23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

MOTION CONTROL STRATEGY OF A ROBOTIC MANIPULATOR WITH FLEXIBLE JOINTS BASED ON FUZZY CONTROL

Karina Assolari Takano
Fabian Andres Lara-Molina
Edson Hideki Koroishi
Federal University of Technology - Paraná, Mechanical Engineering Department, Cornélio Procópio-PR.
ka.assolari@gmail.com, fabianmolina@utfpr.edu.br,edsonh@utfpr.edu.br

Abstract. *This paper aims at designing a motion control of a two-link planar manipulator with flexible joints to control the joint variables of the motors. This strategy is composed of two complementary control loops: a regulation and tracking control loops. The regulation control is applied to attenuate the vibration on the joints for a desired closed-loop balance at a specific configuration. The tracking control loop regulate the overall motion. The dynamic model of the flexible manipulator with rigid links and flexible joints is derived using the Lagrange-Euler principle. The nonlinear dynamics of the robotic flexible manipulator is approximated with a Takagi-Sugeno fuzzy model. The control strategy is based on the Fuzzy Takagi-Sugeno model as well as the Parallel Distributed Compensation. Simulation results show that the proposed control strategy regulates the position of the robot over a desired position within the workspace.*

Keywords: *Flexible Manipulator, Takagi-Sugeno Fuzzy Model, Robot Control, Parallel Distributed Compensation and Set-point Control.*

1. INTRODUCTION

There are several applications in which the dynamics of flexible joints of the robots can not be neglected, e.g. in industrial robots that use reductions or transmissions with flexible elements such as belts, cables, long shafts, or harmonic drivers. The flexible joints provide compliance between the robotic manipulator and objects of the environment, thus enhancing the performance in eventual collisions (Ramirez *et al.*, 2009). The main sources of flexible joints are compliant transmission elements (harmonic drives, shaft windup, bearing deformation, and compressibility of the hydraulic fluid in hydraulic robots and so on) (Chang and Yen, 2011).

The dynamics of the flexible joints of robotic manipulators yield low frequency oscillations when increasing the speed of operation. Some instabilities can produce undesirable oscillations during the robot motion if the flexibility is not considered to compute the control action (Readman, 1994). This condition can also harm the dynamic response and accuracy. Therefore, the flexibility of the joints should be taken into account in the control law to attenuate the effect of the unwanted oscillations(Karkoub and Tamma, 2011).

The control algorithms for flexible robotic manipulator have typically been designed as if they were rigid (Lara-Molina *et al.*, 2014b). This assumption is valid only for slow motion and small interacting forces. Kinematics, dynamics, and control design of robotic manipulator with rigid elements are well defined. Therefore, it is necessary to develop new control algorithms in order to improve the dynamic performance of flexible robots.

According to Benosman and Vey (2004), the four main objectives related with the control of a flexible robotic manipulator are: i) end-effector position regulation, ii) rest to rest end-effector motion in fixed time, iii) trajectory tracking in the joint space (tracking of a desired angular trajectory) and iv) trajectory tracking in the operational space (tracking of a desired end-effector trajectory). A number of control techniques have been used in robotic manipulators with flexible joints in order to achieve these objectives. Nevertheless input/output linearization approach, based on computed torques methods, has been widely used in the literature to control robotic manipulators with flexible joints (Benosman and Vey, 2004). However, alternative control schemes based on fuzzy controllers have been employed to control several mechanical systems. These alternative control is tested in this paper to cope with the nonlinear dynamics of a flexible robotic manipulator as an alternative to the computed torque methods (Lara-Molina *et al.*, 2014a). This control strategy allows working properly in a system with reduced set of state feedback, which involve the most of the cases with flexible manipulators. It occurs due to the physical difficulty of installing the necessary amount of sensors (Chen, 2006).

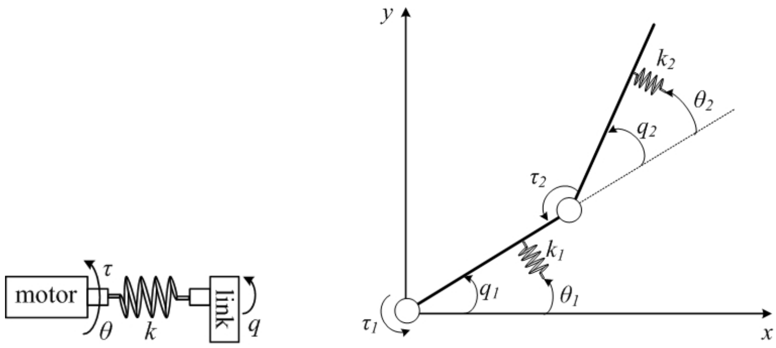
According to Melhem and Wang (2009), the trajectory tracking control of flexible joint robots has been greatly studied by using several control techniques as singular perturbation, sliding mode control, and feedback linearization. The ideal trajectory tracking control should use a full state feedback and know all the terms in the real dynamical system. In the cases of robotic manipulators with flexible joints, the number of sensors would be twice in comparison to the rigid case to measure the full state of the system. In most instances this can not be set because of the physical difficulty to implement this measurement . Therefore, it is highlighted the importance of design controllers working properly with a reduced set of measurements.

This papers aims at designing a set-point regulation control of a two-link planar manipulator with flexible joints considering the joint variables control of the motors based on. The nonlinear dynamics of the robotic flexible manipulator is approximated with a Takagi-Sugeno fuzzy model. The control strategy is based on the Fuzzy Takagi-Sugeno model as well as the Parallel Distributed Compensation. Section 2 presents the complete and simplified dynamic model of the two-link planar manipulator with flexible joints. Section 3 introduces the control strategy which is based on parallel distribution compensation and Takagi-Sugeno fuzzy model. Section 4 suggests a tracking control strategy to future work. In section 5 are presented the results of numerical simulations. Finally, some conclusions are remarked in section 6.

2. MODELING OF THE ROBOT MANIPULATOR WITH FLEXIBLE JOINTS

This section describes the dynamic model of two-link robotic manipulator with flexible joints.

Figure 1(a) shows the model of a flexible joint. With $\boldsymbol{\tau} = \left\{\tau_1 \quad \ldots \quad \tau_n\right\}^T \in \mathbb{R}^{n\times 1}$ the torque applied by the motor after the reduction, $\boldsymbol{\theta} = \left\{\theta_1 \quad \ldots \quad \theta_n\right\}^T \in \mathbb{R}^{n\times 1}$ the angular position of the motor after the reduction and $\mathbf{q} = \left\{q_1 \quad \ldots \quad q_n\right\}^T \in \mathbb{R}^{n\times 1}$ the angular position of the link. The flexible transmission of the motor to the joint is modeled by a torsional spring stiffness k_i.



(a) (b)

Figure 1. Two-link robot with flexible joints: (a) Flexible joint, (b) Planar robot with flexible joints.

Considering that the angular velocity of the motor's rotors is produced only by their own spinning (De Luca and Book, 2008), the reduced dynamic model of a n-link robot manipulator with flexible joint is written as follows:

$$\mathbf{M}_L(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}_L(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{K}(\mathbf{q} - \boldsymbol{\theta}) + \boldsymbol{\tau}_{f\mathbf{q}} = 0 \tag{1}$$

$$\mathbf{B}\ddot{\boldsymbol{\theta}} + \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) + \boldsymbol{\tau}_{\boldsymbol{\theta}} = \boldsymbol{\tau} \tag{2}$$

where the vectors $\dot{\mathbf{q}} = \left\{\dot{q}_1 \quad \ldots \quad \dot{q}_n\right\}^T \in \mathbb{R}^{n\times 1}$ and $\ddot{\mathbf{q}} = \left\{\ddot{q}_1 \quad \ldots \quad \ddot{q}_n\right\}^T \in \mathbb{R}^{n\times 1}$ are the angular velocity and acceleration of the links; $\mathbf{M}_L(\mathbf{q}) \in \mathbb{R}^{n\times n}$ is the inertia matrix of the rigid links; $\mathbf{V}_L(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n\times 1}$ is the vector that contains the Coriolis and centrifugal torques; $\mathbf{B} \in \mathbb{R}^{n\times n}$ is the matrix of inertia of the motor's rotors; $\ddot{\boldsymbol{\theta}} = \left\{\ddot{\theta}_1 \quad \ldots \quad \ddot{\theta}_n\right\}^T \in \mathbb{R}^{n\times 1}$ is the angular acceleration of the motor after the reduction; $\mathbf{K} = diag(k_1, \ldots, k_n)$ is the diagonal matrix of the joint stiffness. The link and motor equations (eqs. (1) and (2)) are dynamically coupled by the elastic torque $\mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$. The direction of the acceleration of gravity is assumed to be oriented along the $-z$ axis, therefore its effects are ignored in the dynamic model of eq (3). Fig. 1(b) shows specifically a two-link robot with flexible joints where n=2.

The rigid model of the manipulator considers an infinite stiffness ($\mathbf{K} \to \infty$), thus $\boldsymbol{\theta} \to \mathbf{q}$, for this case the model can be stated as follows:

$$\mathbf{M}_L(\mathbf{q}) + \mathbf{V}_L(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}_L(\mathbf{q}) + \boldsymbol{\tau}_{f\mathbf{q}} = \boldsymbol{\tau} \tag{3}$$

2.1 Simplified model

The dynamic model of eqs. (1) and (2) is simplified in order to obtain a model to design the controller. Two assumption are considered: the manipulator is considered at a constant configuration, therefore $\mathbf{V}_L(\mathbf{q}, \dot{\mathbf{q}}) = 0$; and additionally, the worst case in which the manipulator is not damped is considered, thus $\boldsymbol{\tau}_\theta = \boldsymbol{\tau}_{f_\mathbf{q}} = 0$. The total dynamic equation can be express by using the matricial notation:

$$\mathbf{M}_T(\mathbf{q})\ddot{\mathbf{z}} + \mathbf{K}_T\mathbf{z} = \boldsymbol{\tau} \tag{4}$$

where, $\mathbf{z} = \begin{Bmatrix} \mathbf{q} & \boldsymbol{\theta} \end{Bmatrix}^T$, $\mathbf{M}_T(\mathbf{q}) = \begin{bmatrix} \mathbf{M}_L(\mathbf{q}) & 0_{2,2} \\ 0_{2,2} & \mathbf{B} \end{bmatrix}$ and $\mathbf{K}_T = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & 0 & -k_2 \\ -k_1 & 0 & k_1 & 0 \\ 0 & -k_2 & 0 & k_2 \end{bmatrix}$. The state-space representation of the system considering the states $\mathbf{x}_1 = \mathbf{z}$ and $\mathbf{x}_2 = \dot{\mathbf{x}}_1$ is state as follows:

$$\begin{Bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{Bmatrix} = \begin{bmatrix} 0_{4,4} & \mathbb{I}_{4,4} \\ -\mathbf{M}_T(\mathbf{q})^{-1}\mathbf{K} & 0_{4,4} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{Bmatrix} + \begin{bmatrix} 0_{4,4} \\ \mathbf{M}_T(\mathbf{q})^{-1} \end{bmatrix} \begin{Bmatrix} 0_{1,2} \\ \boldsymbol{\tau} \end{Bmatrix} \qquad \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{Bmatrix} \tag{5}$$

It is wort to mention that the total inertia matrix of $\mathbf{M}_T(\mathbf{q})$ depends on the position of the rigid link q_2, thus the expression of eq. (5) is equivalent to the state-space-representation:

$$\dot{\mathbf{x}} = \mathbf{A}(q_2)\mathbf{x} + \mathbf{B}(q_2)\mathbf{u} \qquad \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{6}$$

where $\mathbf{u} = \begin{Bmatrix} 0_{1,2} & \boldsymbol{\tau} \end{Bmatrix}^T$. The transfer matrix $\mathbf{G}$ can be obtained for specific values of q_2. Considering the torque of the actuators $\boldsymbol{\tau}$ as the input and the position of the motor $\boldsymbol{\theta}$ as the output, the matrix function is obtained based on the state-space representation of eq. (5), thus:

$$\begin{Bmatrix} \theta_1(s) \\ \theta_2(s) \end{Bmatrix} = [\mathbf{G}(\mathbf{q})] \begin{Bmatrix} \tau_1(s) \\ \tau_2(s) \end{Bmatrix} \tag{7}$$

Where, $[\mathbf{G}(\mathbf{q})] = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$ and s is the variable of Laplace. The first pair of imaginary zeros of $G_{11}(s)$ and $G_{11}(s)$ transfer functions occurs at the so-called locked frequency defined as ω_1 and ω_2 for each joint respectively. The locked frequency is used to tune the controller of motion. Additionally, the controllability of the model in the eq. (6) is verified the output controllability by means of the output controllability matrix $\begin{bmatrix} \mathbf{CB} & \mathbf{CAB} & \mathbf{CA}^2\mathbf{B} & \dots & \mathbf{CA}^{n-1}\mathbf{B} & \mathbf{D} \end{bmatrix}$.

3. CONTROL STRATEGY

This section presents the strategy to regulate the position of robot in a constant configuration based on on the Takagi-Sugeno fuzzy model together with Parallel Distributed Compensation.

3.1 Regulation Control

This problem aims at controlling the motion of the manipulator (with joint elasticity) in a constant configuration (De Luca and Book, 2008). It is worth to mention that this control law should provide asymptotic stabilization of a desired closed-loop steady state. The control law is based on the motor variables position and velocity feedback, thus a constant reference $\boldsymbol{\theta}_d$ (with $\dot{\boldsymbol{\theta}}_d \equiv 0$) should be defined.

$$\boldsymbol{\tau} = \mathbf{K}_P(\boldsymbol{\theta}_d - \boldsymbol{\theta}) - \mathbf{K}_D\dot{\boldsymbol{\theta}}_d \tag{8}$$

Where $\mathbf{K}_P = diag(k_{p_1}, \dots, k_{p_n})$ and $\mathbf{K}_D = diag(k_{d_1}, \dots, k_{d_n})$ are positive-definite and diagonal proportional and derivative n-by-n gain matrices, respectively. In order to obtain sufficient damping in the closed-loop system, the PD gains are select to limit the bandwidth to one third of the locked frequency of each joint.

Considering the simplified dynamic model of the robot (eq. (5)) in order to access the gains of the PD controller of eq. (8), the state-space-representation of control system is presented in eq. (9).

$$\dot{\mathbf{x}} = [\mathbf{A}(\mathbf{q}) - \mathbf{B}\mathbf{F}]\mathbf{x} + \mathbf{B}\overline{\mathbf{F}}\mathbf{u} \qquad \mathbf{y} = \mathbf{C}\mathbf{x} \tag{9}$$

where $\mathbf{F}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{p1} & 0 & 0 & 0 & k_{d1} & 0 \\ 0 & 0 & 0 & k_{p2} & 0 & 0 & 0 & k_{d2} \end{bmatrix}$ is the feedback gain matrix and $\overline{\mathbf{F}} = \begin{bmatrix} k_{p1} & 0 & 0 & 0 \\ 0 & k_{p2} & 0 & 0 \\ 0 & 0 & k_{p1} & 0 \\ 0 & 0 & 0 & k_{p2} \end{bmatrix}$.

Nevertheless, it is observed that the dynamic matrix $\mathbf{A}(\mathbf{q})$ depends on the configuration of the manipulator defined by joint position of the links $\mathbf{q}$, specifically q_2, therefore the locked frequencies of the joints are dependent of the configuration of the robot. This nonlinearity is considered by the control system by using Takagi-Sugeno fuzzy model together with Parallel Distributed Compensation.

3.2 Parallel Distributed Compensation

In this paper, the so-colled Takagi-Sugeno fuzzy model is used to approximate a nonlinear system model (Takagi and Sugeno, 1985). This fuzzy model is described by fuzzy IF-THEN rules which represent local models of the system, thus the fuzzy model is represented as follows:

$$\begin{aligned}&\text{Rule } i: \text{IF } x_1(t) \text{ is } M_{1i} \dots \text{ and } x_n \text{ is } M_{ni}\\ &\text{THEN } \dot{\mathbf{x}} = \mathbf{A}_i\mathbf{x} + \mathbf{B}_i\mathbf{u}\end{aligned} \tag{10}$$

where $i = 1, 2, \dots, r$; r is the number of IF-THEN rules; $\mathbf{x}$ is the state vector; $\dot{\mathbf{x}}$ is the output from the *i-th* IF-THEN rule; $\mathbf{u}$ is the input control vector; For a given pair $[\mathbf{x}, \mathbf{u}]$ the resulting output fuzzy system is inferred as follows:

$$\dot{\mathbf{x}} = \frac{\sum_{\mathbf{i=1}}^{\mathbf{r}} w_i(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i\mathbf{u})}{\sum_{\mathbf{i=1}}^{\mathbf{r}} w_i} \tag{11}$$

Each $\mathbf{A}_i\mathbf{x}$ is denominated a local subsystem. Additionally, w_i is the activation level from application i and is given by:

$$w_i = \prod_{\mathbf{j=1}}^{\mathbf{n}} M_{ij}(x_j) \tag{12}$$

with $M_{ij}(x_j)$ is the grade of membership of the state x_j in M_{ij}; and $M_{ij}$the *j-th* fuzzy set of the *i-th* rule. As an example, the membership functions M_1 and M_2 for the them correspondent rules are shown in Fig. 2.

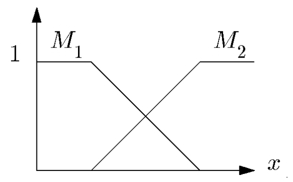


Figure 2. Membership functions of the fuzzy system M_{ij}

The parallel distributed compensation (CPD) is used to design fuzzy controllers to control fuzzy systems. A controller for each fuzzy system is designed, hence the fuzzy controller shares the same fuzzy set with the fuzzy system. For each rule, a linear control design technique is used, thus:

$$\begin{aligned}&\text{Rule } i: \text{IF } x_1 \text{ is } M_{1i} \dots \text{ and } x_n \text{ is } M_{ni}\\ &\text{THEN } \mathbf{u} = -\mathbf{F}_i\mathbf{x}\end{aligned} \tag{13}$$

where $i = 1, 2, \dots, r$ and r is the number of IF-THEN rules. Considering the regulation control of eq. (8), the fuzzy and nonlinear controller is:

$$\mathbf{u} = \frac{-\sum_{i=1}^{r} w_i\mathbf{F}_i\mathbf{x}}{\sum_{i=1}^{r} w_i} \tag{14}$$

4. Tracking Trajectory

The proposed control strategy is based on vibration control strategy presented by Preumont (2002) that suggests the use of a combination of an outer control loop and an inner control loop . The use of outer and inner loop allows a better tracking based on the use of the knowledge of the reference trajectory.

As seen in Fig. 3 two cascade and complementary feedback loops are considered. The inner control loop damps the system in order to attenuate the vibrations during the motion. The outer loop controls the motion of manipulator. The regulation control based on the fuzzy Takagi-Sugeno presented in this diagram was previously presented in the section 3.

The outer loop aims at controling the motion to track a desired trajectory, this controller wil be designed by using Linear Quadratic Regulator (LQR) method.

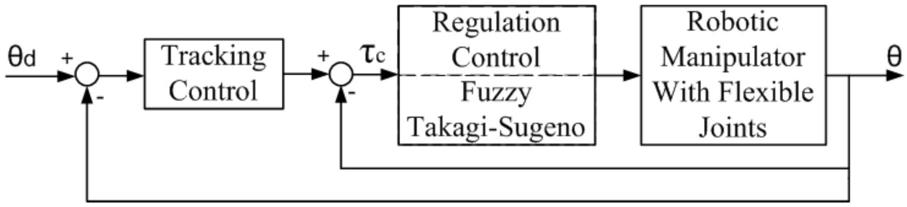


Figure 3. Diagram of tracking trajectory control,

Table 1. Parameters of Flexible Manipulator.

Parameter of first motor/link	Symbol	Value	Parameter of second motor/link	Symbol	Value
Mass of the link	m_1 (Kg)	0.5	Mass of the link	m_2 (Kg)	0.5
Elasticity of the joint	k_1 (N/m)	10×10^3	Elasticity of the joint	k_2 (N/m)	10×10^3
Length of the link	l_1 (m)	0.25	Length of the link	l_2 (m)	0.25
Mass of the motor	$m_{r1}(kg)$	0.5	Mass of the motor	$m_{r2}(kg)$	0.5
Reduction ratio	n_1	100	Reduction ratio	n_2	100
Rotor Inertia of the motor	$I_{r1_{zz}}$ (kg m^2)	14×10^{-6}	Rotor Inertia of the motor	$I_{r2_{zz}}$ (kg m^2)	14×10^{-6}

5. SIMULATION RESULTS

The parameters of the planar manipulator with flexible joints of figure 1(b) are presented in Table 1.

The simulation is performed as state in (Lara-Molina *et al.*, 2012)

Figure 4 shows the locked frequency of the for the two joints ω_1 and ω_2 as function of the configuration of the robot, specifically as function of q_2 that varies between -180^o and 180^o. The variation of locked frequency of the joints is due to the nonlinear dynamics of the robot (eq. 6). As seen, there is an expressive variation of the locked frequencies ω_1 and ω_2, additionally, it is observed that ω_1 is larger than ω_2. Therefore, the variation of the locked frequencies should be taken into account in the design of the control system.

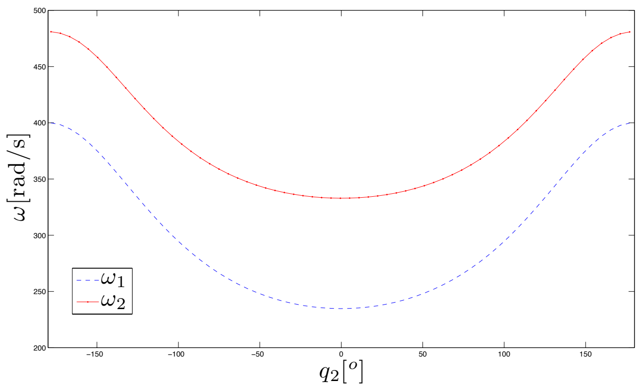


Figure 4. Locked frequencies as function of q_2

Additionally, the Frequency Response Functions (FRF) of the transfer functions $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$ and $G_{22}(s)$ (of the system presented in eq. (7)) are shown in Fig. 5. As seen, the configuration of the robot (the position of the second link q_2) has a great influence in the FRF of the system. When q_2 is equal to 180^o and -180^o the frequency responses are exactly the same because these two configurations yields the same inertia matrix $\mathbf{M}(\mathbf{q})$.

5.1 Application of PDC

In order to apply the PDC technique, a fuzzy model that represents the dynamics of the nonlinear system should be defined. The nonlinear system of eq. (6) is approximated by a Takagi-Sugeno fuzzy model. Few rules are used to minimize the complexity of the model, the system model is approximated by the 2-rule fuzzy model of eqs. (15) and (16):

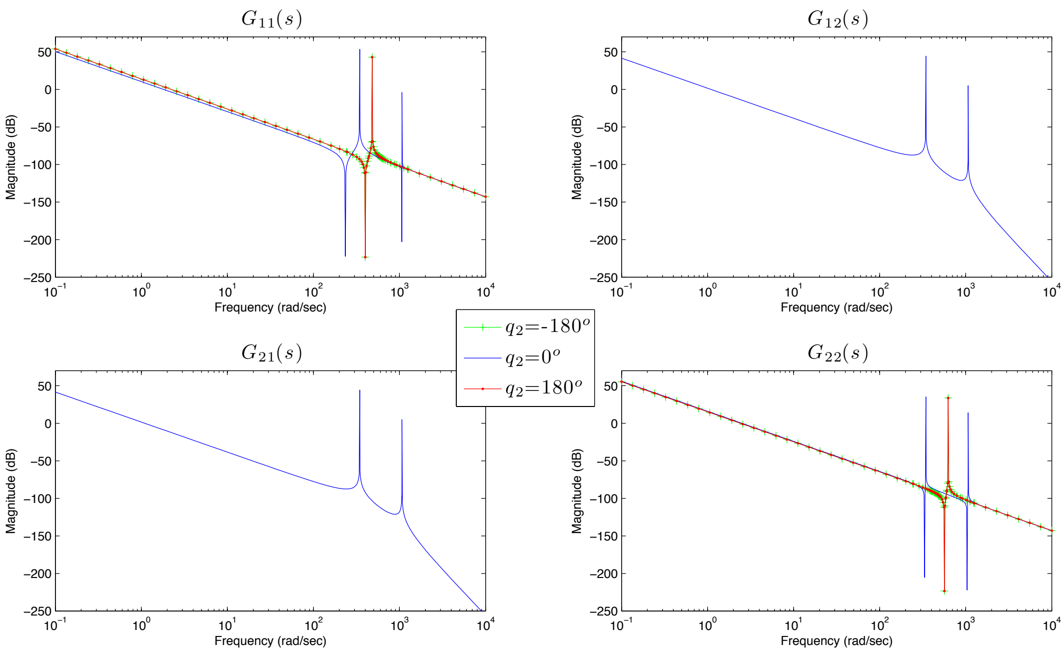


Figure 5. FRF of $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$, $G_{22}(s)$ for q_2=-180^o, q_2=0^o and q_2=180^o.

$$\begin{aligned} &\text{Rule 1 : IF } q_2 \quad \text{is about} \quad 0^o \\ &\qquad \text{THEN } \dot{\mathbf{x}} = \mathbf{A}(0)\mathbf{x} + \mathbf{B}(0)\mathbf{u} \end{aligned} \tag{15}$$

$$\begin{aligned} &\text{Rule 2 : IF } q_2 \quad \text{is about} \quad \pm 180^o \\ &\qquad \text{THEN } \dot{\mathbf{x}} = \mathbf{A}(180^o)\mathbf{x} + \mathbf{B}(180^o)\mathbf{u} \end{aligned} \tag{16}$$

The fuzzy rules are illustrated in fig. 6

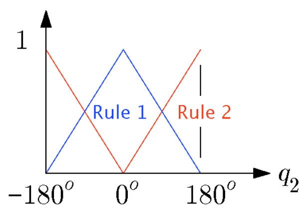


Figure 6. Membership functions of the robot fuzzy model of the robot.

The regulation controller of eq. (9) was tuned, considering the correspondent locked frequencies, for each local model of the fuzzy model of the robot in eqs. (15) and (16), thus the local and linear controllers with gain matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ were obtained. The gains of these linear controllers are: $k_{p1_1} = 3500$, $k_{p2_1} = 3500$, $k_{d1_1} = 60$, $k_{d2_1} = 42$; and $k_{p1_2} = 4000$, $k_{p2_2} = 4000$, $k_{d1_2} = 50$, $k_{d2_2} = 35$, respectively. These local controllers are used in the nonlinear fuzzy controller of eq. (14).

The FRFs of the controlled system are shown in Fig. 7. The FRFs indicate that the bandwidth of the controlled system is less than one third of locked frequencies ω_1 and ω_2 for the configurations evaluated for $-180^o \leq q_2 \leq 180^o$. It is observed that the the peaks of the FRF of the uncontrolled system, previously presented in Fig. 5, have been attenuated. Therefore the PDC controller damps the robot over all the range of the considered values of q_2.

The step position response of the controlled system is evaluated considering a step input of amplitude $5.7{\times}10^{-2\,o}$ at first and second joint. The closed-loop step position response of the joints θ_1 and θ_2 (in Fig. 8) is evaluated for several values of q_2 where $-180^o \leq q_2 \leq 180^o$. The results show that the step response overshoot is less than 10% and the settling time is $0.08s$. Hence, the controller damps and stabilizes the system response adequately.

Finally, the torque applied to the motors $\boldsymbol{\tau} = \begin{bmatrix}\tau_1 & \tau_2\end{bmatrix}^T$ is also evaluated for same values of q_2 that were evaluated in the previous cases where, thus $-180^o \leq q_2 \leq 180^o$. As seen in Fig. 9 the maximum torque applied to the motor is $4Nm$, according as the system reach the steady state the torque goes to zero.

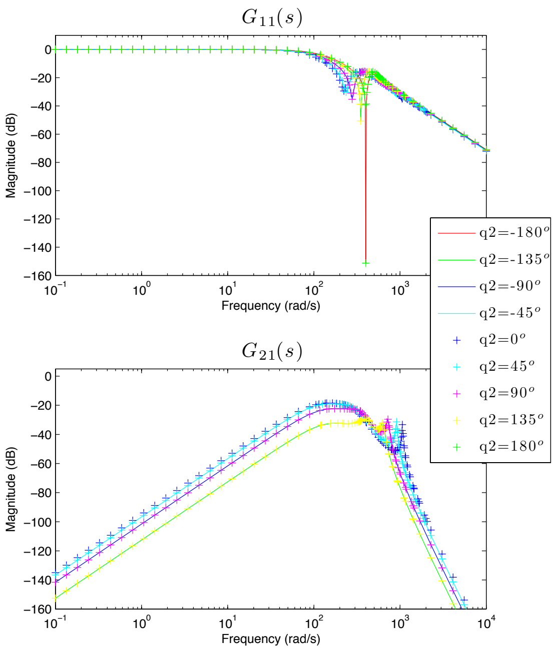


Figure 7. FRF of controlled system.

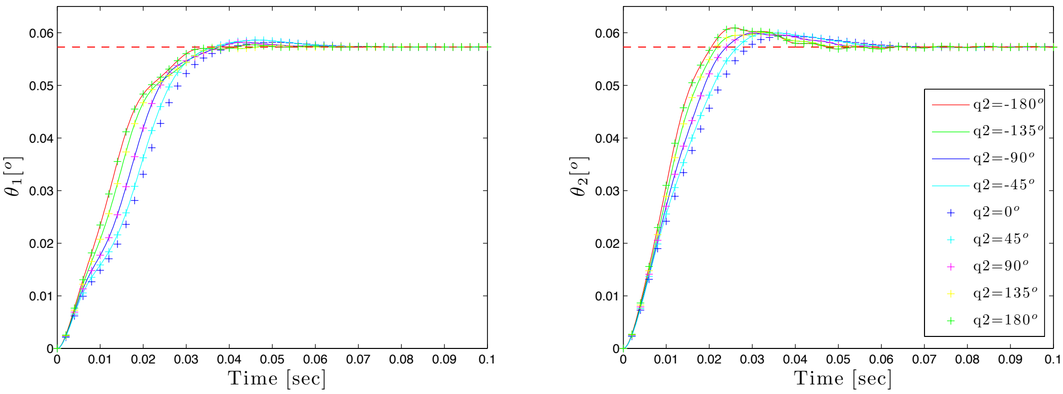


Figure 8. Step position response of the controlled system: θ_1 and θ_2.

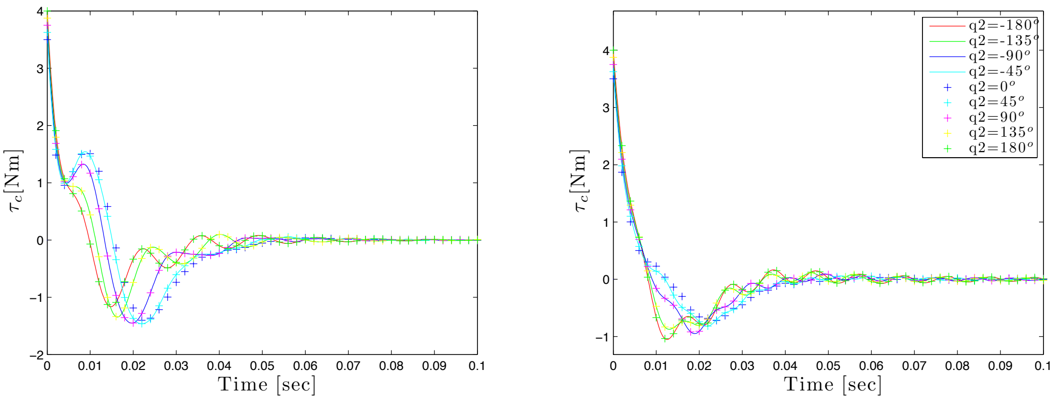


Figure 9. Control input τ_1 and τ_2.

6. CONCLUSIONS

This paper presented a strategy to control the motion of a two-planar robot with flexible joints in a constant configuration. Initially, it is verified that the dynamics in a constant configuration depends on the specific configuration of the robot, this introduces a nonlinearity in the model. The nonlinear model of the robot is approximated with a Takagi-Sugeno fuzzy model which combines the linear local models. The control strategy is based on the Takagi-Sugeno fuzzy model together with Parallel Distributed Compensation, the resulting controller is a fuzzy combination of the linear PD controllers designed for each local linear model of the robots.

The time domain and the frequency response function were used to access the dynamic performance of the robot controlled with the proposed fuzzy controller. The dynamic response of the robot exhibits an adequate behavior in terms of the motion performance. Therefore, the proposed fuzzy controller shows its effectiveness to regulate the position of the robot over a desired position within the workspace.

Some preliminary results using the trajectory tracking control proposed in section 4 have been obtained. Future work will encompass this tracking control strategy based on fuzzy control together with LQR.

7. ACKNOWLEDGEMENTS

The authors express their acknowledgements to the Graduate Program in Mechanical Engineering of the Federal University of Technology - Paraná funded by CAPES.

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