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AN ASSESSMENT OF HDPE VISCOELASTIC BEHAVIOR EFFECTS ON THE BENDING OF OFFSHORE FLEXIBLE PIPES

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Abstract. *Flexible pipes are complex structures composed by several metallic helical layers and polymeric extruded tubes, composed in concentric arrangements. Numerous analytical and numerical models exist nowadays, enabling analysts to assess stress and strain fields in the structural components of flexible pipes. In these models, internal friction is a common feature, being its effect associated to the hysteretic bending behavior. On the other hand, it is well known that flexible pipes bending stiffness is strongly dominated by the extruded external polymeric layer. In spite of that, existing numerical and analytical models do not take into account the viscoelastic mechanical behavior, which characterizes most of the polymeric external layers. The present work addresses this particular point numerically. Results from a recent experimental campaign, meant to assess the viscoelastic response of pipes made of high-density polyethylene (HDPE), are used to generate a representative model. Such a model is then applied in a case study to assess and discuss the viscoelastic effect on bending of a typical flexible pipe, through an innovative Finite Element Analysis approach.*

Keywords: *flexible pipes; bending; HDPE; viscoelastic behavior; experimental investigation; FE analysis*

1. INTRODUCTION

A well-known and proven concept in the offshore oil and gas industry, flexible pipes have been used as flowlines and risers to conveying fluid from the wellhead, at the sea floor, to the floating production unities. These types of structures have been proving their effectiveness over the last decades, especially in the oil fields found along the Brazilian coast, in ultra-deep waters (Novitsky & Sertã, 2002). Flexible pipes are composite structures highly deformable in bending and, at the same time, resistant to internal pressure, external pressure, tension and torsion. Their metallic layers are responsible for the pipe axial and torsional stiffness, while the polymeric layers provide sealing (the internal one) or mechanical protection (the external). Ramos Jr (2001) and Fergestad & Løtveit (2014) describe in details all flexible pipe layers and respective functionalities. Fig. 1 illustrates a typical flexible pipe structure.

Since these pipes are very complex structures, with layers of different materials and geometries, as seen in Fig. 1, predicting their structural response is by no means a simple task and it has been subject of numerous scientific papers. Nowadays, there exist numerous analytical and numerical models that enable analysts to assess stress and strain fields in the structural components of flexible pipes, due to combined loading associated with pressure, traction, torsion, bending, as well as to particular external contact loads as those related to laying operations. Just to list some examples, Witz & Tan (1992) presented analytical expressions to predict the flexible pipe response under moderate loads. Ramos & Pesce (2004) also developed a consistent analytical model to study the behavior of flexible risers, taking into account axisymmetric combined loads and bending. Numerical (FE) models with such a purpose can be easily found in the literature. Examples of FE models that consider the whole flexible pipe with all layers represented, using commercial software, are Cruz (1996) and Sousa (2005).

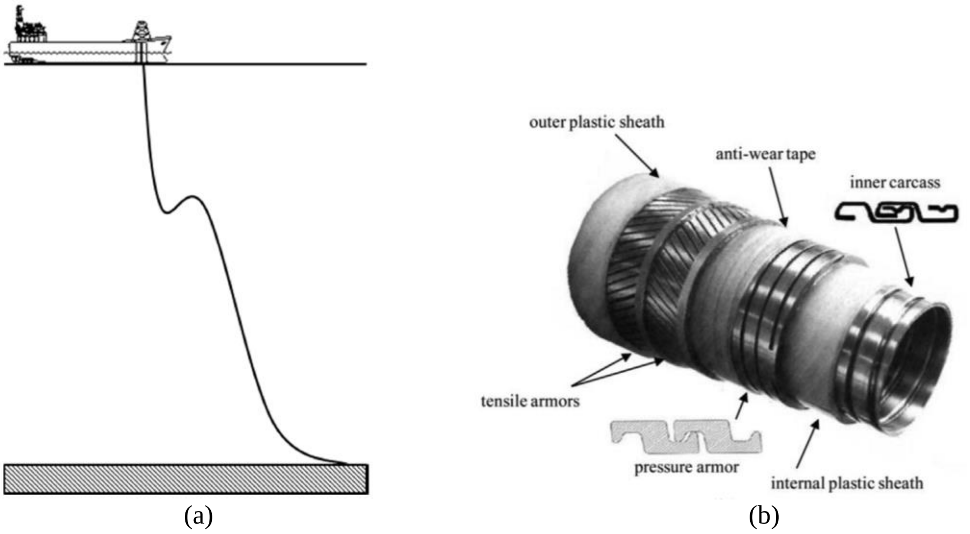


Figure 1. (a) Typical flexible riser (lazy-wave) configuration (Péronne *et al*, 2015);
(b) Typical flexible riser structure (Sousa *et al*, 2012).

During the design phase of a flexible riser, its structural behavior is of major importance, in a global and a local scale. An optimized project of these structures involves many interactions between global dynamic analysis and local structural calculations; Pesce (1997). The global dynamic analysis generates loads used to design the flexible cross-section. Then, considering the flexible riser structure, the design engineer estimates the main mechanical characteristics to be used as inputs into a second global dynamic analysis, closing a step of the design optimization loop.

It is well known, see, e.g., Aranha & Pesce (1997), Pesce & Martins (2005), that the riser global dynamic response is dominated by geometric rigidity, thus controlled by tension. However, locally, at the ends - top and TDZ (Touch Down Zone) - or inside regions of sudden variation of curvature - the lazy-wave hump, bending stiffness effects are highly significant.

As a first step of the design loop, linear bending behavior is usually considered. However, large contact pressures between layers can arise when the riser is subjected to high mechanical loads. Consequently, internal friction between components may play a substantial role. This effect dissipates energy, by adding hysteresis to the bending stiffness behavior. A number of authors describe this phenomenon in details (e.g., Boom & van der Boer, 1992, Saevik, 2011, and Sousa *et al*, 2012), presenting analytical and numerical tools capable to predict the bending stiffness in different situations. Some papers, e.g., Tan *et al* (2007) and Leroy *et al* (2010) estimate the hysteretic cycle due to a cyclic bending load.

As a main input in the global modelling, the bending stiffness of the flexible pipe has a major influence on the resulting curvatures along the line. Thereby, it influences directly the loads applied in the structure. Secondly, it also affects the dynamic analysis in a way that the hysteretic bending behavior can be seen as an energy dissipation factor. Péronne *et al* (2015) describes a refinement of the dynamic global analysis and the fatigue analysis considering the hysteretic bending stiffness (HBS) of flexible risers, showing the importance of considering the energy dissipation in the design project.

As frequently pointed out, the external polymeric sheath dominates the bending stiffness behavior during the second phase of bending, once internal slippage has initiated. On the other hand, it is well known that polymers are commonly viscoelastic materials. Despite this fact, at least to the authors' knowledge, no attempt has been made to assess the rule such a viscoelastic behavior could play regarding flexible pipes bending. Usually, a simple linear elastic constitutive law characterizes the polymers in the existent models. The aim of the present work is to investigate the effect of the external sheath viscoelastic behavior on the local bending response of flexible pipes.

The present paper proposes a Finite Element approach that takes internal friction between layers as well as the viscoelastic behavior of the external layer into account. A linear viscoelastic model for the high-density polyethylene (HDPE) is calibrated using results from an experimental campaign, representing the time-dependent response of the polymeric material. Thereafter, the paper addresses a numerical case study, with a 4" flexible pipe. The structure is modeled in the Abaqus Finite Element software. The case study compares internal friction with viscoelastic effects, by considering them separately and by combining both models. Interesting findings are revealed.

2. HYSTERETIC BENDING BEHAVIOR

This section describes the typical hysteretic bending behavior of flexible pipes found in the literature, due only to the contact interactions between layers. As mentioned previously, many authors describe the bending stiffness of risers depending only on the curvature of the central axis. The hysteretic bending stiffness (HBS) response is ruled by a slip-stick mechanism, activated by the contact pressures between layers. These contact pressures appear when the pipe is subjected to axisymmetric loads (tension, torsion, pressure) and increases as these loads become important. Fig. 2 shows a sketch of a typical HBS response, depending only on the curvature of the pipe.

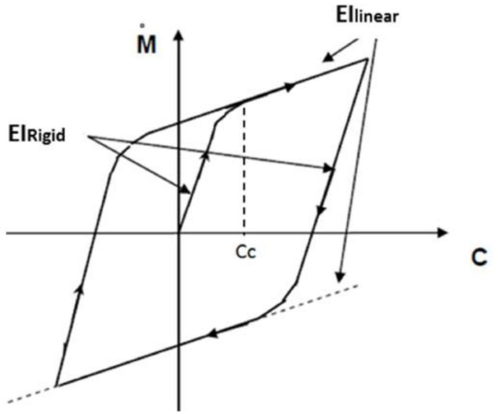


Figure 2. Schematic representation of the hysteretic bending moment vs. curvature curve (Péronne *et al*, 2015).

When the applied curvature is small, the initial friction between layers precludes slipping. Before overcoming this friction limit, the bending load is resisted conjunctly by all metallic and polymeric components. This leads to a quasi-linear bending-to-curvature response, with a high stiffness. The tangent to the curve for this small curvature state is usually called 'no-slip bending stiffness', illustrated as EI_{Rigid} in Fig. 2.

By increasing the curvature, the interlayer friction limit state is eventually overcome, allowing slippage between components to occur. Due to the relative movement between the helically wound wires and their adjacent layers, the metallic layers cease to resist significantly to the bending loads. In this second phase, analogously to a bent helical spring, the metallic layers of the flexible pipe offer a very low bending resistance, making the contribution of the polymeric layers to prevail. This phase, when the polymeric layers regulate the bending stiffness of the riser, is called 'full-slip bending stiffness', illustrated as EI_{linear} in Fig. 2. The curvature value corresponding to the internal friction limit (C_c in Fig. 2) is usually called 'critical curvature'. It is worth mentioning that when the polymers are represented by a linear elastic material law the resultant bending moment vs. curvature function is also linear.

If the flexible pipe is loaded and unloaded, by exceeding the critical curvature, the hysteretic behavior appears, characterized by the (quasi-) bilinear behavior shown in Fig. 2. When the bent flexible pipe is unloaded, reversing the curvature, the metallic layers are prevented from sliding once again, by the interlayer friction, until curvature reaches a new critical value. Therefore, load and unload path in the bending moment vs. curvature diagram are not the same. This mechanism leads to a hysteretic loop, being the area inside the loop proportional to the energy dissipation (per unit of pipe length) in a given bending cycle.

Grealish *et al* (2006), Smith *et al* (2007) and Péronne *et al* (2015) highlight that this hysteretic response has a substantial impact on the fatigue life of flexible pipes, as curvature amplitudes resulting from the global dynamic of the riser depend on that. They point out that a correct estimate of the HBS provides more accurate fatigue predictions. Meanwhile, the authors present different theoretical approaches leading to quite different curves. Furthermore, substantial differences can be found between the coefficient of Coulomb friction used by the authors, with values varying between 0.07 and 0.20.

Complementing all those studies, the present paper intends to address the effects of external polymeric layer viscoelastic behavior on the hysteretic bending stiffness of flexible pipes.

3. HDPE VISCOELASTICITY

Viscoelastic polymers have substantial energy absorption abilities. This kind of material is commonly incorporated in mechanical systems in order to dissipate energy avoiding structural damages. Viscoelasticity has been widely studied for decades; see Ferry (1961), Flügge (1975), Roylance (2001) and Brinson (2008). Such a material behavior is characterized by a strain time rate dependent delayed response of the structure, the force always preceding the displacement, which can be described by a relaxation function and a creep function. In the offshore risers scenario, viscoelastic material has been studied in a number of applications; see Caire *et al* (2005), Vaz *et al* (2009), Vaz & Caire (2010) and Torres & Vaz (2014)).

The high-density polyethylene (HDPE), regularly used for the external layer of flexible pipes, is a typical viscoelastic polymer. Based on Chen (2000), Rabelo (2014) experimentally characterized a HDPE pipe material behavior through relaxation and creep tests, at different load rates.

A well-known analytical model used to represent linear viscoelastic behavior is the one given by the Prony series, presented in Eq. (1),

$$\sum_{i=1}^{N} \alpha_i e^{-\frac{t}{\tau_i}} \tag{1}$$

Considering the strain history, the time dependent stress-strain state of the linear viscoelastic material can be determined. The load and displacement history, what includes the loading time rate, influence the material behavior. The Prony series introduces these dependences in the constitutive equation. The linear viscoelastic constitutive model used to model the HDPE can be represented by a simple physical model composed by a spring and a damper. The spring is the linear-elastic component, and its constitutive equation is the Hooke's law, presented in Eq. (2).

$$\sigma = E\varepsilon \tag{2}$$

The damper introduces the viscous behavior, and its constitutive equation may be written in the form of Eq. (3), where η is the viscosity constant.

$$\sigma = \eta \frac{\partial \varepsilon}{\partial t} \tag{3}$$

A linear viscoelastic constitutive model may be constructed by superimposing the effects of Eqs. (2) and (3). Using the Prony series for the relaxation behavior, a constitutive relation can be written as follows in Eq. (4), being $Y(t)$ the relaxation function.

$$\sigma(t) = Y(t)\varepsilon_0 \tag{4}$$

This relaxation function $Y(t)$ is typically modeled with the Prony series as presented in Eq. (5), where p_i is the i'th Prony constant, τ_i is the i'th Prony retardation constant, and E_0 is the initial value of the elastic modulus of the material. It is easily seen that $Y(0) = E_0$ and that $Y(\infty) = E_0(1 - \sum p_i)$.

$$\frac{Y(t)}{E_0} = 1 - \sum_{i=1}^{n} p_i \left(1 - e^{-\frac{t}{\tau_i}}\right) \tag{5}$$

Experimental results of relaxation and creep tests on HDPE carried out by Rabelo (2014) were used to calculate the Prony coefficients. Fig. 3 illustrates the details of the HDPE specimens. Rabelo (2014) conducted relaxation tests at three different strain levels (0.45%, 1.00% and 2.50%) and creep tests at three different stress levels (3 MPa, 8 MPa and 18 MPa). Fig. 4 illustrates examples of results for each strain and stress levels.

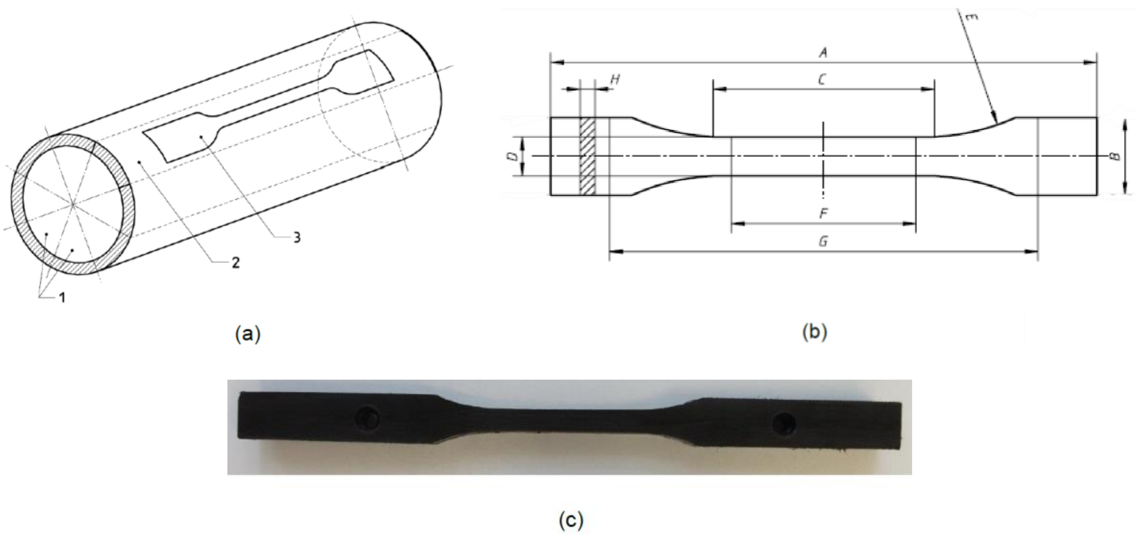
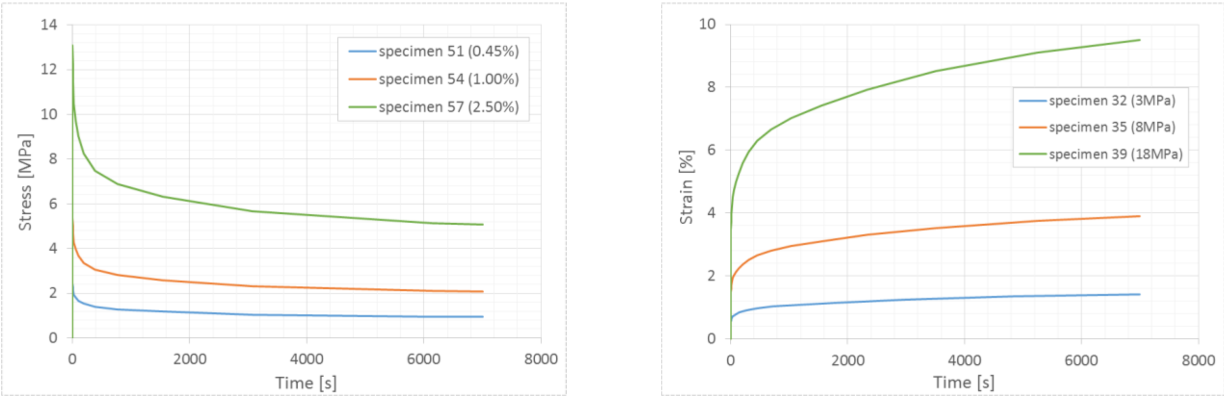


Figure 3. (a) Specimens manufactured from a HDPE tube; (b) Specimens dimensions according to ISO6259-3;
(c) Example of a specimen (Rabelo, 2014).



(a) (b)

Figure 4. Example of the experimental HDPE characterization results, from Rabelo (2014):
(a) Relaxation tests; (b) Creep tests.

The HDPE experimental data was inputted in the Abaqus 6.12-3 software and a truncated Prony series was determined up to the fourth coefficient, as presented in Table 1. The E_0 estimated for the HDPE is presented in the next section, in Table 2.The Prony series model, given by Eq. (5), is illustrated in Fig. 5.

Table 1. Prony series coefficients.

i	p_i	τ_i [s]
1	0.132	0.512
2	0.211	13.4
3	0.185	156
4	0.171	2294

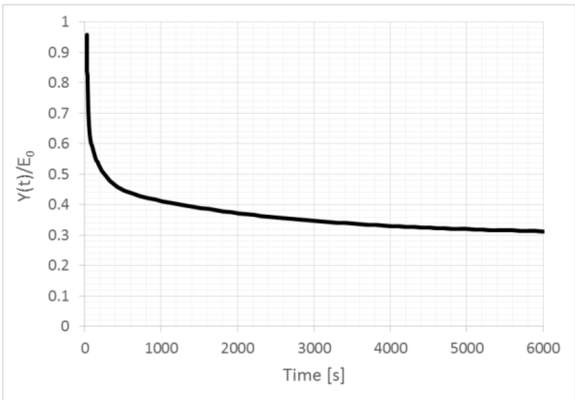


Figure 5. Normalized Prony series calculated with Eq. (5) using the coefficients presented in Table 1.

The linear viscoelastic constitutive model was then used in a Finite Element model to analyze the effect of the viscoelasticity in the bending stiffness of flexible pipes. The proposed approach will be presented in the next section, through a case study with a 4” flowline.

4. CASE STUDY

As mentioned above, the HBS might be affected by the external polymeric layer, usually composed by HDPE, a viscoelastic material, whose time-dependent response may be characterized by a Prony series, as described in the precedent section. The present case study runs a Finite Element model capable to reproduce the HBS of the modeled flexible pipe by taking into account both, the viscoelastic behavior of the external layer and/or the interlayer friction.

The FE model, based on Santos (2013) and Santos *et al* (2015), reproduces a 4” flexible pipe. The main properties of this flexible pipe are presented in Table 2. Fig. 6 shows, at left, a view of the pipe and, at right, a view of its cross-section.

Table 2. Prysmian flowline 4” properties from Santos (2015).

Layer	Thickness [mm]	Lay ang. [degrees]	N. of wires	Ext. diam. [mm]	E_0 [GPa]	ν
Carcass	5.00	-87.97	1	111.60	191	0.3
Inner sheath	5.00	n/a	n/a	121.60	0.284	0.47
Pressure armour	6.45	-88.58	1	134.50	200	0.3
Tensile armour 1	2.00	36.00	46	138.50	200	0.3
Tensile armour 2	2.00	-34.00	49	142.50	200	0.3
Outer sheath	6.00	n/a	n/a	154.50	0.442	0.47

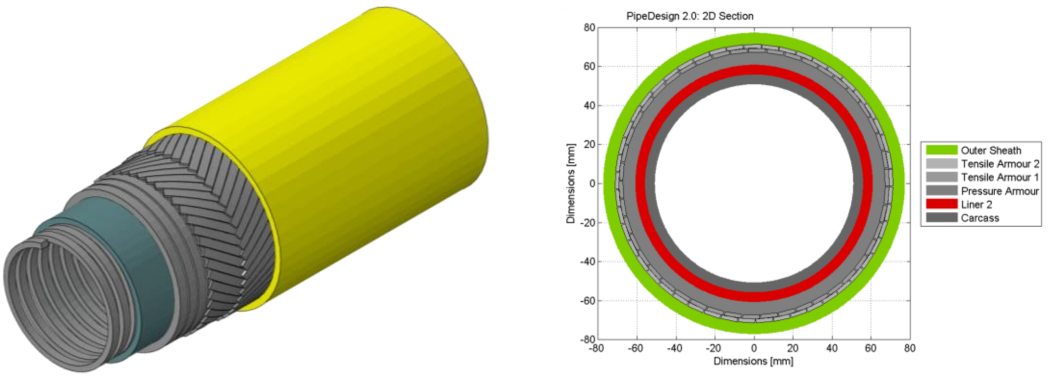


Figure 6. 3D view from the Abaqus FE model of the analyzed 4” flexible pipe and 2D view created with *PipeDesign*, proprietary software.

Manned with the geometries of all components and the coefficients that characterize the material behavior for all layers, the proposed FE model is capable to reproduce the HBS for this flexible riser under specific conditions set by the user. Some more details about the finite element model are presented in Table 3 and in Fig. 7. Another important detail to retrieve the flexible pipe HBS is to keep all layers in contact. In order to ensure the interlayer contact during all the calculations, the modeled pipe is subject to an external pressure load of 1 MPa.

To obtain the hysteretic response, a sinusoidal bending load was applied around the non-deformed configuration. This was done using multi-point constraints (MPC), by imposing the model a rotation at both extremities until the desired curvature is reached. The maximum curvature was chosen as 50% of the value corresponding to the minimum bending radius (MBR) of this particular pipe, which is equal to 1.36 meters. Therefore, curvature in the HBS diagrams varies between -0.37 m^{-1} and 0.37 m^{-1}.

The coefficient of Coulomb friction μ and the material constants are parameters of the model. Thus, the influence of these two inputs in the hysteretic bending stiffness (HBS) of the flexible pipe can be analyzed separately. Considering these two parameters at a time, the model reveals the influence of these nonlinear effects in the bending load and, then, both effects can be modeled together, reproducing a more realistic case.

Table 3. FE model size.

	Elements	**Nodes**
Flexible pipe	37 657	75 904
Contact interactions	31 842	63 690
Total	**69 499**	**139 594**
Degrees of freedom	**323 298**	

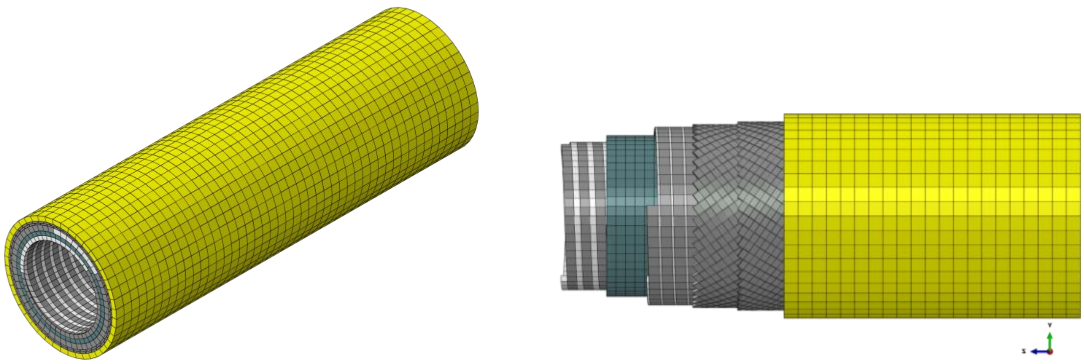


Figure 7. FE model developed based on Santos (2015).

The first analyzed case is a typical one found in the literature: only friction, all materials assigned with elastic constitutive laws, ruled by a Young modulus and a Poisson coefficients ν (parameters presented in Table 2). Figure 8 presents the HBS for different coefficients of friction μ. The black curve indicates the null friction case, while the other three curves refer to $\mu = 0.10,\ 0.15,\ 0.20$. It is worth mentioning that the Young modulus of the HDPE used in this first example corresponds to a case without viscous behavior. In this case, no viscosity means considering $t \to \infty$ in Eq. (5), in other words, considering the long-term Young modulus $Y(\infty) = E_0(1 - \sum p_i) = 0.301E_0$ in the numerical model.

As expected, the simulations with a frictionless contact do not present a hysteretic character. With all materials assigned with linear-elastic constitutive laws, nonlinear effects are not present and the result is characterized by a linear bending stiffness behavior. Introducing a Coulomb's frictional contact, a typical HBS of a flexible pipe, as found in the literature, appears. As expected, the area of the hysteretic cycles increases with the friction coefficient, making explicit the relation between this parameter and the energy dissipation per bending cycle.

This first HBS diagram, only considering Coulomb's frictional contact mechanisms, is not time-dependent. As mentioned above, the bending load was applied sinusoidally, with a period of oscillation of 1 second. Even changing the period of oscillation or the wave format, the same results are obtained, as the contact model does not depend on the load history and no structural dynamics is implied.

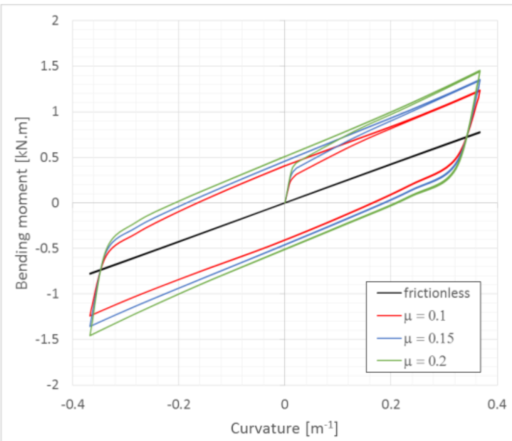


Figure 8. HBS diagram. Coulomb friction model. HDPE with elastic behavior.

Leaving aside the interlayer friction for a while, the next HBS diagram was generated taking into account only the effect of the external layer viscoelasticity. For this second case, the response of the FE model depends on the load history. The bending load was applied varying the oscillation period. The results are presented in Fig. 9 covering four decades of oscillation periods: 1, 10, 100 and 1000 seconds. These large range spans all the typical load periods a real flexible pipe might be subjected to, from vortex-induced vibrations (1-10 seconds), floating platform induced motions (10-500 seconds) to quasi-static loadings; see Pesce and Martins, 2005. The black curve (linear elastic, no friction) previously presented in Fig. 8 is taken in Fig. 9 as a reference.

Two main conclusions can be drawn from the results presented in Fig. 9. Firstly, as expected, when considering the viscoelasticity, the bending stiffness of the flexible pipe is proportional to the load speed. The yellow hysteretic cycle, obtained with an oscillation period of 1 second, has the higher bending stiffness. Decreasing the load speed, for bigger periods, the bending stiffness clearly decreases, until the period of 1000 seconds. For the red curve, the speed load is quasi-static and the viscoelasticity do change considerably the bending stiffness of the FE model. That makes the inclination of the red ellipse comparable to the inclination of the black curve (no friction, linear-elastic materials).

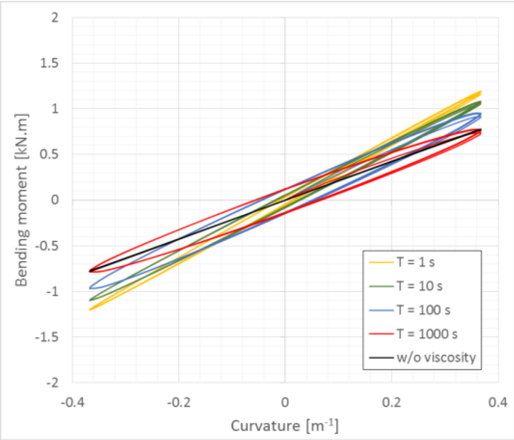


Figure 9. HBS diagram with frictionless interlayer contact and HDPE with viscoelastic behavior. Sinusoidal loading with period T.

A second remark about Fig. 9 is that slow bending cycles reveal larger hysteretic areas, indicating that more energy is dissipated per cycle. The area of the red ellipse obtained by cycling the flexible pipe with a period of 1000 seconds is larger than the other ones and the dissipation of energy considering only the viscoelastic effect is about 50% of those presented in Fig. 8.

A third analysis was made placing both effects together, the frictional interlayer contact and the viscoelasticity of the external HDPE layer. The results, presented in Fig. 10, were produced for a coefficient of friction $\mu = 0.15$ and varying the period of oscillation. The (linear-elastic material, only friction) black curve was taken again as reference.

Fig. 10 shows the same effect noticed in Fig. 9: the 'full-slip bending stiffness' is proportional to the load speed. In addition to that, as expected, the 'no-slip bending stiffness' is not significantly affected by the constitutive law of the external layer, as in this phase curvature is below the critical friction value. Also, the energy dissipated per cycle is slightly affected by load speed. However, viscoelasticity clearly increases, by a large amount, the energy dissipated per cycle if compared to the pure friction contact case. This result shows clearly that the viscoelasticity of the external HDPE layer may contribute substantially to the dissipation of energy during the bending cycle of flexible pipes.

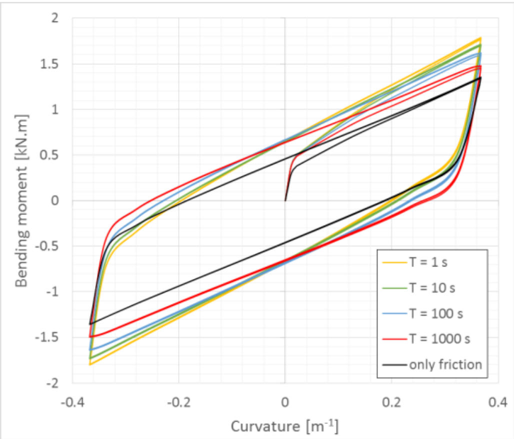


Figure 10. HBS diagram combining frictional (μ=0.15) interlayer contact with HDPE viscoelastic behavior.

Finally, Fig. 11 makes a comparison for each simulated loading period (1, 10, 100 and 1000 seconds). These results compare the following cases: (i) both effects considered; (ii) only friction considered (no viscoelastic effect); (iii) only viscoelasticity considered (no friction). As mentioned, loop areas are proportional to the energy dissipation per cycle and Fig. 11 reveals an interesting conclusion: the sum of the areas corresponding to the isolated effects (ii) and (iii) is smaller than the area of the combined case (i). Or, in other words, combining these two effects is not the same as to summing up the two.

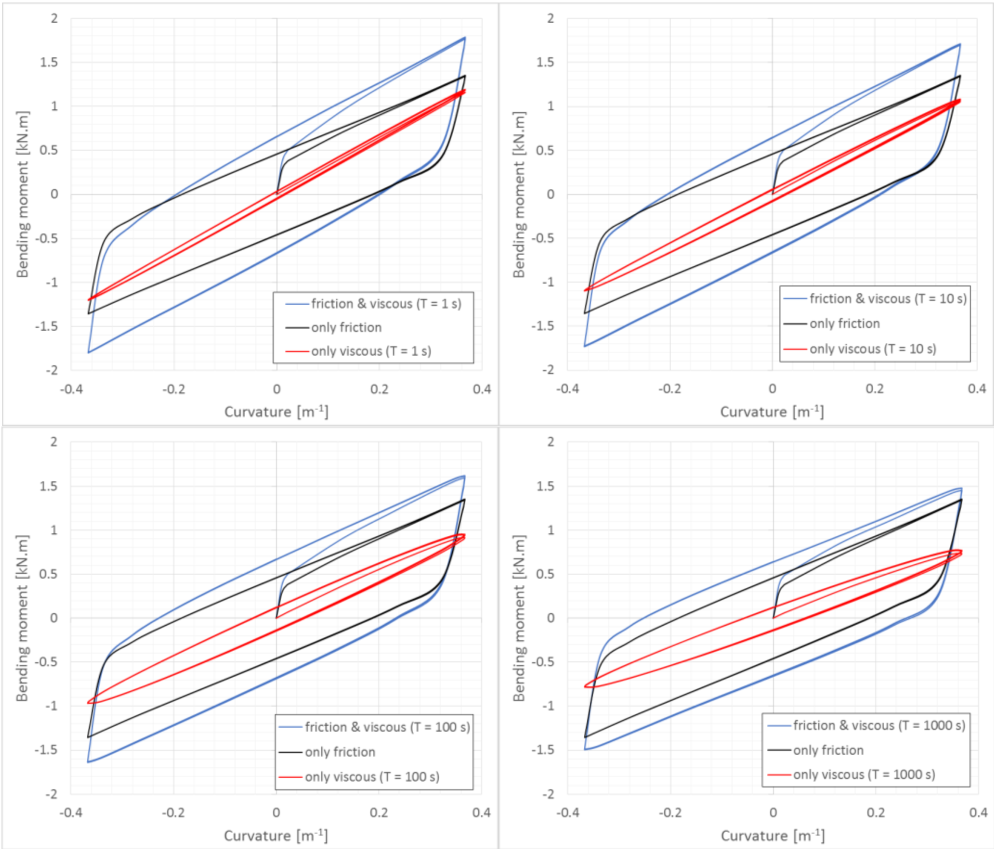


Figure 11. HBS diagrams considering sinusoidal load periods of 1, 10, 100 and 1000 seconds.

Together, the nonlinearities act in a synergistic way, dissipating more energy. As noticed already, the ‘no-slip bending stiffness’ is not affected considerably by the external layer of the riser. The black curves and the blue curves are almost coincident in the ‘no-slip zone’. In contrast, in the ‘full-slip’ regime the external sheath behavior rules the hysteretic mechanism. It can be seen that the inclination of the blue loop in the full-slip zone follows the inclination of the red ellipses. Then, when considering both, interlayer friction and viscoelasticity, the mechanism are synergistically combined, showing that the viscoelasticity affects directly the hysteretic loop and, therefore, the dissipation of energy during bending. As in structural dynamics, structural damping is paramount, this might substantially affect curvature amplitudes and, in consequence, the fatigue life of a flexible pipe riser.

5. CONCLUSIONS

The present investigation study assessed the influence of the viscoelasticity in the hysteretic behavior of flexible pipes subjected to bending. In a first case study step, the well-known typical loop diagrams of bending load vs. curvature were reproduced, considering only the interlayer friction. In a second step, a new (and almost elliptical) hysteretic behavior was obtained in a frictionless simulation, considering a linear viscoelastic constitutive law for the external polymeric layer material. When both effects were considered together, the influence of the viscoelasticity remains apparent. The hysteretic loops change with the load history and their areas are relatively larger than the typical ones corresponding to pure frictional interlayer contact. This paper shows that, at least for the conditions simulated in the case study, the effect of the viscoelasticity of the polymeric external layer is important.

For further work, different combinations of coefficients of friction and speed loads are to be tested. The influence of the FE model length is also to be analyzed. New hysteretic bending stiffness loops can be used as inputs to global dynamic analyses to assess the influence of viscoelasticity on the fatigue life of a flexible pipe.

6. ACKNOWLEDGEMENTS

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