant gains to those values. Compute a decoupling gain matrix tuned at 0 rad/min.
- Design a multivariable feedback controller, such as a lag type diagonal controller, tuned to met the performance specifications.
- Whether high frequency compensation is required, compute a decoupling controller at the desired frequency and introduce a lead type controller to match the system specifications.
- Validate the controller design by some multivariable technique.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE