23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

DYNAMIC MODEL OF A HYDRAULIC STEWART PLATFORM MANIPULATOR IN JOINT SPACE

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Abstract. *Some robotic control strategies as computed torque are designed to be applied in joint space. However, literature on parallel robots commonly presents only task space formulation due to its simplicity. In this sense, this paper presents a joint space representation for the simplified dynamic model of a hydraulic Stewart Platform manipulator considering the nonlinear dynamic properties of these kind of systems, which can be later used for joint state based control purpose. The rigid body equations of motion, neglecting the legs inertia, and hydraulics dynamics, including servo valve models, are used to calculate the state equations in the task space of the platform; and, aiming at control applications, the correspondent Inertia and Coriolis matrices properties are proven.*

Keywords: *Stewart platform, hydraulic dynamics, joint space*

1. INTRODUCTION

Parallel manipulators are closed-loop mechanisms where the assembly of the base with the end-effector has more than one link connected in parallel. When compared to open-loop mechanisms, this kind of robotic manipulator presents the advantage of high load capacity, good dynamic response and accurate positioning (Dasgupta and Mruthyunjaya, 2000). The Stewart platform is one example of parallel manipulators that achieved good acceptance in the industrial field over the last decades. This manipulator, proposed by Stewart, 1965, is described as two plates, a fixed one and a moving one, connected by six linear actuators which can change their lengths in order to position the end-effector (moving base) in different configurations.

Hydraulic Stewart Manipulators are commonly used as motion simulators, where precision motion control is needed. Its application is justified because hydraulically driven manipulators present relatively smaller size-to-power ratio and the ability to apply large force and torque when compared to compatible electrical actuators. Nevertheless, hydraulic dynamics are expressively non-linear; as a result, control schemes must usually consider the hydraulic dynamics to achieve good position tracking performance (Bu and Yao, 2000). Therefore, several researchers sustain that enhanced motion control of hydraulic parallel manipulator can be attained only by sophisticated control schemes, as for example, Sirouspour and Salcudean, 2001, and Kim, *et al.*, 2000, that developed robust adaptive controllers based on the backstepping approach taking into account both the parallel manipulators mechanical dynamics and hydraulic cylinder dynamics in the synthesis process. It is important to remark that, despite these control strategies have been designed to be applied in the joint space of the manipulator, the literature on parallel robots, due to its simplicity, usually presents only the formulation in the task space. Moreover, this two control strategies are based on the Lyapunov Stability Analysis, which generally requires that manipulators dynamic equation complies with a skew-symmetric property of their coefficients. In this context, besides a complete hydraulic Stewart Platform model, this paper presents a method to obtain the formulation of the manipulators dynamics in the joint space calculated through a transformation from the task space formulation, also demonstrating that the skew-symmetric property is maintained in the new joint space expression.

This paper is organized as follows. The geometrical aspects of a Stewart Platform mechanism are briefly described on Section 2. In Section 3, the forward kinematics and the Jacobian matrix are introduced. The dynamic formulation in the task space of the moving platform and the dynamic model of a hydraulic cylinder are presented in Section 4. In Section 5, the joint space formulation is transformed from its task space expression to the joint coordinate system and the skew-symmetric property of the new formulation is established. Finally, in Section 6, the conclusions are presented.

2. STEWART PLATFORM MANIPULATOR DESCRIPTION

The mechanism of a Stewart Platform Manipulator (SPM) is a six-degree-of-freedom (6-dof) parallel manipulator (Stewart, 1965). As already mentioned, this manipulator consists of two bodies, a moving platform and a lower fixed plate (base frame), connected by six prismatic joints (called *legs*). By changing the length of each leg, the moving platform can perform three rotations and three translations. The SPM structure considered in this paper is defined as being *rotationally symmetric* because the joint pairs are placed in intervals of 120-degrees in relation to the base and the platform (Advani, 1998). Figure 1 depicts the general configuration of the SPM:

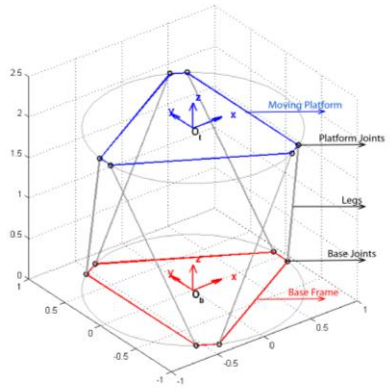


Figure 1. General configuration of SPM (adapted from Salzmann, 2004).

Figure 2a shows base frame parameters, including the base joint points (defined by $\boldsymbol{b}_i$) and the half separation distance between the joint pairs (s_b). Figure 2b shows the moving platform parameters, including the joint points ($\boldsymbol{t}_i$) and the separation distance between the joint pairs (s_p).

a) Base frame parameters and joint points b) Moving base parameters and joint points

Figure 2. Platform parameters -adapted from Salzmann, 2004

Defining radii R_b and R_p of the circumscribed circle at the base and moving platform, and, θ_b and θ_p as the half angles between the base and the top joint pairs, respectively, it is possible to determinate x and y coordinates of each joint point defined in the base frame coordinate system O_b and in the platform coordinate system O_t.

3. KINEMATICS

When all six actuators lengths are known, it is possible, thought forward kinematics procedures, to calculate the position and orientation of the moving platform with respect to the base frame. This procedure does not have a single solution and it was studied extensively on the literature (Gosselin, 1985; Advani, 1998; Merlet, 2004; Yang, *et al.*, 2010). Therefore, aiming to support control application, this work is focused in the inverse kinematics equations of the Stewart Platform.

3.1 Inverse Kinematics

The inverse kinematics of a SPM deals with the procedures for calculating the lengths of all six actuators when a certain geometric configuration, defined by the position and orientation of the movable frame in relation to the base, is given (Advani, 1998; Dasgupta and Mruthyunjaya, 2000; Salzmann, 2004; Merlet, 2006).

Taking into account the coordinate systems defined in Figure 2 and considering the coordinate system O_b attached to the center of the base frame, the position of the center of the moving platform with respect to coordinate system O_b can be defined by a vector $\boldsymbol{c}$ written as:

$$\mathbf{c}=[x_c \quad y_c \quad z_c]. \tag{1}$$

In addition to that, we can describe the three rotations of the platform with respect to base coordinate system as:

1. Rotation about *x-axis* by the angle *roll* α
2. Rotation about *y-axis* by the angle *pitch* β
3. Rotation about *z-axis* by the angle *yaw* γ

The generalized Cartesian position vector can, therefore, be defined as:

$$\boldsymbol{X}=[x_c \quad y_c \quad z_c \quad \alpha \quad \beta \quad \gamma]^T=[\boldsymbol{c} \quad \alpha \quad \beta \quad \gamma]^T. \tag{2}$$

Figure 3 shows the vector relations for each joint and leg of a SPM.

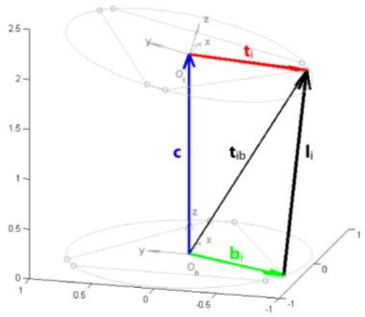


Figure 3. Vector relations for both top and base joints and legs of a SPM.

The vector $\boldsymbol{t}_i$ describes the position of each top joint with respect to the coordinate system O_t. This vector must take into account the rotation of the three angles of the coordinate system with respect to the coordinate O_b of the base frame, while vector $\boldsymbol{b}_i$ represents the position of each base joint connected to the actuator with respect to coordinate system O_b. The set of rotations with relation to the local coordinate axis describing a specific configuration of the mobile base in the three-dimensional space are commonly referred as Euler rotations. In this paper, the first rotation is made around the *yaw* axis, followed by a rotation around the *pitch* axis and, finally, around the *roll* axis. Using the transformation matrix $\boldsymbol{R}$ correspondent to these three rotations it is possible to obtain the coordinates of a vector that relates the system of reference of the platform with the system of reference of the base. Therefore, the vector $\boldsymbol{t}_{ib}$ describes the top joint position with respect O_b, as expressed in Eq. (3):

$$\boldsymbol{t}_{ib} = \boldsymbol{R}\cdot\boldsymbol{t}_i+\boldsymbol{c}. \tag{3}$$

Finally, it is possible to derive the length of each actuator denoted by $\boldsymbol{l}_i$ using both vectors $\boldsymbol{t}_{ib}$ and $\boldsymbol{b}_i$:

$$\boldsymbol{l}_i = \boldsymbol{t}_{ib}-\boldsymbol{b}_i. \tag{4}$$

The absolute value of this vector is:

$$l_i = \| \boldsymbol{l}_i \| \tag{5}$$

In addition, the generalized link displacement vector can now be defined as:

$$\boldsymbol{q}=[l_1 \quad l_2 \quad l_3 \quad l_4 \quad l_5 \quad l_6]^T. \tag{6}$$

3.2 Jacobian Matrix

The Jacobian Matrix $\boldsymbol{J}(X)\in R^{6x6}$ is the transformation matrix that turns the velocity vector expressed in Cartesian space $\dot{\boldsymbol{X}}$ into the Joint space $\dot{\boldsymbol{q}}$, and can be expressed by Eq. 7.

$$\dot{\boldsymbol{q}} = \boldsymbol{J}\dot{\boldsymbol{X}}. \tag{7}$$

Lebret et al., 1993, derived the Jacobian Matrix expression as a product of two matrixes, $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$, defined as

$$\mathbf{J}_1=\begin{bmatrix} \boldsymbol{u}_1^T & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{u}_2^T & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{u}_3^T & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{u}_4^T & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{u}_5^T & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{u}_6^T \end{bmatrix} \tag{8}$$

where $\boldsymbol{u}_i=\boldsymbol{l}_i/\|\boldsymbol{l}_i\|$ is the legs unit vector and 0_{3x1} is a 3x1 vector. A given line of matrix $\boldsymbol{J}_{2_i}$ is defined as:

$$\boldsymbol{J}_{2_i}=\left[\boldsymbol{I}_{3x3} \quad S(\hat{\imath})\boldsymbol{R}_x(\alpha)\boldsymbol{R}_y(\beta)\boldsymbol{R}_z(\gamma)\cdot\boldsymbol{t}_i \quad \boldsymbol{R}_x(\alpha)\boldsymbol{S}(\hat{\jmath})\boldsymbol{R}_y(\beta)\boldsymbol{R}_z(\gamma)\cdot\boldsymbol{t}_i \quad \boldsymbol{R}_x(\alpha)\boldsymbol{R}_y(\beta)\boldsymbol{S}(\hat{k})\boldsymbol{R}_z(\gamma)\cdot\boldsymbol{t}_i\right], \tag{9}$$

where I_{3x3} is a 3x3 identity matrix, $S(\cdot)$ is an skew-symmetric operand, expressed as

$$\mathrm{S}(\vec{v})=\begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}, \qquad \boldsymbol{v}=[v_1 \quad v_2 \quad v_3]^{\mathrm{T}}, \tag{10}$$

and $\boldsymbol{R}_x(\alpha)$, $\boldsymbol{R}_y(\beta)$ and $\boldsymbol{R}_z(\beta)$ are respectively, the rotation matrixes around the *x*, *y* and *z* axes:

$$\mathbf{R}_x(\alpha)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{11}$$

$$\mathbf{R}_y(\beta)=\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \tag{12}$$

$$\mathbf{R}_z(\gamma)=\begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

Finally, the Jacobian matrix results from the following product:

$$\boldsymbol{J}(\boldsymbol{X})=\boldsymbol{J}_1\boldsymbol{J}_2. \tag{14}$$

4. DYNAMICS

Precise position and trajectory tracking controllers of hydraulically driven robot manipulators are usually structured using model-based control strategies usually taking into account both the mechanical system and hydraulic dynamics. In these circumstances, to achieve a high performance control, it is important to model the dynamics of the entire system as closely to the physical model as possible. Accordingly, aiming at easing to accomplish this goal, this paper addresses the SPM dynamics as two subsystems, the rigid body model of the platform and the actuators hydraulic model.

4.1 Rigid body dynamics of the Platform on task space

Arai, *et al.*, 1990, describe a duality between the mechanism properties of parallel and serial manipulators. Fundamentally, this duality expresses that what might be a simple analysis for one type of manipulator can be a complex task for the other. For example, to derive the rigid body dynamics of a serial robot it is easier on the joint space, while for parallel mechanisms is a difficult endeavor. On the contrary, to deduce the rigid body dynamics on task space is a straightforward process for parallel manipulators, but not so simple for serial type. Therefore, due to the duality, the rigid body dynamics for SPM are stated on the task space, even though, most control strategies are designed for joint space actuation, causing, in this circumstances, the need of deducing the joint space formulation, as follows.

The dynamics of the SPM can be expressed by a second-order nonlinear differential equation (Sirouspour and Salcudean, 2001):

$$\boldsymbol{M(X)}\ddot{X}+\boldsymbol{V}(X,\dot{X})\dot{X}+\boldsymbol{G(X)}=\boldsymbol{J}^T(X)\boldsymbol{\tau}, \tag{15}$$

where, $\boldsymbol{X} \in R^6$ is a coordinates vector, $\boldsymbol{\tau} \in R^6$ is the generalized actuator force vector, $\boldsymbol{J(X)} \in R^{6x6}$ is the Jacobian matrix, $\boldsymbol{M(X)} \in R^{6x6}$ is the inertia matrix, $\boldsymbol{V}(X,\dot{X}) \in R^{6x6}$ is the Coriolis matrix and $\boldsymbol{G(X)} \in R^{6x6}$ represents gravitational effects.

Lebret, *et al.*, 1993, proposed a method to derive $\boldsymbol{M(X)}$, $\boldsymbol{V}(X,\dot{X})$ and $\boldsymbol{G(X)}$ using the Lagrange formulation. Using this methodology and neglecting the effects of the legs inertia, Kim, *et al.*, 2000, obtained expressions for all the matrixes coefficients. The resulting expression for $\boldsymbol{M(X)}$ is:

$$\mathbf{M}(X)=\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_xC_\beta^2C_\gamma^2+I_yC_\beta^2S_\gamma^2+I_zS_\beta^2 & (I_x-I_y)C_\beta C_\gamma S_\gamma & I_zS_\beta \\ 0 & 0 & 0 & (I_x-I_y)C_\beta C_\gamma S_\gamma & I_xS_\gamma^2+I_yC_\gamma^2 & 0 \\ 0 & 0 & 0 & I_zS_\beta & 0 & I_z \end{bmatrix} \tag{16}$$

where, I_x, I_y and I_z are the inertia moments with respect of the x, y and z axis, m is the mass of the platform, and the operators C and S represent, respectively, cosine and sine functions, and their subscripts indicates the angle. The Coriolis matrix expression is:

$$\mathbf{V}(X,\dot{X})=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_1\dot{\beta}+K_2\dot{\gamma} & K_1\dot{\alpha}+K_5\dot{\beta}+K_3\dot{\gamma} & K_2\dot{\alpha}+K_3\dot{\beta} \\ 0 & 0 & 0 & -K_1\dot{\alpha}+K_3\dot{\gamma} & K_4\dot{\gamma} & K_3\dot{\alpha}+K_4\dot{\beta} \\ 0 & 0 & 0 & -K_2\dot{\alpha}-K_3\dot{\beta} & -K_3\dot{\alpha}-K_4\dot{\beta} & 0 \end{bmatrix}, \tag{17}$$

where the terms K_1, K_2, K_3, K_4 e K_5 are defined as:

$$K_1=-C_\beta S_\beta\left(C_\gamma^2I_x+S_\gamma^2I_y-I_z\right), \tag{18}$$

$$K_2=-C_\beta^2C_\gamma S_\gamma\left(I_x-I_y\right), \tag{19}$$

$$K_3=\frac{1}{2}C_\beta\left(C_\gamma-S_\gamma\right)\left(C_\gamma+S_\gamma\right)\left(I_x-I_y\right), \tag{20}$$

$$K_4=C_\gamma S_\gamma\left(I_x-I_y\right), \tag{21}$$

$$K_5=-C_\gamma S_\gamma S_\beta\left(I_x-I_y\right), \tag{22}$$

and the gravitational term is written as:

$$\mathbf{G}(\boldsymbol{X})=[0 \quad 0 \quad mg \quad 0 \quad 0 \quad 0]^T. \tag{23}$$

Lewis, *et al.*, 2006 give special attention to the coefficients of Eq. 15, remarking the following properties:

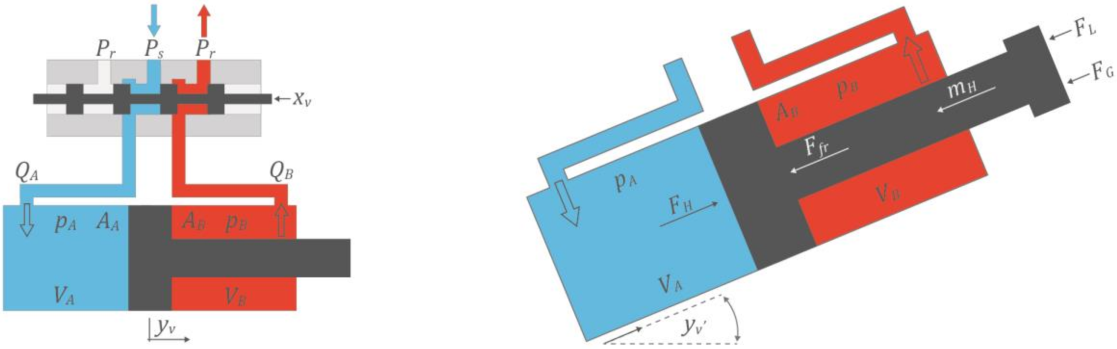
(1) Inertia matrix $\boldsymbol{M(X)}$ is symmetric and positive definite. Also, it is bounded above and below.

(2) The derivative of $\boldsymbol{M(X)}$ and the Coriolis vector are related in a skew-symmetric form as $\dot{\boldsymbol{M}}\text{-}2\boldsymbol{V}$.

It is important to observe that, to obtain the above skew-symmetric form, the Coriolis matrix must be obtained using a particular factorization (Lebret, *et al.*, 1993).

4.2 Hydraulic Actuator Model

A five-way asymmetric servovalve connected to a hydraulic cylinder configuration, as shown in Figure 4a, is used to command each DOF. The neutral position of the piston, which is chosen to be the symmetrical position, is considered as $y_v(0)$ (Merritt, 1967).



a) Five ways servovalve with cylinder.

b) Force balance diagram.

Figure 4. Hydraulic system diagram.

Assuming that there is no leakages, the chambers pressure dynamics can be written as (Merritt, 1967):

$$\dot{p}_A=\frac{\beta_c}{V_A}(Q_A\text{-}A_A\dot{y}_v);\ \dot{p}_B=\text{-}\frac{\beta_c}{V_B}(Q_B\text{-}A_B\dot{y}_v) \tag{24}$$

where the sub-indexes A and B refer, respectively, to the chambers A and B of the piston, $V_A=V_{A0}+A_Ay_v$ and $V_B=V_{B0}\text{-}A_By_v$ are the forward and return chambers volume, V_{A0} and V_{B0} are the chambers initial volumes, p_A and p_B are the chambers pressure, β_c is the bulk modulus, Q_A and Q_B are the flow rates at each port of the two chambers, A_A and A_B are the ram areas of the two chambers, and, y_v is the displacement of the piston. The spool displacement is named x_v while the supply and tank reference pressures are, respectively, p_s and p_r. So, the hydraulic oil flow rate at the forward and return chambers can be expressed as:

$$Q_A=\begin{cases} K_{vA}\frac{u_v}{U_{cn}}\sqrt{p_S\text{-}p_A}, & x_v\geq 0 \\ K_{vA}\frac{u_v}{U_{cn}}\sqrt{p_A\text{-}p_R}, & x_v<0 \end{cases};\ Q_B=\begin{cases} K_{vB}\frac{u_v}{U_{cn}}\sqrt{p_B\text{-}p_R}, & x_v\geq 0 \\ K_{vB}\frac{u_v}{U_{cn}}\sqrt{p_S\text{-}p_B}, & x_v<0 \end{cases} \tag{25}$$

where K_{v_A} and K_{v_B} are called flow gains of the forward and return loops, respectively; u_v voltage control input and U_{cn} is the nominal voltage control input. The piston force balance can be derived applying Newton's second law, and results in:

$$F_H-F_{fr}-F_L-F_G=m_H\ddot{y}_v, \tag{26}$$

where, $F_H=p_AA_A-p_BA_B$ is the hydraulic force originated by the difference of pressures and ram areas between both chambers, F_{fr} is the friction force acting on the cylinder rod, F_L is the load force related to the platform-rod interaction, F_G is the weight component on the actuator axis, and m_H is the mass of the rod, while the vector τ is the one formed by each of the six actuators load force.

5. JOINT SPACE MODEL

As previously stated, several control strategies are designed in the joint space, while, despite of that, dynamic equations of parallel manipulators are commonly expressed in Cartesian space, as in Eq. 9.

The procedure to rewrite Eq. 9 in the joint space is straightforward. Considering the definition of the Jacobian matrix, expressed in the Section 3.2, it is possible to write this relation in a reverse order:

$$\dot{\boldsymbol{X}}=\boldsymbol{J}^{1}(X)\dot{\boldsymbol{q}} \tag{27}$$

where $\boldsymbol{J}=\boldsymbol{J}(\boldsymbol{X})$. Now, the time derivative of Eq. 18 is taken in order to obtain the relation between accelerations in Cartesian and in Joint spaces, it is possible to describe the acceleration in Cartesian space as:

$$\ddot{\boldsymbol{X}}=\boldsymbol{J}^{1}\ddot{\boldsymbol{q}}\text{-}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\boldsymbol{J}^{1}\dot{\boldsymbol{q}}, \tag{28}$$

With both, acceleration and velocities relations in Cartesian and Joint space, one can substitute these equations into the dynamic equation of the parallel manipulator (Eq. 9). After algebraic manipulations, the equation becomes:

$$\boldsymbol{J}^T\boldsymbol{M}\boldsymbol{J}^{1}\ddot{\boldsymbol{q}}+\boldsymbol{J}^T\left(\boldsymbol{V}\text{-}\boldsymbol{M}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\right)\boldsymbol{J}^{1}\dot{\boldsymbol{q}}+\boldsymbol{J}^T\boldsymbol{G}=\boldsymbol{\tau}, \tag{29}$$

where, $\boldsymbol{M}=\boldsymbol{M}(\boldsymbol{X})$, $\boldsymbol{V}=\boldsymbol{V}(\boldsymbol{X},\dot{\boldsymbol{X}})$ and $\boldsymbol{G}=\boldsymbol{G}(\boldsymbol{X})$. With the dynamic of each actuator considered, the Newton's second law applied to the leg results (Valdiero, 2004):

$$\boldsymbol{F}_H - \boldsymbol{F}_{at} - \boldsymbol{\tau} - \boldsymbol{F}_G = \boldsymbol{M}_H\ddot{\boldsymbol{q}} \tag{30}$$

where, $\boldsymbol{F}_H$ is the hydraulic force, $\boldsymbol{F}_{at}$ is the friction force, $\boldsymbol{\tau}$ is the torque, $\boldsymbol{F}_G$ is a gravitational force and $\boldsymbol{M}_H$ is a diagonal $6x6$ matrix that describes the masses m_{H_i} of each leg in the form:

$$\mathbf{M}_{\mathrm{H}} = diag(m_{H_1}, m_{H_2}, m_{H_3}, m_{H_4}, m_{H_5}, m_{H_6}) \tag{31}$$

Substituting $\boldsymbol{\tau}$ from Eq. 29 into Eq. 30, the dynamic equation of a parallel manipulator can be written in the Joint space as:

$$\overline{\boldsymbol{M}}\ddot{\boldsymbol{q}}+\overline{\boldsymbol{V}}\dot{\boldsymbol{q}}+\overline{\boldsymbol{G}}+\boldsymbol{F}_{at}=\boldsymbol{F}_H, \tag{32}$$

where, $\overline{\boldsymbol{M}}=\overline{\boldsymbol{M}}(\boldsymbol{X})=\boldsymbol{J}^T\boldsymbol{M}\boldsymbol{J}^{1}+\boldsymbol{M}_H$, $\overline{\boldsymbol{V}}=\overline{\boldsymbol{V}}(\boldsymbol{X},\dot{\boldsymbol{X}})=\boldsymbol{J}^T\left(\boldsymbol{V}\text{-}\boldsymbol{M}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\right)\boldsymbol{J}^{1}$ and $\overline{\boldsymbol{G}}=\overline{\boldsymbol{G}}(\boldsymbol{X})=\boldsymbol{J}^T\boldsymbol{G}+\boldsymbol{F}_G$.

Equation 32 describes the SPM dynamic equations in the Joint space considering the mass of each hydraulic piston. It is important to remark that this equation still depends on the Cartesian variables $\boldsymbol{X}$ and $\dot{\boldsymbol{X}}$.

5.1 Skew-symmetric property

To verify the closed loop control stability through the Lyapunov analysis, the skew-symmetric property of the system dynamics usually must be complied. In this sense, it is important to demonstrate that the skew-symmetric property (Section 4.1) is also present in the new joint space formulation as long as J remains nonsingular (Slotine and Li, 1987). This poof procedure is based on a method presented in Lewis, *et al.*, 2006, which uses the following identity regarding the Jacobian matrix:

$$\frac{d}{dt}\left(\boldsymbol{J}^{1}\boldsymbol{J}\right)=0, \tag{33}$$

$$\frac{d}{dt}\left(\boldsymbol{J}^{1}\right)=\text{-}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\boldsymbol{J}^{1} \tag{34}$$

Using Eq. 23 and the definitions of matrixes $\overline{\boldsymbol{M}}$ and $\overline{\boldsymbol{V}}$ from Eq. 24 and Eq. 25, and expanding Eq. (36), results:

$$\boldsymbol{J}^T\left(\dot{\boldsymbol{M}}\text{-}2\boldsymbol{V}+\left[\boldsymbol{M}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\text{-}\left(\boldsymbol{M}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\right)^T\right]\right)\boldsymbol{J}^{1}, \tag{35}$$

which can be used to verify the skew-symmetric property with an arbitrary vector w as follows:

$$\boldsymbol{w}^T\boldsymbol{J}^T\left(\dot{\boldsymbol{M}}\text{-}2\boldsymbol{V}+\left[\boldsymbol{M}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\text{-}\left(\boldsymbol{M}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\right)^T\right]\right)\boldsymbol{J}^{1}\boldsymbol{w} \tag{36}$$

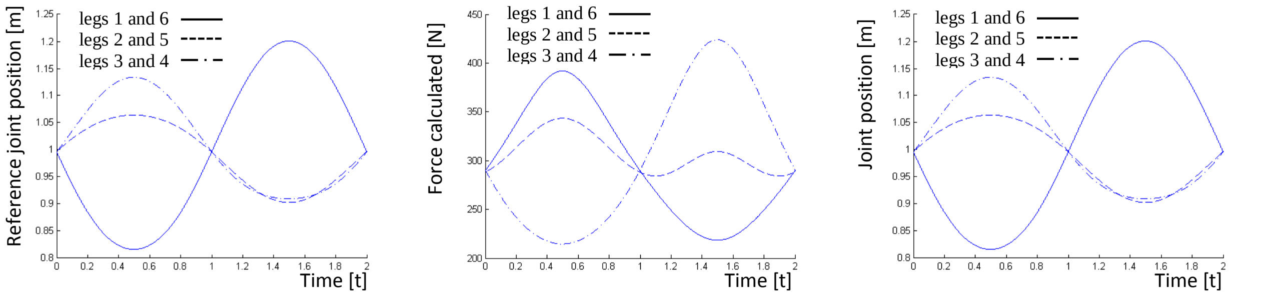
$$\left(\boldsymbol{J}^{1}\boldsymbol{w}\right)^T\left(\dot{\boldsymbol{M}}\text{-}2\boldsymbol{V}\right)\left(\boldsymbol{J}^{1}\boldsymbol{w}\right)+\left(\boldsymbol{J}^{1}\boldsymbol{w}\right)^T\left[\boldsymbol{M}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\text{-}\left(\boldsymbol{M}\boldsymbol{J}^{1}\dot{\boldsymbol{J}}\right)^T\right]\left(\boldsymbol{J}^{1}\boldsymbol{w}\right)=0 \tag{37}$$

The first term of the Eq. 39 is skew-symmetric, being, consequently, equal to zero. By processing the multiplications and applying the commutative property of scalar products, of the second term, it can be proven that this term also results null. By this analysis, the skew-symmetric property of the newly derived $\dot{\overline{\boldsymbol{M}}}\text{-}2\overline{\boldsymbol{V}}$ expression was proven.

6. NUMERICAL APPLICATION

A numerical simulation is presented aiming to demonstrate the applicability of the model scheme proposed in this work in a robot control problem. The test consists of applying the dynamic expression of the SPM mechanical system in joint space (Eq. (32)) to obtain the hydraulic force from each actuator ($\boldsymbol{F}_H$ vector) necessary to reproduce a single pitch trajectory defined as $\beta(t) = 20\sin(\pi t)$ [m], where t represents the time. Furthermore, a new computation that reconstructs the input trajectory from the calculated $\boldsymbol{F}_H$ vector applied in a direct dynamic model. The configuration parameters of the simulated Stewart Platform are $R_b = R_p = 0.95$ [m], $\theta_b = \theta_p = \pi/20$ [rad], initial height of 0.6 [m],

$m = 100$ [kg], $m_p = 3$ [kg], $I_x = I_y = 16.88$ [Kg.m^2], and $I_z = 32.26$ [kg.m^2]. The friction effect is neglected and the gravity is considered $g = 9.8$ [m/s^2]. The test results are presented on Figure 5.



a) Reference joint trajectory b) Joint hydraulic force c) Response joint trajectory [m]

Figure 5. Simulation results**.**

7. CONCLUSION

This work deals with the Stewart Platform hydraulic actuated control problem. Bibliography research showed that, despite joint space formulation be necessary for applying important robot control strategies, the specialized literature on parallel robots commonly presents only task space formulation. Therefore, in this paper it was proposed a method based on the factorization method presented in Lebret, *et al.*, 1993, to process the rigid body dynamics expression on the task space. Furthermore, it was shown that proposed formulation furnishes Inertia and Coriolis matrixes compliant with the skew symmetric property, appropriate to be applied in robotic control strategies designed to be applied in joint space.

A simple numerical example is presented, showing that the inverse dynamics equations obtained by means of the proposed modelling strategy allow to obtain the necessary force vector in joint coordinates (which must be supplied by the hydraulic actuators) that, when applied in open loop in the correspondent direct dynamic model, leads to the desired trajectory.

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