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SKELETAL MUSCLE FORCE GENERATION: PARAMETER IDENTIFICATION OF DIFFERENT MATERIAL MODELS TO A CONTRACTION COMBINATION

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Abstract. *Skeletal muscle tissue provides force generation, balance and locomotion for the human body. The main characteristics of this tissue is that they are composed of sarcomers, which contract upon electrical stimulus. Based on experimental observations, the muscle mechanical behavior exhibit a passive contributions, due to collagen fibers and intrinsic elasticity of the muscle fiber, as well as an active contribution, due to sarcomer contraction, that increases the tissue stiffness. Many authors have been investigating the muscle mechanical behavior, since the numerical simulation of the observed mechanical response is not trivial. In these studies the attention is given to individual contractions (isometric, eccentric or concentric), but not the combination of them, which are responsible to the most common movements of a skeletal muscle. In general, these models can be applied only to one specific kind of muscle contraction, not being able to predict different contraction combinations. Then, the main objective of this paper is to study and compare the mechanical response of different constitutive models capable to simulate isometric, eccentric and concentric contractions for large strains. In this paper the most promising models are presented and tested to an isometric-eccentric-isometric contraction. Also, to have a fair comparison between the models a parameter identification procedure was performed. The results show that none of the models were capable of representing the expected stress observed in the contraction combination, indicating a need of a new constitutive model that can represent these basic muscle mechanical behaviors. This result was not surprising, since no model was proposed to account for muscle contraction combination.*

Keywords: *Skeletal muscle, active-passive behavior, contraction, comparative analyses*

1. INTRODUCTION

In the human body all movement and force production rely on the coordination of muscles complex actions, which depend on the interaction of the muscle system, nervous system and skeletal system. In a simplified form, muscle is responsible for force generating, bones are responsible for structural support and the nervous system is responsible for controlling intensity and timing for force generation.

Muscles are tissues capable of generating force due the existence of specialized cellular substructures. Among these, the sarcomer is the most important as it acts as the tiniest unit responsible for muscle contraction. The sarcomer is composed of filaments of actin and myosin which slide upon each other when contracted, thus shortening the sarcomer length. To begin the sarcomer contraction, and shortening, the nerve must give an action potential, which spread spread through the muscle fibers as an electrical stimuli.

The muscle mechanical response does not rely only in the active stress, due the activation potential. Each muscle fiber that composes the muscle tissue is surrounded by up to three collagen fiber layers. This arrangement gives the muscle an passive behavior, a mechanical passive response of the tissue intrinsic properties, just like others soft tissues.

The muscle mechanical force representation depends also in the contraction kind that it is subjected, which can be classified according to length changes during activation. When muscle contracts and its length doesn't change, it's called an isometric contraction. When the muscle shortens during the contraction, it's called a concentric contraction. Finally, when the muscle length increases during the contraction, it's called an eccentric contraction. Each contraction kind have a specific mechanical response and when the muscle is subjected to the combination of these three basic contractions the mechanical response is amended. When a muscle is subjected to a sequential combination of contractions such as isometric-eccentric-isometric, there is an observed force gain when compared the final force with an purely isometric contraction force in the same final length (Rassier, 2003).

The main difficulty involved on the numerical modeling of muscle reside in the combination of different contractions kind, specially isometric contractions with concentric or eccentric contractions. Skeletal muscles have two well established models: one that relates the force produced in a certain muscle length (Huxley's force-length relation), and another that relates the force produced in different shortening velocities (Hill's force-velocity relation). Theorized by Huxley (1957), and experimentally verified by (Gordon et al., 1966), the force-length relation predict muscle isometric force for different lengths. Originally this relation was composed by linear regions (Huxley, 1957), however nowadays many authors change the expressions to better fit experimental data (Ehret et al., 2011; Tang et al., 2009;Kojic, 1998; Lu et al., 2010; Hernández-Gascón et al., 2013). The other well established muscle model was developed by Hill (1938), to reproduce the force generation decay with increasing shortening speed, observed experimentally. It was used a logarithmic force-velocity relation to characterize the concentric force production. This relation has been extended by many authors (Ehret et al., 2011; Tang et al., 2009;Kojic, 1998; Lu et al., 2010) to relate the force production with increasing muscle lengthening speed (eccentric contractions).

In literature many models combine the force-length and force-velocity relation aiming to represent muscle force production in different loading conditions. However, these results are specific to a certain kind of contraction, not existing a known model capable of represent all muscle mechanical responses, or even a simple realistic skeletal muscle movement based on an isometric-eccentric-isometric contraction. Then, in this paper, different muscle models are subjected to a parameter identification of experimental results of a isometric-eccentric-isometric contraction. This comparison aims to find qualitative and numerical response of the models representative capacities of the mechanical response related to skeletal muscle force generation along time.

2. MODEL FORMULATION

Some models found in the literature had their representation capacities previously studied. Here the most prominent models to reproduce the muscle mechanical behavior under contraction combination are presented.

MODEL PROPOSED BY BÖL: The model proposed by Böl (Ehret et al., 2011) aimed primarily to reproduce the active and passive stress of skeletal muscles, and secondly to represent the stress under isometric contractions. When there is no activation the model respond only with passive stress, as can be seen in Eq. (1) when the parameter w_a is null. The active behavior of muscle is condensed within w_a, including force-velocity and force-length relations, as well as the activation parameter. When the muscle is activated w_a is not null, magnifying the passive response to account for passive and active contributions. The Second-Piola Kirchhoff stress tensor S can be written as (Ehret et al., 2011):

$$\mathbf{S} = \frac{\mu}{4}\left\{e^{\alpha(\tilde{I}_p-1)}\left[\frac{w_0}{3}\mathbf{I} + (w_p + w_a)\,\mathbf{N}\right] - e^{\beta(\tilde{K}-1)}\mathbf{C}^{-1}\left[\frac{w_0}{3}\mathbf{I} + w_p\mathbf{M}\right]\mathbf{C}^{-1}\right\} - p\mathbf{C}^{-1} \tag{1}$$

where μ, α, β, w_0 e w_p are material parameters, $\mathbf{I}$ is the identity matrix, $\mathbf{C}$ is the right Cauchy-Green strain tensor, p is the hidrostatic pressure and $\tilde{I}_p$ and $\tilde{K}$ are pseudo-invariants. The Cauchy stress tensor σ can be obtained from the Second-Piola Kirchhoff stress tensor $\mathbf{S}$ (Holzapfel, 2000).

Should be noted that $\tilde{I}_p$ is a function of the activation parameter w_a, which in turn depends on the stretch of the muscle fiber, activation function, force-length relation and force-velocity relation. Further details are found in Ehret et al. (2011).

MODEL PROPOSED BY HERNÁNDEZ: The model proposed by (Hernández-Gascón et al., 2013) aim to represent isometric and isotonic contractions, and is based on a anisotropic incompressible hyperelastic formulation. It is assumed that the sarcomer can contract independently in different positions in the muscle, which means that the total deformation gradient of the muscle may not be equal to the active deformation gradient. Thus, there is a multiplicative decomposition of the isochoric deformation gradient $\overline{\mathbf{F}}$ into an active isochoric deformation gradient $\overline{\mathbf{F}}_a$ and a passive isochoric deformation gradient $\overline{\mathbf{F}}_p$. Using this mathematical framework the Cauchy stress tensor is obtained from a hyperelastic constitutive expression (Holzapfel, 2000). The free energy potential ψ is defined as:

$$\psi = \psi_{vol}\left(J\right) + \overline{\psi}_p\left(\overline{I}_1, \overline{I}_2, \overline{I}_4\right) + \overline{\psi}_a\left(\overline{J}_4, \overline{\lambda}_a,\right) \tag{2}$$

where ψ_{vol}, $\overline{\psi}_p$ and $\overline{\psi}_a$ are, respectively, the volumetric part, the isochoric passive part and the isochoric active part, J is the Jacobian, $\overline{I}_1$ e $\overline{I}_2$ are, respectively, the first and second isochoric principal invariants. $\overline{J}_4$ and $\overline{I}_4$ are the isochoric pseudo-invariants and λ_a is the active stretch related to $\overline{\mathbf{F}}_a$ in the fiber direction. The active and passive isochoric potential are defined, respectively, as:

$$\overline{\psi}_a\left(\overline{J}_4, \overline{\lambda}_a,\right) = f_1\left(\overline{\lambda}_a\right) f_2\left(f_r, t\right) \overline{\psi}_a'\left(\overline{J}_4\right) \tag{3}$$

$$\overline{\psi}_p\left(\overline{I}_1, \overline{I}_2, \overline{I}_4\right) = c_1\left(\overline{I}_1 - 3\right) + \overline{\psi}_{pf} \tag{4}$$

where f_1 a the force-length relation, f_2 is an activation function, f_r is an activation parameter and $\overline{\psi}'_a$ is a function related to the maximal isometric force. In this model the potential $\overline{\psi}_{pf}$ is defined as:

$$\overline{\psi}_{pf} = \begin{cases} 0 & \overline{I}_4 < \overline{I}_{4_0} \\ \frac{c_3}{c_4}\left(exp\left(c_4\left(\overline{I}_4 - \overline{I}_{4_0}\right)\right) - c_4\left(\overline{I}_4 - \overline{I}_{4_0}\right) - 1\right) & \overline{I}_4 > \overline{I}_{4_0} e \overline{I}_4 < \overline{I}_{4_{ref}} \\ c_5\sqrt{\overline{I}_4} + \frac{1}{2}c_6 \ln\left(\overline{I}_4\right) + c_7 & \overline{I}_4 > \overline{I}_{4_{ref}} \end{cases} \tag{5}$$

where c_1, c_3, c_4, c_5, c_6, c_7, $\overline{I}_{4_0}$ and $\overline{I}_{4_{ref}}$ are material parameters.

It is important to point that the active stress depends in the activation function f_2 and the force-length relation f_1. More details about the stress evaluation can be found in Hernández-Gascón et al. (2013)

MODEL PROPOSED BY LU The model proposed by (Lu et al., 2010) considers the muscle as fibers equally dispersed in a matrix, where the fiber mechanical behavior is defined by the Hill's model. Lu introduced an additional term on the Hill's model, that correspond to a damping element in parallel with the Hill's model. This model was developed to represent active and passive behavior under isometric or eccentric contractions, based in force-length and force-velocity relation. Then, the stress can be calculated as (Lu et al., 2010):

$$\sigma = \hat{\sigma} + 2J^{-1}\left[\alpha_4 \mathrm{dev}\left(\overline{\mathbf{B}}\overline{\mathbf{dB}}\right) + \alpha_5 \mathrm{dev}\left(\overline{\mathbf{F}}\mathbf{N}\overline{\mathbf{F}}^T\overline{\mathbf{dB}} + \overline{\mathbf{B}}\overline{\mathbf{dF}}\mathbf{N}\overline{\mathbf{F}}^T\right)\right] \tag{6}$$

where α_4 and α_5 are material parameters to account for viscous effects and $\hat{\sigma}$ correspond to:

$$\hat{\sigma} = \frac{1}{J}\left[\frac{b}{2}\left(2\overline{\mathbf{B}} - \frac{2}{3}\overline{I}_1\mathbf{I}\right) + (\sigma_p + \sigma_s)\left(\overline{\lambda}_f(\mathbf{N}) - \frac{1}{3}\overline{\lambda}_f\mathbf{I}\right)\right] + \frac{2}{D}(J-1)\mathbf{I} \tag{7}$$

where b and D are material parameters, $\overline{\mathbf{B}}$ is the isochoric left Cauchy-Green, σ_p is the passive stress in the muscle fiber, σ_s is the active stress in the muscle fiber, $\overline{\lambda}_f$ the stretch in the muscle direction, $\mathbf{N}$ is the result to $\mathbf{n}\otimes\mathbf{n}$, where $\mathbf{n}$ is the direction of the muscle fiber. The passive and active stress components can be, respectively, obtained by (Kojic, 1998):

$$^{t+\Delta t}\sigma_p = \sigma_0 f_{PE}\left(^{t+\Delta t}\overline{\lambda}_f\right) \tag{8}$$

$$^{t+\Delta t}\sigma_s = \sigma_0 f_t(t+\Delta t) f_\lambda\left(\overline{\lambda}_f\right) f_v\left(\dot{\lambda}_m\right) \tag{9}$$

where $\sigma_0 f_{PE}$ is the passive stress, σ_0 is the stress in the optimum strain, f_t is the activation function, f_λ is the force-length relation and f_v is the force-velocity relation.

Further details are found in (Lu et al., 2010).

MODEL PROPOSED BY VAN LOOCKE This model was developed to represent the passive behavior of muscle under concentric contractions, similar to Rehorn et al. (2014) who model passive eccentric contractions. Van Loocke et al. (2008) proposed a model using Prony series, which is capable to fit the viscoelastic mechanical behavior of skeletal muscle. Since this model aims to represent the passive behavior of muscle, there is no activation function, nor force-length or force-velocity relations. The expression for the Cauchy stress is:

$$\sigma_t(t) = p_{\infty j}\sigma_j^e(t) + \sum_{i=1}^{N}\int_0^t p_{ij} exp\left(\frac{-(t-\tau)}{\tau_{ij}}\right)\frac{\partial\sigma_j^e\left[\epsilon_j(\tau)\right]}{\partial\tau}d\tau \tag{10}$$

where ϵ_j is an appropriate strain measure, $p_{\infty j}$ and $\sigma_{\infty j}$ are material parameters.

3. PARAMETER IDENTIFICATION

This study is based on the experimental results presented by Rassier et al (2003). The authors present the stress response of a contraction sequence of a real skeletal muscle, providing also the muscle movement. This experiment recorded the stress produced by frog's muscles fibers for two isometric and one isometric-eccentric-isometric contractions. The isometric contractions provides the stress reference results as the muscle is maintained in the initial and final stretches through the whole active stress measure. The contraction combination begins in the initial stretch, where the muscle is activated. Then, the muscle is lengthened to the final stretch and held constant until the end of the experiment. The stretch history can be seen in Fig. (1). The muscle was activated at 0.5 seconds and deactivated at 6.5 seconds for all three contractions.

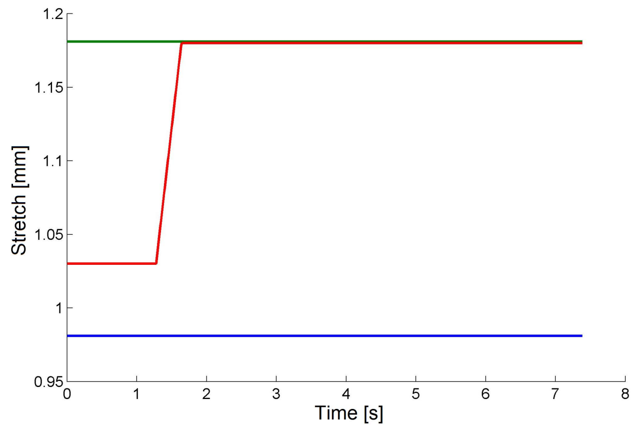

Figure 1. Stretch History for different contractions. Blue line: isometric contraction at smaller strain, green line: isometric contraction at greater strain, red line: contraction sequential combination. Adapted: Rassier *et al.*, 2003

None of the studied models presented before were proposed or tested to the contraction of interest of this study. Then, a parameter identification is necessary in order to allow the models comparison. The parameter identification used a built-in routine *lsqcurvefit* (MATLAB, 2011), that use a optimization procedure based on least-squares. Given the high number of material parameter to identify, the non-linearity of the equations, and the method dependence of initial parameter approximations, many different initial guesses were used for initial parameter approximations. To evaluate the adequacy of the models response, the error measure used was the square difference sum. The best response of the models can be seen in the Fig.(2) to Fig.(5). The material parameter found can be seen in Tab. (1) to Tab. (4).

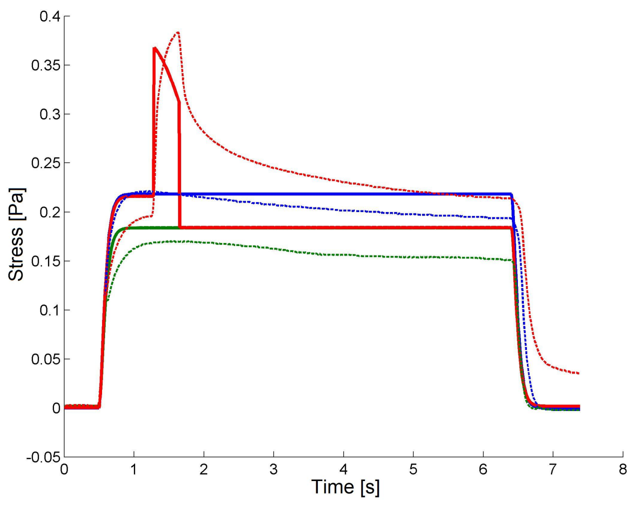

Figure 2. Best result for the mode model proposed by Böl. Dashed lines - experimental results, solid lines - model results. Blue line: isometric contraction at smaller strain, green line: isometric contraction at greater strain, red line: contraction sequential combination.

Regarding the isometric stress representation, no model could predict the force decay during contraction. The models proposed by Hernández, Lu and Böl presented only a constant isometric force along all the contraction time. As can be

Table 1. Parameter identification - Model proposed by Böl

λ_{min}	λ_{opt}	F_i	T_i	I_i	P_{opt}	μ
0.6159	0.9898	0.3427	0.051	0.0120	0.2380	0.03990
α	β	w_0	Na	$\dot{\lambda}_m^{min}$	k_e	k_c
7.3515	0.0031	0.8072	1.0615	-0.0026	1000	1

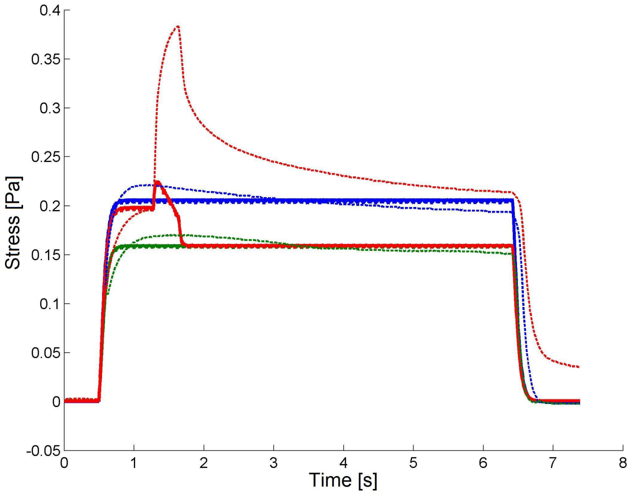

Figure 3. Best result for the mode model proposed by Hernández-Gascón. Dashed lines - experimental results, solid lines - model results. Blue line: isometric contraction at smaller strain, green line: isometric contraction at greater strain, red line: contraction sequential combination.

Table 2. Parameter identification - Model proposed by Hernández-Gascón

n_{imp}	r	$\tilde{T}$	c	$\tilde{P}$	ξ	λ_{opt}	P_0	v_0
50	0.1	0.04244	0.002	0.34136	0.32951	0.98317	0.3194	6.0028
ν	I_{40}	I_{4REF}	c_1	c_3	c_4	c_5	c_6	
0.34619	4.4424	2.4726	0.001	3.5544	4.4782	3.0637	-4.7596	

Table 3. Parameter identification - Model proposed by Lu

λ_{opt}	l_1	l_2	l_3	l_4	a_1	a_3	a_3	α	β	A
1.0399	0.3	0.8	1	2.2233	2	1.1280	0.9696	0.1005	0.0643	0.7950
σ_0	k_c	k_e	b	d	D	$\dot{\lambda}_m^{min}$	η_1	η_2	S	n_2
0.2216	1.8004	1.0309	0.0413	0.09736	0.01	-49.9965	0.0009	0.001	8.7222	0.93677

Table 4. Parameter identification - Model proposed by Van Loocke

k_1	k_2	k_3	p_1	τ_1	p_2	τ_2
0.0296	0.0086	0.0002	0.4951	0.9126	0.098043	0.8563
p_3	τ_3	p_4	τ_4	p_5	τ_5	
0.0992	0.5460	0.0992	0.6358	0.0941	1.0693	

seen especially in Fig. (3), this incapacity led to a isometric stress that is an average of the stress developed experimentally. The model proposed by Van Loocke was not able to represent activation, so the isometric contraction acted as a relaxation

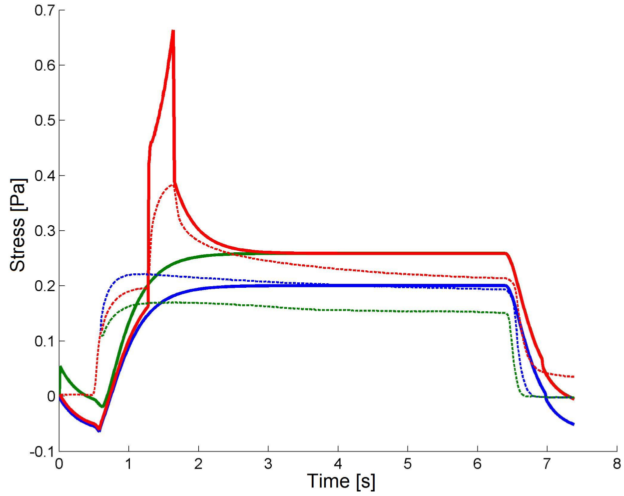

Figure 4. Best result for the mode model proposed by Lu. Dashed lines - experimental results, solid lines - model results. Blue line: isometric contraction at smaller strain, green line: isometric contraction at greater strain, red line: contraction sequential combination.

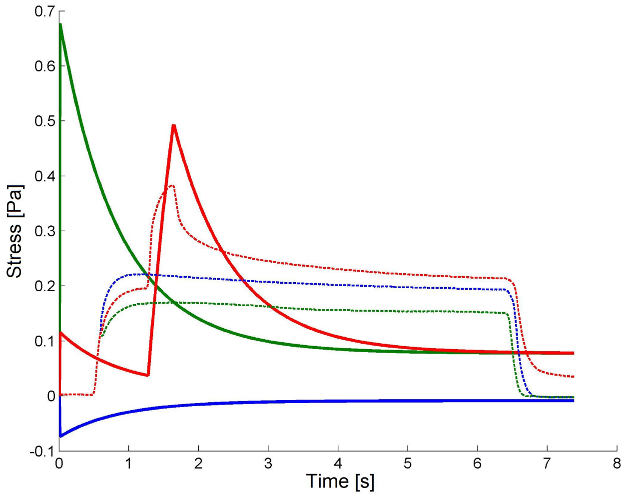

Figure 5. Best result for the mode model proposed by Van Loocke. Dashed lines - experimental results, solid lines - model results. Blue line: isometric contraction at smaller strain, green line: isometric contraction at greater strain, red line: contraction sequential combination.

test, an inadequate result.

Regarding the contraction combination representation, no model could predict the force gain. This is the main deficiency of all four models. The model proposed by Böl have a force-velocity dependence, as can be seen as soon as the movement start. Once the movement began as instant velocity is imposed, and based in the force-velocity relation the stress make a sudden rise. While the muscle lengthens the force steadily increases. However, once the movement stops the force leaps again to the isometric force at the final length, a result that is no adequate.

The model proposed by Hernández presented no dependence of the force-velocity relation, so once the movement began the stress basically scaled from one length to the other. In the beginning and ending of the moment little leaps were evaluated, but they are due the evolution of the internal variables.

The model proposed by Lu presented a inadequate result. Firstly, before activation the stress in the muscle decreases, because of the internal variable evolution of the Hill model. Once the muscle is activated it takes time to evolve, and once the movement begins and stops there is no evidence of force gain.

The model proposed by Van Loocke could no present any relation with the expected result. The main advantage of this model was that once the movement stops, the internal variable evolution could allow stresses difference from the isometric contraction. But as can be seen in Figure (5) this was not the case.

4. CONCLUSION

In this paper is presented a comparison of the mechanical response of different skeletal muscle models to a contraction sequence. Since the studied models were originally proposed to different contractions of those of interest, a parameter identification procedure was performed. It was verified that the models proposed by Hernández, Lu and Böl, could only represent a constant isometric force along time, no being adequate to represent the response observed numerically. In these cases it was due to the isometric stress recorded suffered from muscle fatigue, which was not taken into account in the muscle models. The main disadvantage of all models was the incapability of representing the force gain after a contraction combination such as the sequence isometric-eccentric-isometric studied. When the movement began all models had the stress increased, however once the movement stops the stresses decreased to the isometric stress of the final length. Since this is a basic and realistic muscle contraction, it is clear that the skeletal muscles models found in literature can not be applied with success in whatever contraction sequence. This paper shown that all studied models fail into representing the isometric-eccentric-isometric sequence. These observations demonstrate that skeletal muscle needs the development of constitutive models that can represent muscle fatigue, force gain and force loss, even in more complex contraction combinations.

5. ACKNOWLEDGEMENTS

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