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A NEW ADAPTIVE CONTROL SYSTEM FOR PLANTS WITH VARIABLE TIME DELAY

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Abstract. *Adaptive schemes are designed to produce reliable controllers under plant uncertainties, non-linearity and slow varying parameters. In model reference adaptive control schemes, however, perfect model tracking formally depends on some conditions that usually are too restrictive for the class of plants under consideration. The control law is in general achieved through Lyapunov's and Popov's theorems, some initial assumptions appear in almost every of those approaches; the most restrictive is that the plant needs to be strictly positive real (SPR). This work presents an alternate technique, the dynamic model reduction adaptive control (DMR-AC) that does not require the SPR plant condition. This work considers DMR-AC applied to plants with variable time delays.*

Keywords: *direct model reference adaptive control, delay time control.*

1. INTRODUCTION

Adaptive control schemes have been delivered as the solution for model uncertainties and to cope with slow plant parameter variations. Among a constantly increasing number of adaptive control techniques, direct model reference adaptive control play an important role in the area. However, several problems remain unsolved, among them: Current results for direct model reference adaptive control yield asymptotic stability only for strictly positive real (SPR) plants.

This paper presents a new model reference adaptive controller for poorly known systems. Conditions for asymptotic stability are established without constraining the plant to be strictly positive real (SPR). It is shown that the proposed Dynamic Model Reduction Adaptive Control (DMR-AC) scheme is asymptotic stable inside of a relatively large neighborhood of the nominal plant dynamics. Simulation results illustrate the regulation and tracking outstanding performances of the proposed scheme for the case of plants with variable time delays. Section 2 shows the basis of the proposed controller; Section 3, presents the stability analysis; Section 4 presents simulation results to illustrate the controller performance under plants with time delays; finally, in Section 5 presents final comments and conclusions are presented.

2. THE DINAMIC MODEL REDUCTION - ADAPTIVE CONTROL (DMR-AC) SCHEME

The main objective of adaptive schemes is to produce a robust controller under plant uncertainties, nonlinearities, and time varying parameters. In direct model reference adaptive control schemes, however, perfect model tracking depends on some conditions that are not always valid for the class of plants under consideration. The derivation of the control law for these schemes is not unique. Several derivations based on Lyapunov and Popov's theorems have been proposed in the literature. However, some basic assumptions appear in almost every variation of them; the most restrictive is the one that the plant must remain strictly positive real (SPR) for all time.

This section presents the dynamic model reduction adaptive control (DMR-AC) technique. A dynamic model reduction is performed in the frequency domain. The derivation of the control law is based on Lyapunov's method. The results are given for the case in which the dimension of the plant may be much larger than the dimension of the model.

Figure 1 shows the block diagram of the proposed adaptive control law based on the Dynamic Model Reduction (DMR) approach. Three state space realizations can be seen: The plant defined by [A, B, C], the reference model by [Am, Bm, Cm], and the projection model by [Ap, Bp, Cp]. In the case of getting a projection into the reference model coordinates, one can chose [Ap=Am, Bp=Bm, Cp=Cm] as shown in Fig. 1.

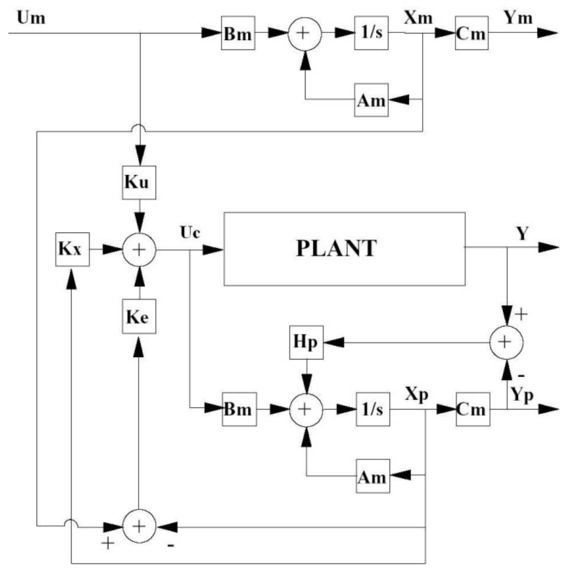

Figure1. The DMR-AC Block Diagram.

3. DERIVATION OF THE CONTROL LAW:

Let the plant be defined by

$$\dot{x} = A\,x + B\,u \quad ; \qquad y = C\,x \tag{1}$$

and let the dynamic projection state space realization be defined by

$$\dot{x}_p = A_m x_p + B_m u_p + \varepsilon_y ; \;\; y_p = C_m x_p \;\&\; \quad \varepsilon_y = H_p (y - y_p) \tag{2}$$

It should be noticed that the particular case in which dim x = dim xp and [A, B, C] = [Ap, Bp, Cp] is just the state estimator case and for some properly designed gain matrix Hp , the estimation error asymptotically converges to zero.

In the general case, however, dim x >> dim xp and an exact solution of the DMR problem cannot be actually obtained. In the context of this work, perfect tracking means that yp(t) = y(t) (εy = 0) or yp(ω) = y(ω) (for all ω in the domain of interest of the spectrum of u(ω).

Nevertheless, it can be shown that the tracking error can be made as small as desired, and in this case:

$$\lim_{t\to\infty} \varepsilon_y = \lim_{t\to\infty} (\, y - y_p \,) \cong 0 \tag{3}$$

Thus, the DMR problem (creating a dynamic projection of the plant output, y, on the reference model coordinates) is reduced to find a matrix Hp, such that, the transfer function from y to yp be as “flat” as possible over the frequency range of the plant frequency response. This problem can be solved as a pole-zero placement one, and can be obtained by classical control techniques.

The DMR adaptive controller proposed here implements the control law from output measurements to eliminate the dimensionality problem, the DMR scheme takes the dynamic projection of the plant output into the projection model coordinates and uses the dynamic projection state vector to implement the controller adaptation law, as it will be shown next. Let the reference model be defined as

$$\dot{x}_m = A_m x_m + B_m u_m \;\; ; \qquad y_m = C_m x_m \tag{4}$$

then an error equation can be written as

$$e_x = x_m - x_p \tag{5}$$

differentiating Equation (5) one gets

$$\dot{e}_x = \dot{x}_m - \dot{x}_p \tag{6}$$

The control law is implemented by the time varying matrices [Ku(t)] and [Kx(t)] and the constant matrix [Ke] such that:

$$u = K_u(t)u_m + K_x(t)x_p + K_e\, e_x = K(t)z + K_e\, e_x \tag{7}$$

the primary objective is to find the adaptation mechanism for *K(t)* such that

$$\lim_{t\to\infty}(y_m - y_p) = 0 \tag{8}$$

for the moment let us assume that Equation (3) holds, that is $\lim_{t\to\infty} y_p = y$ thus

$$\lim_{t\to\infty}(y_m - y_p) = \lim_{t\to\infty}(y_m - y) \tag{9}$$

A Lyapunov's function candidate can be chosen as

$$V = e_x{}^T P\, e_x + trace\left\{ \Delta K(t)\ \ \Delta K(t)^T \right\} \geq 0\ ;\ P > 0 \tag{10}$$

taking its first derivative, one gets

$$\dot{V} = e_x{}^T \left(A_c{}^T P + P\, A_c \right) e_x + 2\, trace\left\{ (\Delta\dot{K}(t) - B_p{}^T P\, e_x z^T)\Delta K(t)^T \right\};\ P > 0 \tag{11}$$

where

$$A_c = A_m - B_p\, K_e \tag{12}$$

Notice that $\Delta\dot{K}(t) = \dot{K}(t)$, then, $\dot{V} \leq 0$ requires that

$$\dot{V} = e_x{}^T \left(A_c{}^T P + P\, A_c \right) e_x \leq\ 0 \tag{13}$$

$$\dot{K}(t) = \Delta\dot{K}(t) = B_p{}^T P\, e_x\, z^T \tag{14}$$

Equation (13) is satisfied solving the Lyapunov's Equation given by

$$A_c{}^T P + P\, A_c = -Q\ \ ; \qquad Q >\ 0 \tag{15}$$

Equation (14) yields the adaptation law as

$$\dot{K}(t) = B_p{}^T P\, e_x\, z^T \Rightarrow K(t) = K(t_o) + \int_{t_o}^{t} B_p{}^T P\, e_x(\tau)\, z^T(\tau)\, d\tau \tag{16}$$

for such an adaptation law Equation (13) becomes

$$\dot{V} = -e_x{}^T Q\, e_x \leq\ 0\ \ ; \qquad Q >\ 0 \tag{17a}$$

it should be notice that

$$\dot{V} = -e_x{}^T Q\, e_x =\ 0\ \ for \qquad e_x =\ 0 \tag{17b}$$

Equations (3) and (17) are sufficient conditions for asymptotic stability.

4. STABILITY ANALYSIS

The derivation of the adaptation law was performed heuristically assuming that

$$\lim_{t \to \infty} \varepsilon_y = \lim_{t \to \infty} (y - y_p) = 0 ; \tag{3}$$

however, this might not be the usual case, in general, Equation (17) has the form:

$$\dot{V} = -e_x{}^T Q\, e_x - 2e_x{}^T P\, \varepsilon_y ;\;\; Q > 0 \;\&\; \varepsilon_y \neq 0 \tag{18}$$

Equation (18) shows that asymptotic stability of the proposed adaptive scheme depends on the tracking performance of the dynamic projection. However, if Equation (3) holds then Equation (18) becomes

$$\dot{V} = -e_x{}^T Q\, e_x \leq 0 \;\; ; \qquad Q > 0 \tag{20}$$

and

$$\lim_{t \to \infty} (y_m - y_p) = \lim_{t \to \infty} (y_m - y) = 0 \tag{21}$$

that means asymptotic stability.

Heuristic Remarks

Since the dynamic projection tracking response is designed independently of the adaptive control law, it is always possible to design a dynamic projection with an appropriate frequency response such that Equation (3) holds. Besides that, dynamic constrains through low-pass filters can be imposed to the plant input to improve the dynamic projection tracking performance. In the time domain, Equation (18) suggests the existing of an unstable limit cycle (encircling the origin) as shown in Figure 2.

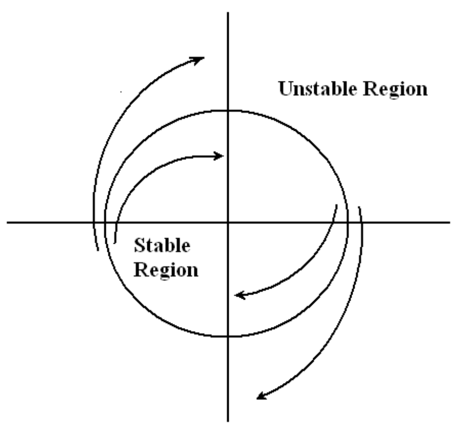

Figure 2. Unstable Limit Cycle.

The inside region corresponds to a stable system and the outside region to an unstable one. A nice result in this case is that the stability region can be increased by the proper design of the dynamic projection. In this case, the better the dynamic projection tracking the larger the stability region.

5. SIMULATION RESULTS

To assess the controller performance an experiment was set. The reference and the projection models are chosen to have the desired plant dynamics and are kept unchanged through this experiment.

The dynamic projection frequency response was designed to have a flat response up to 100 rad/s, which can be obtained by choosing H_p as:

$$H_p = \begin{bmatrix} 125.225 \\ 50.000 \end{bmatrix}$$

Following the procedure presented in Galvez (2010) the adaptation law was defined with

$$K_e = [\,100 \;\; 150\,]$$

and solving Eq. (16) for a matrix Q > 0, it was found that

$$Q = \begin{bmatrix} 5.05 & 9.92 \\ 9.92 & 168.36 \end{bmatrix} \qquad \Leftrightarrow \qquad P = \begin{bmatrix} 50 & 0.025 \\ 0.025 & 0.558 \end{bmatrix}$$

The parameters of the nominal plant (a minimal phase linear system with no time delay) drift from their nominal values through the experiment to reflect possible parameters degradation. In the following Examples the static gain, time delay and damping ratio have been changed through the experiment as follows: Static Gain (Kss) = [1, 1.8]; Time Delay (τ) = [0, 1] and Damping Ratio (ζ) = [0.75, 0.25].

Figure 3 shows the case that corresponds to the model matching condition in which the plant, the dynamic projection and the reference model share the same dynamics. Figures 4, 5, 6 and 7 present simulation results for various combination of parameters . It should be noted that the plant in Examples 3 and 4 (Figs. 6 and 7) has unstable phase and gain margins, yet the DMR-AC still delivers a stable result.

Every figure shows at the top the model and plant time responses, at the middle the projection tracking response (plant and projection time responses) and at the bottom the control signal produced by the adaptive control law.

Example1: The Model/Nominal Plant at Model Matching Conditions

$G_m(s) = G(s) = \dfrac{1\,e^{-0.0s}}{s^2 + 1.5s + 1}$	Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
	$-0.75 \pm j0.66$	0.75	1.00	Inf	180

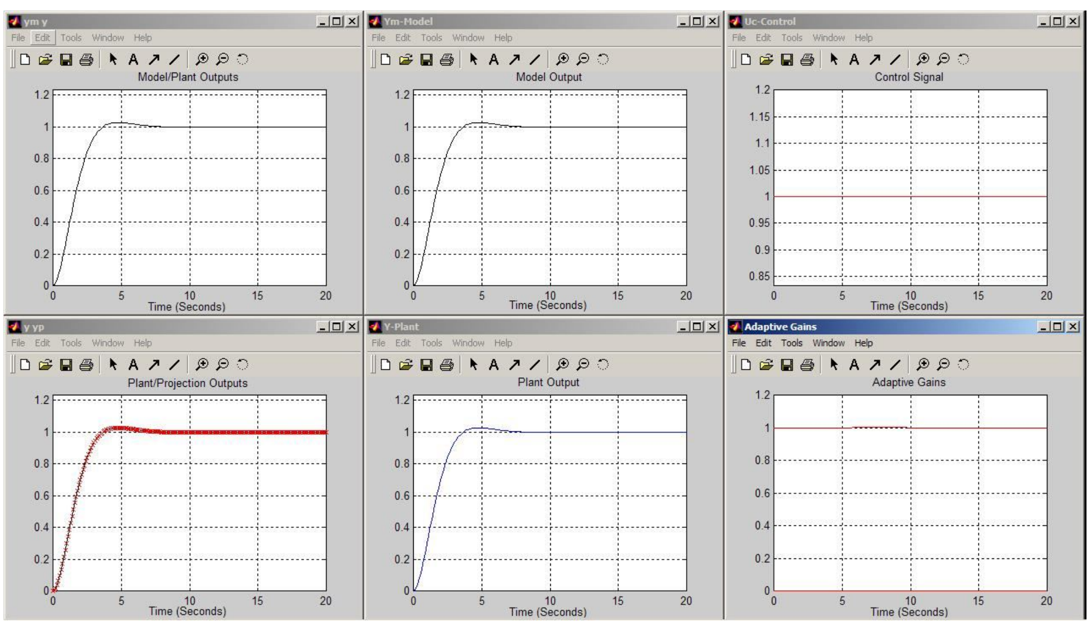

Figure 3. Results for the Model Matching Conditions for Example1 (Nominal Plant).

Example2: The Time Varying Plant with Time Delay

$G(s)=\dfrac{1.5\,e^{-0.25s}}{s^2+1.5s+1}$	Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
	$-0.75 \pm j0.66$	0.75	1.00	Inf	180

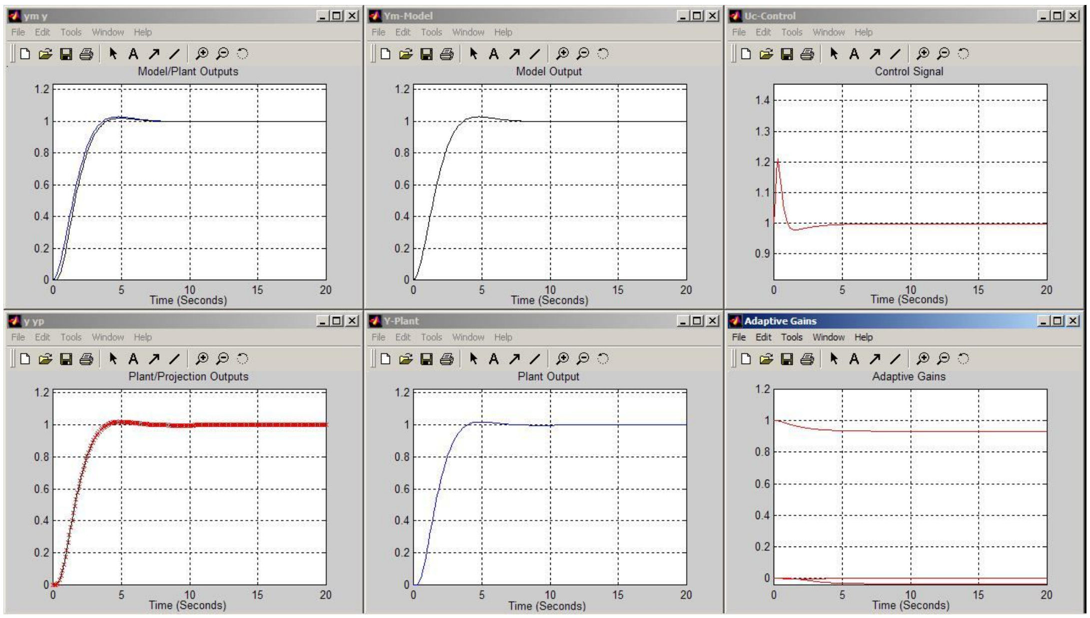

Figure 4. Simulation Results for Example2.

Example3: The Time Varying Plant with Large Time Delay

$G(s)=\dfrac{1.5\,e^{-0.75s}}{s^2+1.3s+1}$	Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
	$-0.65 \pm j0.76$	0.65	1.00	12.61	75.79

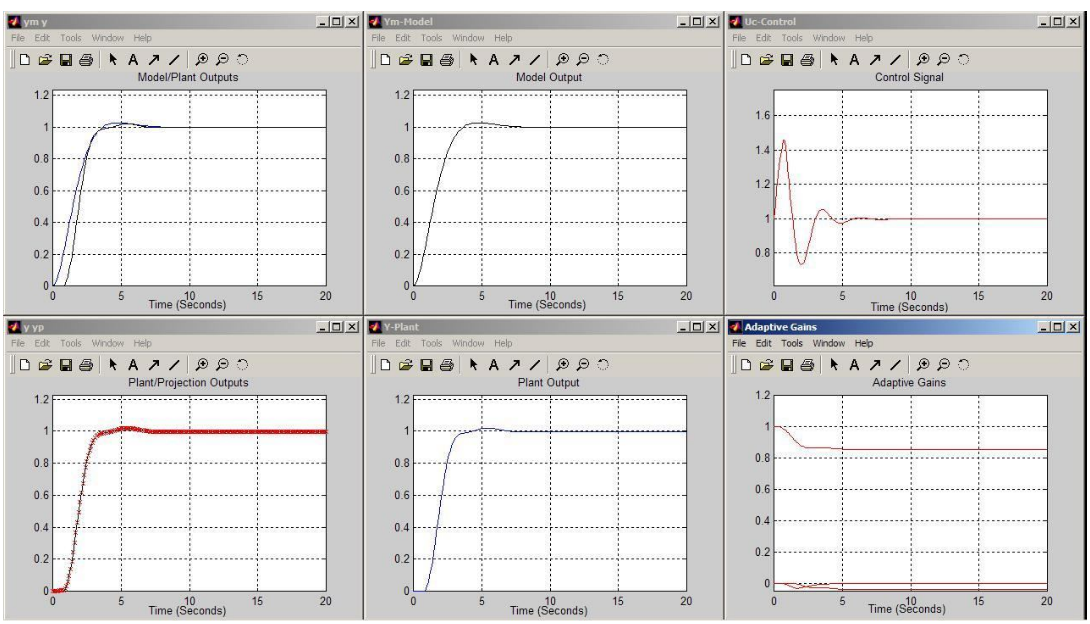

Figure 5. Simulation Results for Example3.

Example4: The Time Varying Unstable Plant

$G(s) = \dfrac{1.8\ e^{-1.0s}}{s^2 + 1.3s + 1}$	Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
	$-0.65 \pm j0.76$	0.65	1.00	-0.44	-5.29

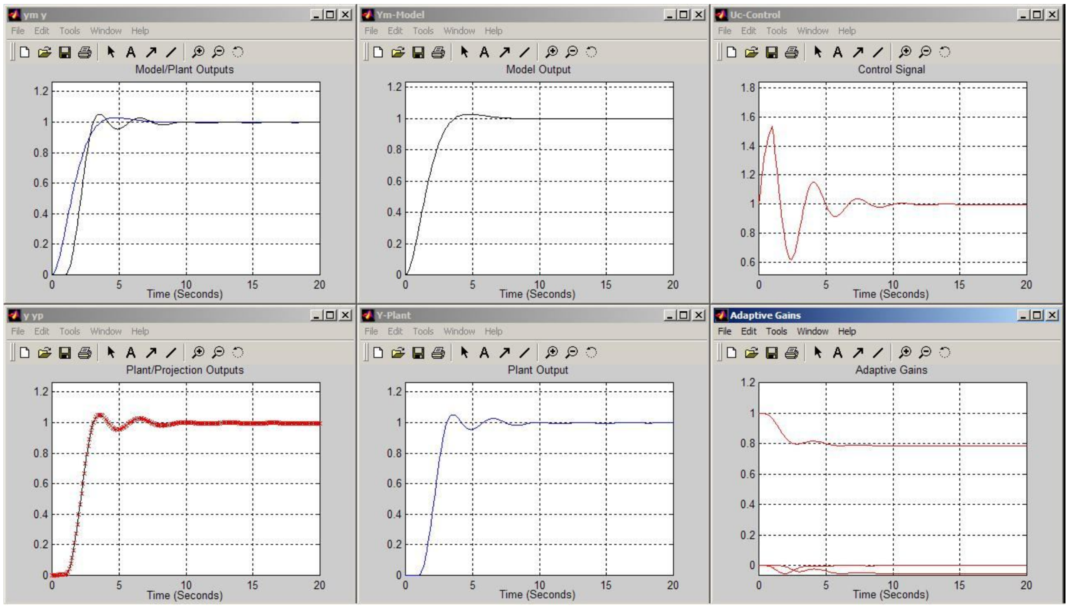

Figure 6. Simulation Results for Example4.

Example5: The Time Varying Unstable Plant

$$G(s) = \frac{(-1.27x10^9 s + 3.59x10^{10})e^{-0.5s}}{s^7 + 34.3s^6 + 5.15x10^4 s^5 + 5.05x10^5 s^4 + 1.93x10^8 s^3 + 3.65x10^8 s2 + 3.00x10^{10} s1 + 8.98x10^9}$$

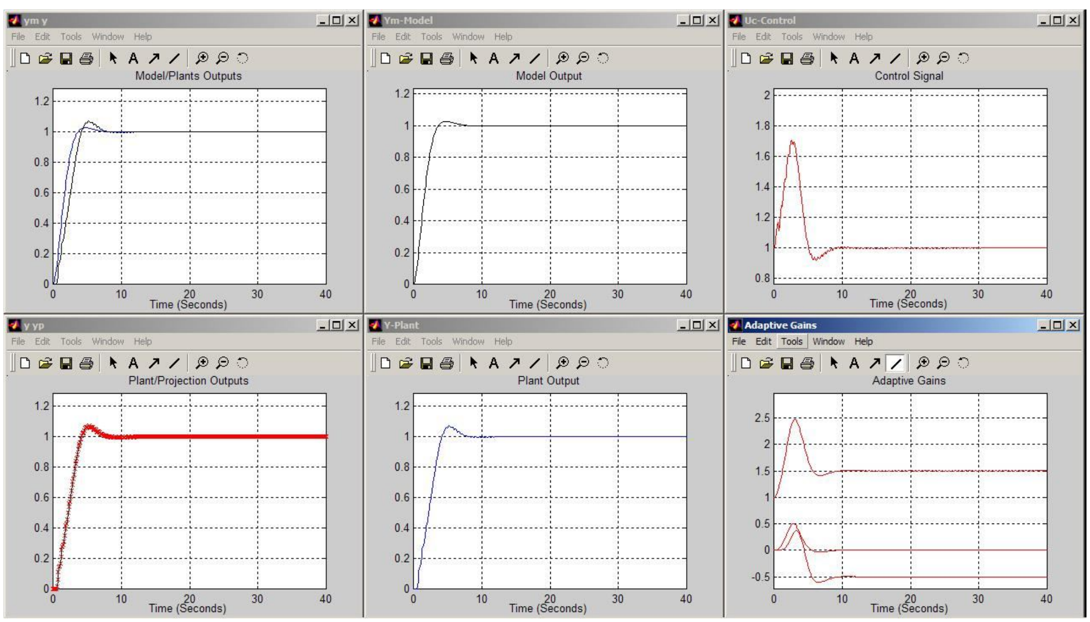

Figure 7. Simulation Results for Example5.

Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
-0.30	1.00	0.30	0.05199	-1.216
$-0.65 \pm j12.7$	0.0510	12.8		
$-3.35 \pm j62.3$	0.0537	62.4		
$-13.0 + j217.0$	0.0598	217.0		

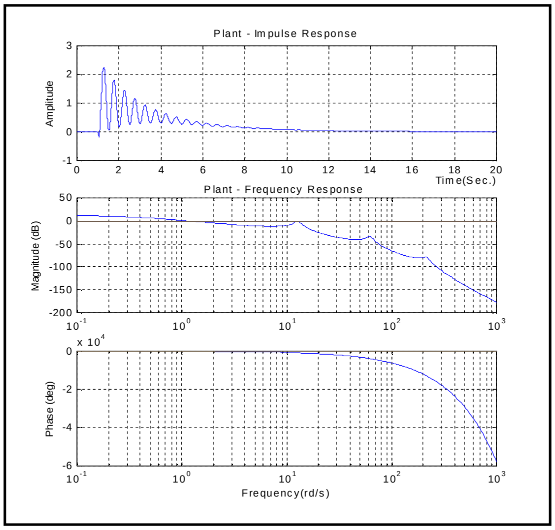

Figure 8. Plant Dynamics for Example5.

6. FINAL COMMENTS AND CONCLUSIONS

This paper presented a new adaptive control technique applied to plants with varying time delay. A benchmark plant was used to perform an experimental analysis of the Dynamic Model Reduction - Adaptive Controller (DMR-AC) scheme. The plant dynamics was changed from a stable model to an unstable one, through the experiment. The result shown it as an outstanding alternative for the control of plants with variable time delays even in the case of unstable plants. The derivation of the DMR-AC gains adaptation law has been performed based on the Lyapunov's method without constraining the plant to be strictly positive real. The results are for the general case in which the dimension of the plant is larger than the dimension of the reference model. It has been verified that the dynamic projection state vector can be used to overcome the dimensionality problem in the derivation of adaptation laws for adaptive schemes. Conditions for asymptotic stability have been verified. Finally, it has verified that by slightly relaxing formality one can reach new controller structures and substantially improve the performance of direct adaptive controllers.

7. ACKNOWLEDGEMENTS

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