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SIMPLIFIED STEADY-STATE NUMERICAL SOLUTION OF MULTIPHASE FLOW IN OIL PRODUCTION WELLS

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Abstract. *Multiphase flow systems are formed along an oil well production system. In this work, the analysis of the flow behavior through the production string is performed through a mathematical model, with the aid of a computer program. The mathematical model is comprised of equations for fluid properties, single and multiphase flows and the mass and momentum conservation laws. The results are useful as a design and optimization tool for oil well production operations. In general, commercial programs are closed packages and not suitable to small private companies. The methodologies used in those commercial packages are fixed, which restricts their use for certain class of problems. Among the applications are the plot of pressure-distribution curves and the optimization of gas-lift installations. In addition, a parametric study is here performed, showing the influence of some governing parameters in the production flow rate, like tubing diameter, fluid properties etc. The results obtained were compared with pertinent literature and are physically reasonable, showing that the software developed correct and promising.*

Keywords: *Multiphase flow, numerical simulation, oil well, gas-lift*

1. INTRODUCTION

Oil production requires the reservoir fluids, which are initially in the porous media in the reservoir high pressure environment, to flow vertically through the tubing. As the oil travels upward the production string, to the surface, pressure diminish, leading to the release of light hydrocarbons, which were dissolved in the liquid and are gassy at atmospheric conditions. Under high pressures, gas dissolves preferentially in oil, rather than in water, due to chemical affinity. Therefore, the mixture may contain only liquid, live oil and connate water, within the reservoir. Nevertheless, as the fluid flows towards the surface, pressure diminish and gas evolves from solution, leading the liquid to shrink in volume. The gas fraction keeps increasing, until atmospheric conditions are reached, when all the gas is released from the oil. According to Danielson et al. (2000), the multiphase flow systems that are formed along an oil production well are quite complex and a good estimate of their behavior is essential for the design and operation of offshore production systems.

Numerical simulators are important tools for the oil industry because they can provide a good estimate of the flow behavior, allowing the optimization of the process and a better choice of the equipment that should be used. In general, a commercial software, used to simulate multiphase flow in oil wells, is very expensive. Moreover, the methodology used in a commercial software to analyze the physical problem and the numerical solution are fixed and closed, which restricts its use to certain classes of problems, since it is not possible to change or improve the numerical solution.

There are some largely used Solvers such as OLGA, PIPESIM and MARLIM. OLGA's dynamic two-phase flow model was firstly developed inside a project for STATOIL Norwegian company in 1983 and nowadays OLGA is a reference in transient flow regime simulation (Bendiksen, et al., 1991). PIPESIM is a steady-state multiphase flow simulator, developed and commercialized by the company SCHLUMBERGER, used for the design and analysis of oil and gas production systems and comprises a large group of empirical correlations and mechanistic models. The thermodynamic behavior of the fluid can be assessed through a black-oil model or by a compositional model (Schlumberger, 2010). MARLIM

(Multiphase Flow and Artificial Lift Modeling) is a software developed by the Brazilian company PETROBRAS and its use is restricted to the company. It is based on mathematical algorithms for steady state flows. Among recent researches in the area, the work of Nascimento (2013) can be highlighted for the development of a multiphase flow simulator based on a mechanistic model.

This article describes the development of a simplified multiphase flow solver, with application to oil production optimization and parametric studies. The developed solver is suited for vertical wells that produces low-volatile oils with density from 825 to 964 kg/m^3 and gases with densities from 0,723 to 1,164 kg/m^3. The computational language used to develop the computer program was Visual Basic for applications, which is a Microsoft's high-level object-oriented application development environment for the Windows platform.

2. MODEL DISCRIPTION

2.1 Mathematical model

There are two classical models to describe the mass exchange between liquid and gas phases: the black oil model and the compositional model. The compositional model, while rigorous, is extremely complex to set up and solve. Such a model may be necessary for some highly volatile oil systems (Peaceman, 1977). The black oil model, herein adopted, refers to oils with API degree lower than 40 and undergo relatively small changes in composition within the two phase envelop (Brill, 1999). These changes are neglected and it is assumed that no mass transfer occurs among the water phase and the other two phases (Peaceman, 1977).

An important phenomena that occurs when gas and liquid flow together in a pipe is due to the low density of gas when compared to the liquid, which causes the gas to flow at a higher velocity than the liquid phase, what leads the liquid to occupy a smaller cross-sectional tubing area. The parameter that represents the *in situ* liquid fraction within a cross-sectional area is called liquid holdup (H_L).

The flow can be modeled through the equations of conservation of mass and momentum. Assuming the flow is steady-state and unidimensional the pressure drop, $\Delta p/\Delta z$, is calculated by

$$\Delta p/\Delta z = (2f\rho_m v_m^2)/d + \rho_m g + \rho_m[\Delta(v_m^2/2)/\Delta z] \tag{1}$$

where ρ_m is the mixture density, v_m is the mixture velocity, f is the fanning friction factor, d is the tubing diameter, g is the acceleration caused by gravity, z is the length of the pipe or tubing, $(2f\rho_m v_m^2)/d$ is the pressure drop component due to frictional loss, $\rho_m g$ is pressure drop component due to the hydrostatic loss and $\rho_m[\Delta(v_m^2/2)/\Delta z]$ is the pressure drop component due to acceleration.

The model is closed by using a mechanistic model to solve for the liquid holdup as a function of the fluid properties such as liquid density (ρ_l), liquid superficial velocity (v_{sl}), surface tension, oil viscosity and mixture velocity ($H_L = f(d, p, \rho_l, v_{sl}, v_{sg}, \sigma, \mu_o, v_m)$).

The correlation used in the mathematical model to evaluate H_L is the modified Haggedorn & Brown method (Economides, 1994). Other correlations can be used in the program. Bellow, some intermediate properties that were necessary to calculate in the model will be presented.

For black oils with associated gas, a simplified parameter has been defined to account for the gas that dissolves or evolves from solution in the oil. This parameter, the solubility ratio R_s, can be measured experimentally, or determined from empirical correlations (Brill, 1999). It is defined as the volume of gas (measured at standard conditions) dissolved at a given pressure and reservoir temperature in a unit volume of stock-tank oil (Peaceman, 1977). That is:

$$R_s = V_{gsc}/V_{ostb} \tag{2}$$

where R_s is the solution gas-oil ratio, V_{gsc} is the volume of gas at standard conditions and V_{ostb} is the volume of oil measured in the stock tank.

The solubility ratio, R_s, can be calculated through empirical correlations, such as Standing correlation (Standing, 1947):

$$R_s = \gamma_g[p/((18)(10^{yg}))]^{1/0.83} \tag{3}$$

where γ_g is the gas density relative to air, p is the pressure and y_g is a parameter defined as:

$$y_g = (0.00091)\, T - 0.0125\, \gamma_{API} \tag{4}$$

where T is the temperature and γ_{API} is the API grade that can be calculated by :

$$\gamma_{API} = 141.5/\gamma_o - 131.5 \tag{5}$$

where γ_o is the oil density relative to water.

The volume of the oil phase at reservoir temperature and pressure is not V_{ostb} but somewhat larger, since the dissolved gas causes some swelling of the liquid hydrocarbon. The formation volume factor for oil, B_o, is defined as the ratio of the volume of oil plus its dissolved gas, $V_{o,}$ (measured at reservoir conditions) to the volume of the oil component (measured at standard conditions) (Peaceman, 1977):

$$B_o = V_o/V_{ostb} \tag{6}$$

B_o can be obtained through the Standing empirical correlation:

$$B_o = 0.972 + (0.000147)\, F^{1.175} \tag{7}$$

where F is an internal parameter, defined by:

$$F = R_s(\gamma_g/\gamma_o)^{0.5} + 1.25\, T \tag{8}$$

Similarly, the gas formation volume factor, B_g, is the ratio of the volume of free gas (all of which is gas component), measured at reservoir conditions, V_g, to the volume of the same gas measured at standard conditions, V_{gsc}. B_g can be obtained by :

$$B_g = Z\,(Tp_{sc})/(pT_{sc}) \tag{9}$$

where Z is the compressibility factor, T is the temperature, p is the pressure and p_{sc} is the pressure at standard conditions.

To estimate the pressure in which the first bubble of gas appears, named bubble pressure, it was used the Standing correlation (Beggs, 1987):

$$p_b = 18(R_{sb}/\gamma_g)^{0.83}\, 10^{yg} \tag{10}$$

where p_b is the bubble pressure, R_{sb} is the dissolved gas oil-ratio at bubble pressure, γ_g is the gas density relative to air and y_g is an internal parameter previously define in Eq. (4).

When the oil contains dissolved gas, it is called “live oil” and after all gas has emerged (standard conditions) the oil is called “dead oil”. The viscosity of both dead and live oils were calculated through Beggs & Robinson correlations (Beggs, 1987). The water viscosity is considered constant and equal 1 cp.

$$\mu_{do} = 10^x - 1 \tag{11}$$

$$x = T^{-1.163}\, e^{(6.9824 - 0.04658\, \gamma API)} \tag{12}$$

$$\mu_o = A\mu_{do}^{\;B} \tag{13}$$

$$A = 10.715(R_s+100)^{-0.515} \text{ and } B = 5.44(R_s+150)^{-0.338} \tag{14}$$

where μ_{do} is the “dead oil” viscosity, μ_o is the “live oil” viscosity, x is a parameter calculated by Eq. (12), A and B are internal parameters defined in Eq. (14).

There is an imbalance of molecular forces at the interface between two phases. This is caused by physical attraction between molecules. This imbalance of forces is known as surface tension. The surface tension varies with temperature and is calculated by (Beggs, 1987):

$$\sigma (T) = \sigma_{68} - (T - 68) (\sigma_{68} - \sigma_{100})/32 \tag{15}$$

where T is the temperature, $\sigma (T)$ is the interfacial tension at any temperature, σ_{68} is the interfacial tension at 68°F (20°C) or less and σ_{100} is the interfacial tension at 100°F (37,8°C) or greater.

2.2 Computational model

The program simulates the vertical flow behavior by discretizing the tubing in a certain number of cells, small enough to consider that the properties are constant in all points within the cell. The well schematic used in this study is shown in Fig.1. The top of the tubing is divided into cells to illustrate the strategy of calculation adopted for the entire column.

The methodology consists in dividing the known length of the string in small cells. The pressure at tubing's head must be an entry data, because it will be the initial condition. An increasing in pressure (ΔP) must be defined, so the pressure at the end of a cell will be the initial pressure plus ΔP. Using the average pressure within the cell, all fluid properties are calculated and the cell's length can be calculated through the equation for pressure drop, Eq. (1). Then, the same is done with the next cells in a loop until the length reaches the value of the total tubing length. At this point, the bottom hole pressure p_{wf} is obtained.

With the same logic it is possible to obtain, through the developed solver, the tubing pressure instead of the bottom hole pressure, given the tubing length. In this case, the initial pressure will be P_{wf}.

The pressure gradient along the tubing can be plotted as a curve that allows the study of the pressure drop behavior of the system. The solver is configured to plot the pressure drop curves for gas-liquid ratio from 100 to 1400 scf/bbL, with a step of 100 scf/bbL. The gas-liquid ratio represents the amount of gas present in the oil and it is defined by the gas volume per unit volume of oil, both measured at standard condition. This is an important factor in pressure drop studies.

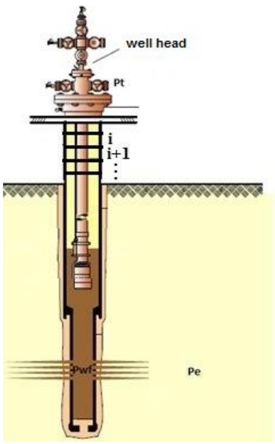

Figure 1. Well scheme used in the study

All functions where developed in VBA as subroutines to calculate the flow properties. The input variables required for the simulation are: flow rate, tubing diameter, inner pipe roughness, gas relative density, API grade, temperature and pressure at initial cell, tubing length, pressure step and water cut.

3. SIMULATION RESULTS

3.1 Validation

Gilbert (1954), published in his study, a set of gradient pressure curves for different combinations of flow rate and tubing diameter. These curves were plotted using experimental data of oils produced in Californian wells with γ_{API} between 15 and 40 which corresponds to densities from 825 to 964 Kg/m^3. According to his conclusions, the governing parameters of the gradient pressure are the tubing diameter, the flow rate, the gas liquid rate and the pressure itself. Gilbert curves are still largely used and referenced.

In order to validate the program and the mathematical model, gradient pressure curves were obtained for a flow of 400 barrels/day considering a 2.75 inches tubing. This curve was compared to the corresponding Gilbert curves for 400 bbl/day and 2.75 inches tubing. Both curves are showed in Fig. 2. Once a variety of wells were used to obtain Gilbert curves, is not possible to have a precise value for all the input parameters. In these cases, common values find in most wells were used as inputs to generate the curves in order to compare. The API grade used was 25, the water cut was set to zero and the tubing length was considered to be 6000 ft.

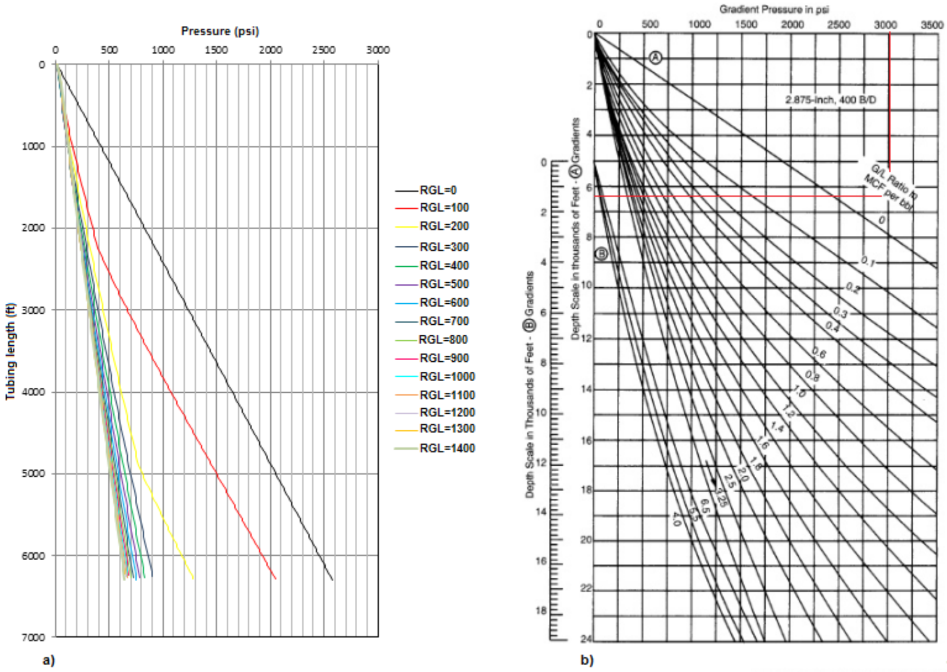

Figure 2 (a) Gradient pressure curves obtained with the developed program (b) Gilbert gradient pressure curves for comparison (reproduced from Gilbert, 1954)

The results were compared by finding p_{wf} using both methodologies and analyzing the differences. For the correspondent RGL curves, the pressure value was obtained for depth equal to 6000 ft (y axis). The result is summarized in Tab. 1.

Table 1. Simulation results in comparison with results obtained with Gilbert curves shown in Fig. 2(b)

Depth (ft)	RGL (scf/bbl)	Pressure obtained with Gilbert curves (psi)	Pressure obtained by the program (psi)	Difference (%)
6000	0	2250	2307	2,5
6000	200	1200	1352	12,7
6000	400	900	776	13,8
6000	600	750	694	7,5
6000	800	650	651	0,2

6000	1000	600	624	4,0
6000	1200	500	607	21,4
6000	1400	480	595	24,0

It is possible to observe that both curves show the same trend: the higher the gas-liquid ratio, the larger the curve slope and then, the pressure drop decays faster. By comparing the absolute values for the bottom hole pressure, it must be taken into consideration that Gilbert curves are an approximate behavior of pressure distribution based in a set of experimental data that comprises a large range of oils. Therefore, the values were found to be satisfactory.

3.2 Applications and parametric studies

The developed program provides the inflow performance relationship (IPR), the tubing performance relationship (TPR) and the real production flow rate. IPR is a curve that relates the flow rate with the difference between the bottom hole pressure (p_{wf}) and the reservoir static pressure. Physically it represents the fluid capacity to flow into the well. In this case, it is necessary to be given as an input, a parameter called productivity index, inherent to each well. TPR physically represents the capacity of the fluid to flow to the surface, given the pressure difference between p_{wf} and the tubing head pressure (p_t). The well production rate is an equilibrium between those curves, and can be determined at their intersection. Figure 4 shows the results obtained for the input data shown in Tab.2.

Table 2. Input data for the simulation

Variable	Value	Unity
Flow rate (Q)	300	STBO/dia
Diameter (d)	2	in
Inner roughness (e)	0.000288	in
Gas density (γ_g)	0.75	-
API grade (γ_{API})	30	°API
Initial temperature (T_{ini})	120	°F
Tubing pressure (P_t)	250	psia
String length (H_{colu})	6000	ft
Pressure step (dp)	10	psia
Well productivity index (J)	1	(STBO/dia)/psi
Reservoir pressure (P_{res})	2500	psia
Water cut (W)	10	%

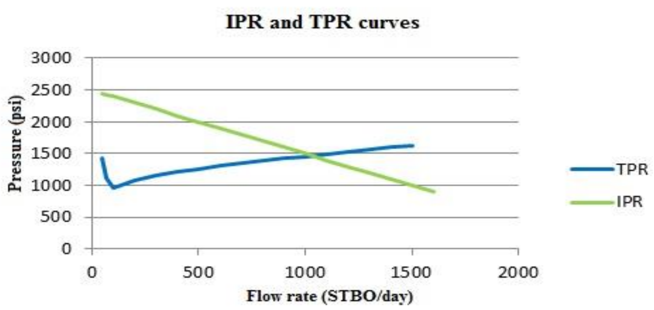

Figure 4. IPR and TPR curves ploted by the program.

By analyzing the results presented in Fig. 4, it is noticed that in this case the oil production flow rate would be approximately 1000 bbl/day.

Another useful tool provided by the developed solver is a way to determine the optimal gas-liquid ratio (RGL) and the optimal TPR, which is the one that will intercept the IPR curve, giving the optimal flow rate. For the conditions presented in Tab. 2, the TPR curves for liquid-gas ratio from zero to 1400 scf/STBO were obtained, and were plotted with the IPR curve showed in Fig. 4. The results are shown in Fig. 5a.

It is evident, by observing the interception of the IPR curve with each TPR curve, that the higher the gas-liquid ratio, the higher the flow rate. This occurs because larger quantity of gas in the flowing mixture causes lower dense mixtures, decreasing the hydrostatic component of pressure drop (see Eq. (1)), which is directly proportional to the density. So, if the density decreases, hydrostatic loss also decreases. By zooming into the crossings region, it is possible to observe that the TPR curve, that intercepts the IPR curve giving the highest flow rate, is the TPR curve for gas-liquid ratio of 1100 scf/STB, not the TPR curve for gas-liquid ratio of 1400 scf/STB. Therefore, the optimal gas-liquid ratio is 1100 scf/STB. The fact that it is not the highest gas-liquid ratio that gives the highest flow rate can be explained by the mixture velocity, which increases, as the amount of gas increases, since gas travels faster than water due to its lower density. The frictional component of pressure drop depends on squared velocity, as showed in Eq.(1). So, as the velocity increases, the frictional component of pressure drop increases in a squared rate. Summarizing, with the increase of gas, mixture density decreases and velocity increases. The amount of gas present in solution must be high enough to decrease density, but small enough to not increase velocity. There must be an equilibrium.

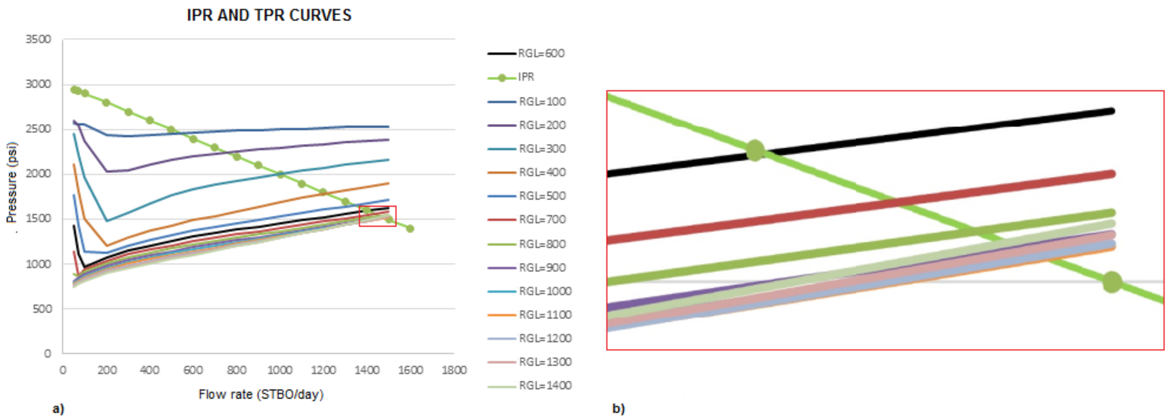

Figure 5. a) IPR curve and TPR curves plotted by the program. b) zoom of a)

Sometimes the reservoir pressure is not sufficient to bring the oil to the surface. Otherwise, if a larger production flow rate is desirable, it is possible to inject gas at the bottom of the string to decrease the mixture density and then improve the flow rate. This artificial lift method, known as gas-lift, has been applied successfully worldwide to increase well production (Yakoot, M.S. et. al, 2014)

Through the optimal TPR curve, it is possible to determine the optimal oil flow rate, which is, in this case, 1480 STB/day. By having the optimal gas-liquid ratio (RGL) and the optimal oil flow rate, it is possible to calculate the proper gas-lift injection flow rate that will be the difference between the optimal gas flow rate and the gas flow pre-existent in the well, which is given by:

Optimal RGL = (1100 scf/STB) (1480 STB/day) = 1628000 scf/day

Well RGL = (600 scf/STB) (1400 STB/day) = 888000 scf/day

Gas-lift injection rate = 1628000 – 888000 = 740000 scf/day

Another interesting study conducted with the developed solver is the influence of the tubing diameter in the oil production flow rate. The simulation was performed for diameters from 1.5 to 3.5 in. The TPR curves were plotted and the result is shown in Fig. 6

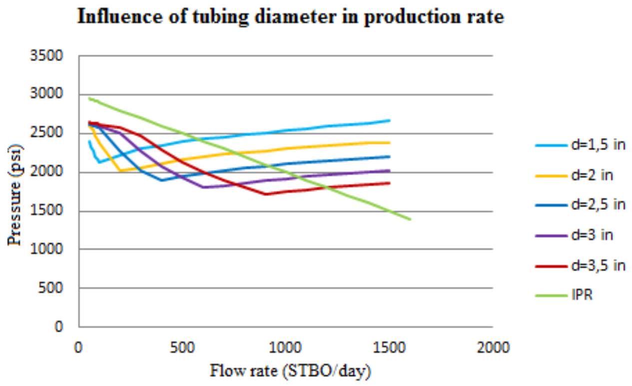

Figure 6. Influence of tubing diameter in oil production rate

The results show that for large diameters the flow rate is higher. This can be explained by the decrease in the pressure drop due to frictional pressure loss as the diameter increases.

4. CONCLUSION

In this work we presented the development of a steady-state numerical solution for multiphase flow in oil production wells. The software was validated using results from the literature, and the effect of tubing diameter and gas-liquid ratio on the production rate was analyzed. It is observed that, for the conditions presented, the production rate increases as the tubing diameter increases, but the effect of the gas-liquid ratio is not monotonic.

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6. RESPONSIBILITY NOTICE