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A VARIATIONAL CONSTITUTIVE MODEL FOR ACTIVE BEHAVIOR OF SKELETAL MUSCLES

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Abstract. *Skeletal muscles are capable to provide force due to their components, which are formed mainly by bundles of specialized cells called muscle fibers. The mechanical response of skeletal muscle can be represented by two anisotropic contributions, a passive contribution that is related to the intrinsic muscle elasticity and an active contribution that dependents of an electrical stimulus of the muscle fibers. The numerical simulation of such materials have been studied for many years, but still it is a challenging task, where the development of an adequate constitutive model is a genuine challenge. Then, this paper propose an anisotropic model capable to take into account the muscle activation, when it is submitted to an electrical stimulus and finite strains. This approach is based on the Hill model. This model consists of an elastic element in series with a contraction component, and both in parallel with a second elastic element. The constitutive model is based on a variational framework, in which a local minimization provides the contractile variable updates for each load increment. The numerical results show that the proposed model can represent the muscle passive and active behavior, and may be used to represent skeletal muscles.*

Keywords: *Skeletal muscle, active-passive behavior, contraction*

1. INTRODUCTION

Unlike any other soft tissues, the muscle tissue is capable of produce force, given an electrical stimulus. Working in coordination with the skeletal and the neural systems, the skeletal muscle is responsible for the human motion. This tissue is very organized, where muscle fibers are grouped in bundles up to three different levels in one or many directions. The muscle fiber is composed of sarcomeres, along with other structures responsible for maintaining these specialized cells functioning. The sarcomer is the smallest contraction unit of the muscle, who is activated when a electrical stimulus is spread. Upon activation the sarcomer tend to shorten it's length [Huxley, 1957]. This microscopical behavior can be seen macroscopically as either a shortening of the muscle, or as a increase in the strength necessary to maintain the muscle in the same length.

When the muscle is not activated (there is no action potential), the mechanical response of the muscle is as any soft tissue, and is called as passive response. When the muscle is activated, in a purely mechanical point of view, the sarcomer increases the rigidity, thus the muscle responds with a combination of the passive response with the active response [Gordon *et al*, 1966]. There are two main features of this behavior that must be observed, the first concerns the shift from the passive to the active/passive state. Given the internal mechanism of activation, the sarcomer does not respond instantly with all the active force: there is a increase of the force until a plateau is reached [Ter Keurs *et al*, 1978]. The second main feature of the active and passive contributions is that their relation is not constant when the muscle stretch increases. The passive stress increases with increasing stretch, as any soft tissue, while the active response has a well defined peak [Gordon *et al*, 1966]. The features described above are the most basic to represent the muscle mechanical behavior, other muscle features can be seen in [Rassier *et al*, 2003; Herzog *et al*, 2005; Van Loocke *et al*, 2009, McGowan *et al*, 2010; Lu *et al*, 2010; Rehorn *et al*, 2014].

The muscle response has been investigated since last century, trying to capture some of the specific features of the muscle behavior. To this date, there is no model found in literature that represent all the features of the muscle response. Most of the present models are based in the Hill's model. This propose was developed in 1932 by Hill to relate the velocity of muscle contraction and the loading imposed. Initially this model had two elastic component in parallel [Hill, 1932], however as the muscle knowledge grew a third component was inserted to account for muscle activation. There is a loose definition of the Hill's model, regarding only the use of two elastic components and one active component, in any combination [Fung, 1990]. One of the most usual configurations consist of an elastic element in parallel with an elastic element in series with a active element. This configuration allows that when the muscle is not activated only the elastic element in parallel responds, and upon activation there is an increase in force and tension. The relation between the active and elastic components in series is fundamental to determine the evolution of the internal variables and the evolution of the active force: many models have been developed only by changing this relation.

The study of muscle behavior is very complex, and can assist many areas of interest. One of the most important and difficult task regarding bone remodeling is the determination of the loading applied to the bone. The understanding of the muscle characterization is the first step to conceive a model that account for muscle properties changes due to training. Such model could help determining a training to an athlete, or even a exercises to motion recovery patients. The understanding of forces that occur during motion could also help in the design or the choice in prostheses. All that could be achieved with a adequate model to represent the muscle constitutive relations.

A constitutive model that aims to represent the skeletal muscle mechanical response should be capable of handling finite strains, muscle fiber anisotropy and a representation of the internal changes that happen during sarcomer contraction. Variational constitutive models can represent finite strains and fiber anisotropy, and are known as a flexible way to evaluate the evolution of internal variables [Vassoler *et al*., 2012]. Therefore the aim in this paper is to propose a simple set of equations using variational constitutive relations to represent muscle active and passive responses. The muscle will be considered as fibers equally dispersed in a matrix, where the anisotropic contribution will be modeled as a Hill's model.

2. VARIATIONAL CONSTITUTIVE FRAMEWORK

The variational constitutive relations use a mathematical expression to define a free energy functional, with conservative and dissipative contributions. This pseudo-potential energy is defined at each load step [Fancello *et al*., 2006], and the actual second Piola-Kirchhoff stress can be computed as :

$$\mathbf{S}_{n+1} = 2\frac{\partial \Psi\left(\mathbf{C}_{n+1};\xi_{n+1}\right)}{\partial \mathbf{C}_{n+1}} \tag{1}$$

where Ψ is the pseudo-potential, $\xi = (\mathbf{C}, \mathbf{Q})$ is the set of external and internal variables, where $\mathbf{C}$ is the right Cauchy-Green and $\mathbf{Q}$ is the internal variables set.

The incremental pseudo potential can be written as [Stainier, 1999]:

$$\Psi\left(\mathbf{C}_{n+1};\xi_{n+1}\right) = \min_{\mathbf{Q}_{n+1}} \quad \left\{ W\left(\xi_{n+1}\right) - W\left(\xi_{n}\right) + \Delta t \psi(\overset{\circ}{\mathbf{Q}};\xi) \right\} \tag{2}$$

and W is the conservative contribution of the pseudo-elastic potential and ψ is the dissipative contribution of the pseudo-elastic potential.

The stress can be obtained in a hyperelastic-like manner. Considering the volumetric and isocoric split the second Piola-Kirchhoff stress tensor can be written as:

$$\mathbf{S}_{n+1} = J_{n+1}^{-2/3} Dev\left(2\frac{\partial \Psi\left(\mathbf{C}_{n+1};\xi_{n+1}\right)}{\partial \overline{\mathbf{C}}_{n+1}}\right) + J_{n+1}\frac{\Psi\left(\mathbf{C}_{n+1};\xi_{n+1}\right)}{\partial J_{n+1}}\mathbf{C}_{n+1}^{-1} \tag{3}$$

where:

$$Dev\left(\bullet\right) = \left(\bullet\right) - 1/3\left[\left(\bullet\right) : \mathbf{C}\right]\mathbf{C}^{-1} \tag{4}$$

$$J = \det\left(\mathbf{F}\right) \tag{5}$$

$$\overline{\mathbf{C}} = J^{-2/3}\mathbf{C} \tag{6}$$

To represent the mechanical behavior of the skeletal muscle the rheological model, presented in Figure (1), was used to define the pseudo-elastic energy functional. In order to obtain the desired mechanical behavior, this paper proposes the conservative contribution to the pseudo-elastic potential as:

$$W = \Psi_V(J) + \overline{\Psi}_M(\overline{\mathbf{C}}) + \overline{\Psi}_{PE}(\overline{I}_4) + \overline{\Psi}_{SE}\left(\overline{I}_{4SE}\right) + \overline{\Psi}_{SC}\left(\overline{I}_{4SC}, f_a\left(t\right)\right) \tag{7}$$

The first contribution was the volumetric $\Psi_V\left(J_{n+1}\right)$, depending only in the jacobian J. The isochoric contributions were divided into an isotropic matrix contribution $\overline{\Psi}_M\left(\overline{\mathbf{C}}_{n+1}\right)$ and the fiber anistropic contributions. The fiber were

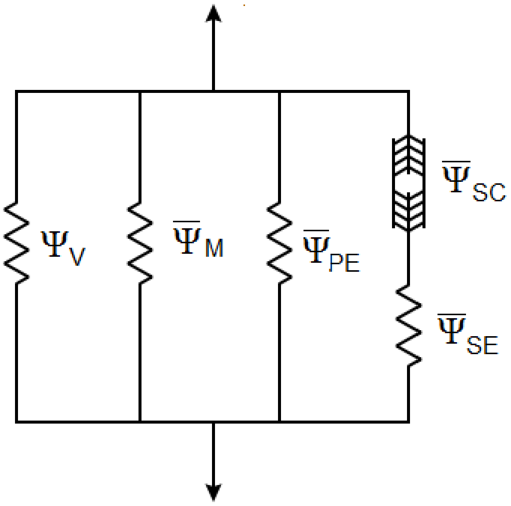

Figure 1. Rheological representation of the proposed model

modeled as a Hill's model with the purely elastic contribution $\overline{\Psi}_{PE}\left(\overline{I}_4\right)$ and the active branch contributions $\overline{\Psi}_{SE}\left(\overline{I}_{4SE}\right)$ and $\overline{\Psi}_{SC}\left(\overline{I}_{4SC}, f_a\left(t_{n+1}\right)\right)$, depending in the modified fourth invariant of each stretch. The contractil pseudo-potential also has a dependence of a function f_a that will later be discussed.

The volumetric energy functional was taken to represent a quasi-incompressibility state:

$$\Psi_V(J) = \frac{K}{2}(J-1)^2 \tag{8}$$

The Neo-Hookean energy functional was taken for the matrix as Lu *et al*, 2010:

$$\overline{\Psi}_M(\overline{\mathbf{C}}) = \frac{\mu_1}{2}(\overline{I}_1(\overline{\mathbf{C}}) - 3)^2 \tag{9}$$

where K and μ_1 are material parameters and $\overline{I}_1$ is the modified first invariant.

The objective of this first approach is to reproduce the Hill's model behavior within the variational framework. Unlike other fiber-reinforced materials, skeletal muscle fiber behaves in a peculiar way. Muscle active behavior have a known length dependence, and many authors define the reference configuration as the length where the maximum muscle active force is produced. The elastic energy functional in parallel, for the Hill's model, can be written as:

$$\overline{\Psi}_{PE}(\overline{\mathbf{C}}, \overline{I}_4) = \begin{cases} \frac{\mu_2}{2}(\overline{I}_4(\overline{\mathbf{C}}, \mathbf{a}_0) - I_{4ref})^2 & \text{if } \overline{I}_4 > I_{4ref} \\ 0 & \text{if } \overline{I}_4 \leq I_{4ref} \end{cases} \tag{10}$$

where μ_2 and I_{4ref} are material parameters, $\overline{I}_4$ is the modified fourth invariant and $\mathbf{a}_0$ is the fiber direction in the reference configuration.

The peculiar behavior in skeletal muscle fiber is that a muscle fiber can produce force with compressive stretches, due to sarcomer internal arrangement. The second branch in Hill's model can determined by:

$$\overline{\Psi}_{SE}(\overline{I}_{4SE}) = \frac{\mu_3}{2}(\overline{I}_{4SE}(\overline{\mathbf{C}}, \mathbf{a}_0) - 1)^2 \tag{11}$$

$$\overline{\Psi}_{SC}(\overline{I}_{4SC}) = \mu_4 f_a(t)\frac{\sqrt{2\pi}}{2}\mu_5 erf\left(\frac{\sqrt{2}}{2\mu_5}\left(\overline{I}_{4SC}(\overline{\mathbf{C}}, \mathbf{a}_0) - \mu_6\right)\right) + cf_a(t) \tag{12}$$

where $\overline{I}_{4SE}$ is the modified fourth invariant of the series elastic element, $\overline{I}_{4SC}$ is the modified fourth invariant of the series contractile element, μ_i and c are material parameters and f_a is the activation function. Also, since the fiber acts only in one direction, we can write the relation:

$$\overline{I}_4 = \overline{I}_{4SC}\overline{I}_{4SE} \tag{13}$$

It is must be noted that equation 12 was developed to represent the energy associated with the active stress, as presented in [Hernández-Gascón *et al.*, 2013] force-length relation.

The main feature in muscle representation is the dependence on the activation of the muscle, given in these equations by f_a. Muscle activation can be seen in various ways, as the action potential the nervous systems delivers, as the electrical stimulus that spread along the muscle fiber, or as the calcium that is pumped inside the sarcomer to begin the contraction. In this proposal is made the same assumption of other works, in which the activation function is known and depend only on material parameters and the activation/deactivation times. This function maps the usual input form the nervous system to reproduce a condition where the muscle produces the maximum force at that length, a tetanized condition. One way of representing this activation is using a saturation law [Tang *et al.*, 2009]:

$$f_a = \begin{cases} 0 & t < t_0 \\ h_t\left(t, t_0\right) & t_0 \leq t < t_1 \\ h_t\left(t_1, t_0\right) - h_t\left(t_1, t_0\right) h_t\left(t, t_1\right) & t \geq t_1 \end{cases} \tag{14}$$

$$h_t\left(t_i, t_b\right) = \{1 - exp\left[-\mu_7\left(t_i - t_b\right)\right]\} \tag{15}$$

where t_0 is the time when the muscle is activated, t_1 is the time when the muscle is deactivated and μ_7 is a material parameter.

The insertion of the activation function in equation 12 provides a mathematical structure where the energy may no be zero in the active component when the muscle is in the reference configuration. If the muscle is activated, i.e. $f_a \neq 0$, there is a input of energy, which is converted to mechanical work, and as expected the energy is not null. However, if the muscle is not activated, i.e. $f_a = 0$, the energy in the active component will be zero at any muscle length.

The stress on the active Hill's model depend in the evolution of the stretch in the active and passive elements in series. The solution of equation 2 at each time increment corresponds to the solution of the internal variable $\overline{I}_{4SE}$. In this case, it is given by:

$$\frac{\partial W}{\partial \overline{I}_{4SC}} + \frac{\partial W}{\partial \overline{I}_{4SE}} = 0 \tag{16}$$

Expression 16 may be easily solved by easily solved by a Newton-Rhapson procedure, providing elastic strech of the component in series with the contractile element, i.e. $\overline{I}_{4SE_{n+1}}$, at each time increment. Finally, the second Piola-Kirchhoff stress tensor can be obtained by

$$\mathbf{S} = J^{-2/3} Dev\left(2\mu_1\left(\overline{I}_1 - 3\right)\mathbf{I} + 2\mu_2\left(\overline{I}_4 - I_{4ref}\right)\mathbf{a}_0 \otimes \mathbf{a}_0 + 2\mu_3\left(\overline{I}_{4SE} - 1\right)\mathbf{a}_0 \otimes \mathbf{a}_0\right) + JK\left(J - 1\right)\mathbf{C}^{-1} \tag{17}$$

3. RESULTS

An uniaxial numerical test is presented solely to evaluate the behavior of the proposed model. The material parameters used in the numerical example are shown in Table 1.

Table 1. Parameters used in model the numerical test

μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	K	I_{4ref}	c
0.0001	1	10	10	0.3	0.6	5	10000	1.1	3.5888

The result of the stress development (Figure 2) shows the model activation for two different stretches (1.2 and 1.4) in two different times 0.5 s and 2 s, as anticipated by the parameters. Before the activation the model behaved as expected, only presenting the passive stress. Once the model is activated the active stress rises according the activation function, until the plateau of the maximum active force at that length. This result shows the same behavior obtained by [Tang *et al.*, 2009; Lu *et al.*, 2010; Böl *et al.*, 2011; Ehret *et al.*, 2011; Hernández-Gascón *et al.*, 2013]. The active and passive stress response with respect to the evolution of the total stretch is presented in Figure 3. The results are in agreement with the original Hill model and those observed experimentally in [Ehret *et al.*, 2011].

4. CONCLUSIONS

In this paper was proposed a first approach of a skeletal muscle model using the variational constitutive framework introduced by [Stainier, 1999] . The model presents the well-known multiplicative decomposition of volumentric and

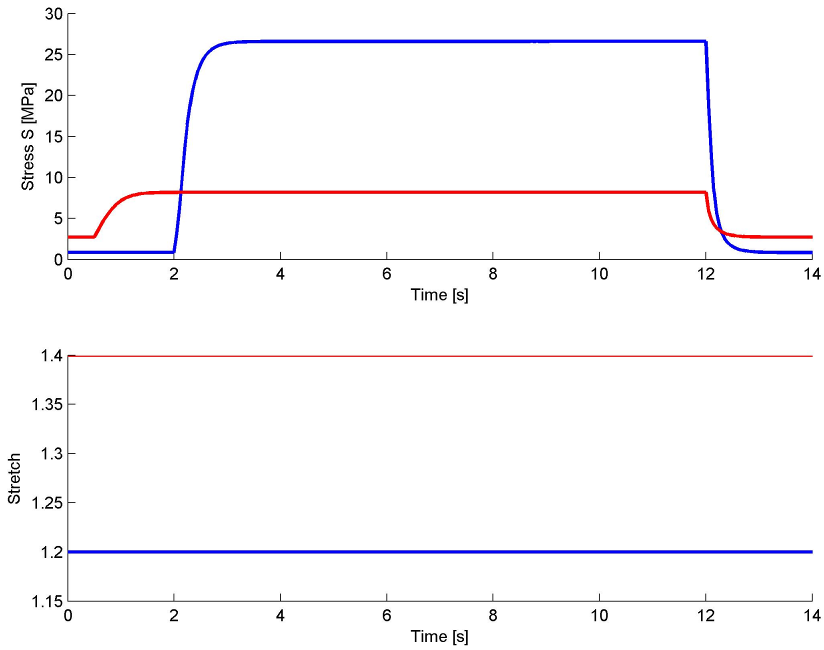

Figure 2. Development from passive to active and passive state

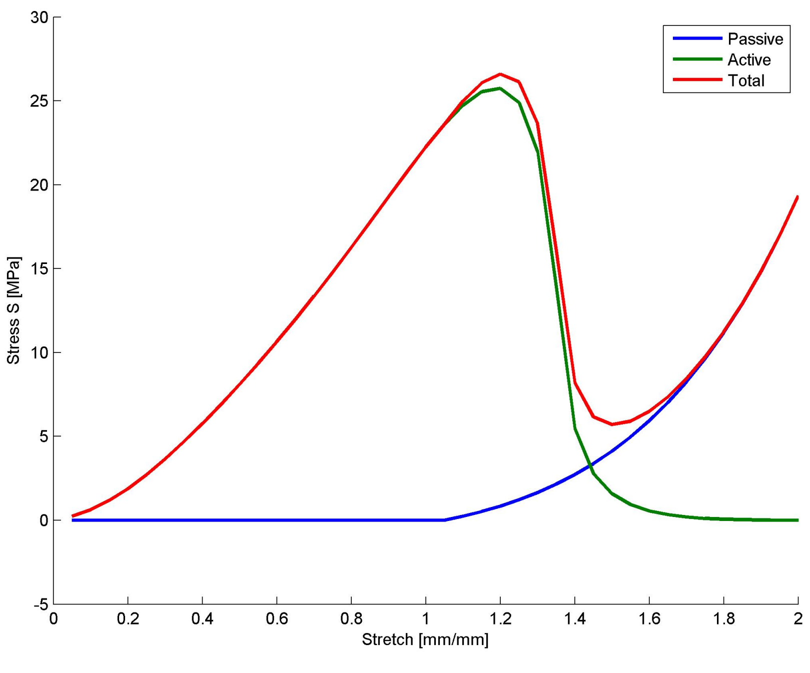

Figure 3. Active and passive stress for different stretches

isochoric contributions, where the isochoric contribution was divided in matrix and fiber contributions. A special attention was given to the fiber representation to reproduce similar behavior of the Hill's model, mainly regarding the application of skeletal muscle fiber considerations. In order to introduce the activation of the muscle, a dependence of a activation function was included on the potentials to increase the active stress from zero to the plateau at the muscle length. Finally, free energy functionals found in literature were used to verify if the expected behavior of the muscle could be represented. The mechanical response of skeletal muscle was qualitatively reproduce, representing appropriately the muscle active and passive stress as well as the activation. The versatility of the variational framework can be used to choose different energy functionals, where more adequate functionals require further investigations.

5. ACKNOWLEDGEMENTS

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